FM Theory & Applications

By Musicians for Musicians

by

Dr. John Chowning
and David Bristow

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This book grew out of an original intention to prepare a short tutorial on the fundamental theory of FM synthesis. As you can see, it didn't stay short, and we would like to thank those people and institutions who have supported us in achieving a complete text. At IRCAM in Paris, where Pierre Boulez has generously encouraged this work, our thanks to David Wessel for his continuous support and Stephen McAdams for his useful background information on acoustics; to Emmanuel Favreau and Dan Timis for their help with some of the figures, and to Robert Gross and Michelle Dell-Prane for their assistance in mastering the main-frame computer and not least in maintaining our computer link between IRCAM and CCRMA, Stanford in the latter stages of our work.

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FOREWORD

John Chowning – the inventor of FM Sound Synthesis Technique – and David Bristow – the gifted DX Series FM Voice Maker – have collaborated to produce this extremely useful "theory meets practice" book.

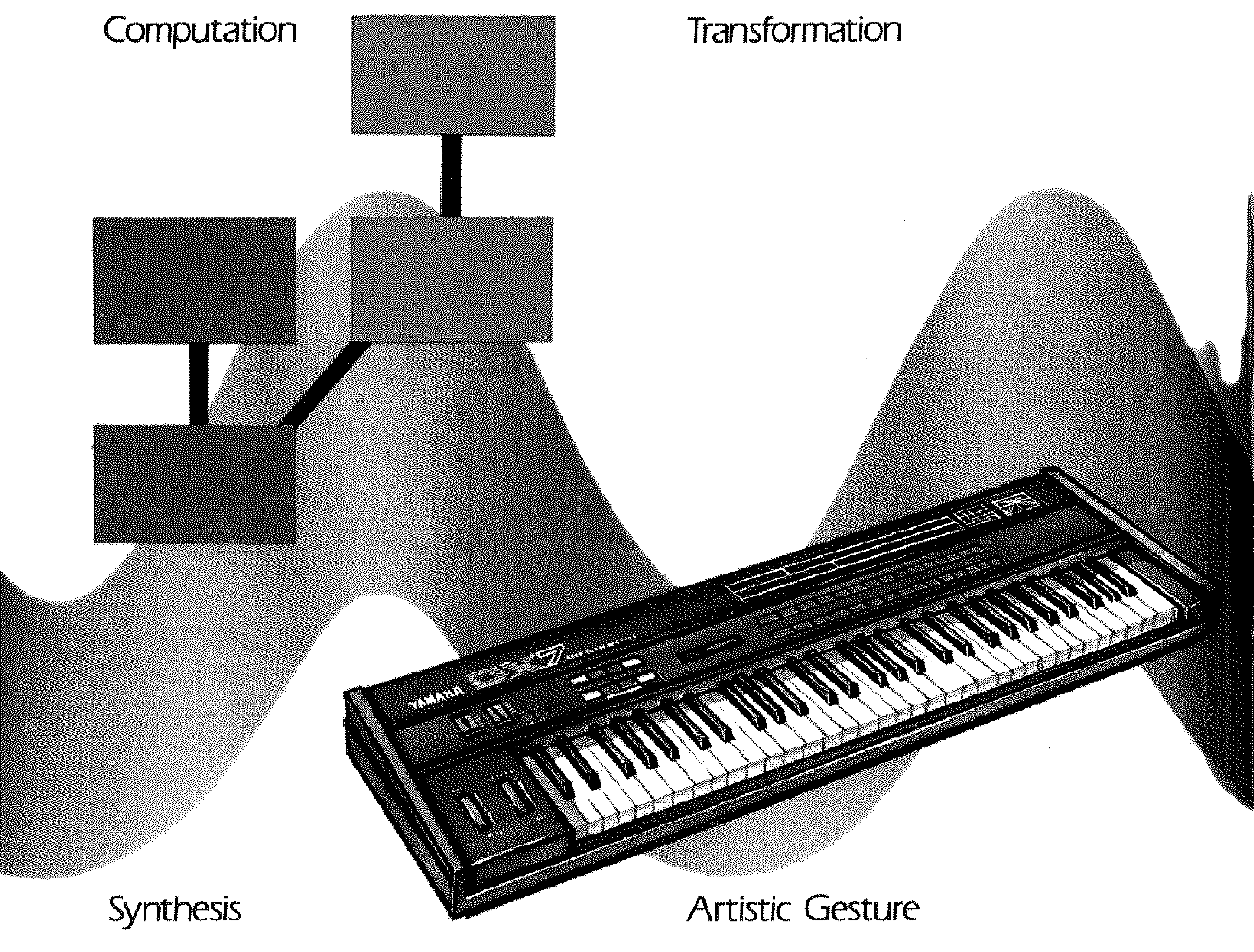
Their emphasis is on putting FM theory to work, enabling musicians to obtain more expressively potent control over their FM instruments. One of the really wonderful things about this book is that otherwise inaccessible information from acoustics and psychoacoustics is clearly explained and applied to musical voicing practice.

David Wessel
IRCAM, Paris

CHAPTER 1

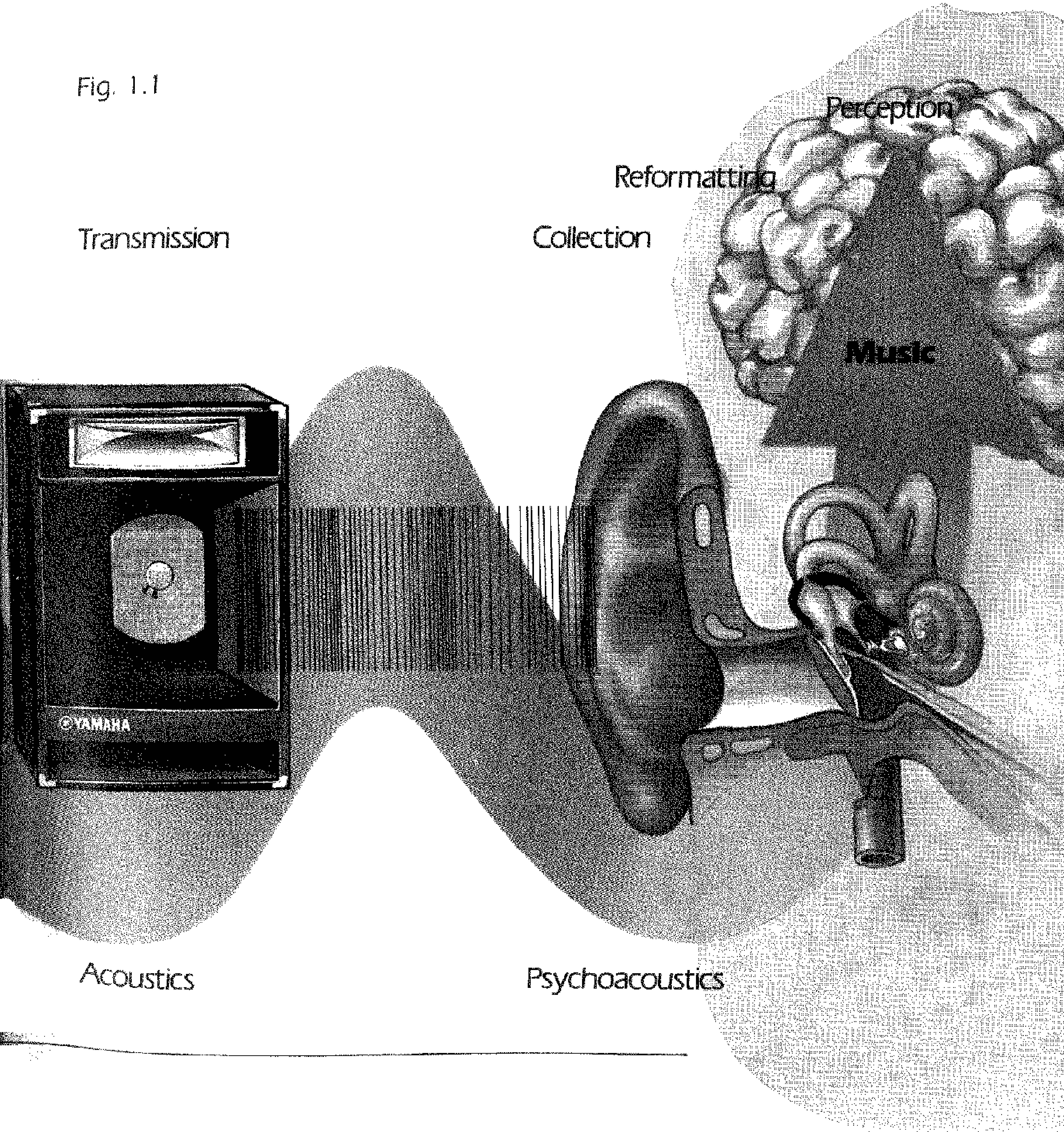
AN INTRODUCTION

The remarkable acceptance of the "X" Series musical instruments produced by Yamaha has led to an equally remarkable need for understanding how the theory of frequency modulation (FM) synthesis can be effectively used in the creation of musical sounds. In this book we have gleaned from the experience of many musicians and composers over many years what we believe to be the most effective means of understanding and use. There is much to learn that we can only touch upon in this text — about acoustics, psychoacoustics (auditory perception), basic mathematics and indeed music itself. But touch upon them we must, for if we are to gain a useful understanding of FM synthesis we must broaden our scope. The required additional effort will surely be worth it, since the knowledge acquired will be of more general use than simply understanding basic FM synthesis. We will rely on your diligence to read the book intelligently and seek additional sources, for which direction will be given.



In our belief that it is of utmost importance always to rely upon our ears for a practical and comprehensive understanding, we encourage you to read this book with an "X" Series synthesizer at hand. As a tool we have found it to be a most extraordinary aid in elucidating acoustic and psychoacoustic phenomena, as well as providing the basis of understanding FM. You will see that our strategy is rather different from other texts on FM in that we are not presenting a recipe book of "patches," but rather a means to understanding, creating, and modifying them. If we are to be positive in our approach to synthesis and FM, a consideration of the whole musical process is important; an awareness of the nature of the transmission and perception of musical sounds will be of great value when considering the details of FM sound synthesis. The picture below is to remind us that the total concept we call

Fig. 1.1



music covers a large range of topics, and the first chapters of this book are designed to lay a broad foundation upon which we can build our understanding of FM synthesis.

In talking about music, we are talking mainly of an aural experience, yet this book on sound synthesis can only be visual. That is the nature of books! However, some steps can be taken to augment the visual nature of the book by following the sound examples outlined for "X" Series instruments from Yamaha. All the exercises are easy to follow and relate directly to the text, so following them will help to strengthen the connection between sound and the written word or spectral diagram (see Fig. 1. 2), as well as lead to some good "patches." Try to get in the habit of doing the sound exercises while following the text. There are some simple "X"-ample boxes here in the introduction with easily followed instructions. You may find it useful to store these examples, as they may be recalled during the text or compared with one another. Although parameter values are given in terms of the DX7 synthesizer, you will find conversion tables at the back of the book to allow you to make the same experiments on other "X" Series instruments. We assume some working knowledge of the synthesizer being used. It is our intention that, with the use of some simple (very simple) arithmetic, you will arrive at a clear understanding of the practical theoretical basis of FM as well as the musical basis. Furthermore, throughout the text there will be intended but useful redundancy, in other words, we will often explain the same thing in different ways.

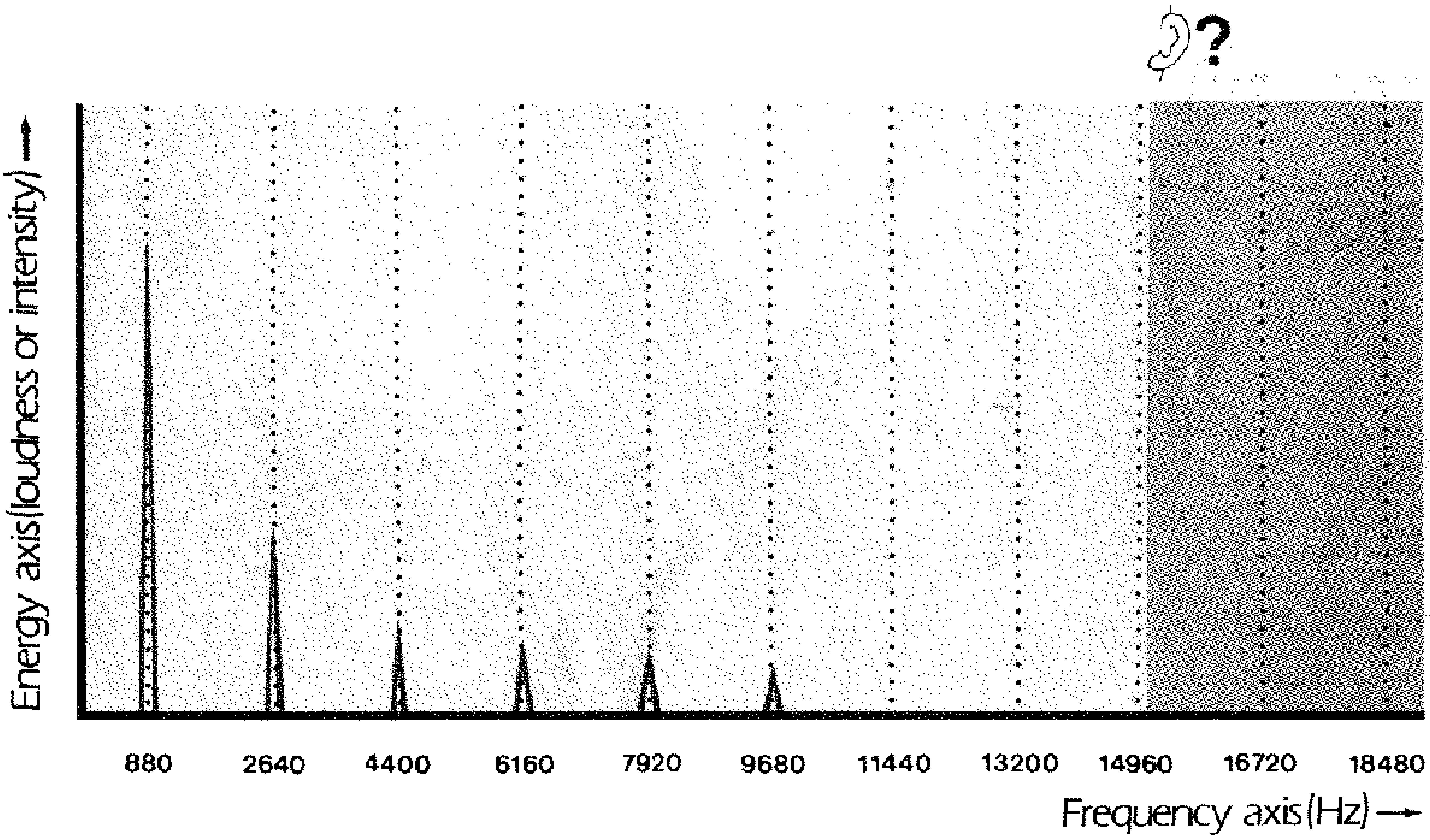


Fig. 1.2

This shows a spectrum taken from a sound on the DX7 synthesizer. The vertical lines show at what frequency simple components exist (frequency is measured in cycles per second, or more commonly called Hertz – Hz). The height of the line represents its energy.

And what of "spectral diagram" mentioned before? The spectral diagram is important because it can provide a link between aural experience and a visual representation of the same, but what exactly does it mean? Well, it helps to describe a timbre, or tone colour, of a complex sound by showing lines which represent the energy (loudness) and the frequency (pitch) of different simple components within that timbre.

The idea of a timbre being composed of many different frequency components is not at all confined to the world of electronic music. You have probably talked of "overtones" or "partials" or "harmonics" when referring to the timbre or colour of a sound yourself, but it was a scientist, Joseph Fourier, who in the early nineteenth century gave a mathematical basis to this idea, one that musicians had been talking about for centuries and indeed still are. This is explained more fully in Chapter 2 but simply means that certain complex sounds, such as that from a violin or trumpet, could be thought of as the sum of a collection of much simpler tones, at frequencies which are whole number (or integer) multiples of the fundamental frequency. A spectrum then, as in Fig.1.2, shows clearly the actual frequencies at which partials exist for a given sound, and the energy of each. The "X"-ample below shows how to make the sound from which the spectrum in Fig.1.2. was obtained. (This particular "X"-ample is not an exercise in itself, but it represents the form that the exercises ("X"-amples) will take throughout the rest of the text. However, please note the settings of the function controls as these occur in many of the coming exercises).

"X"-ample 1.1

	FREQUENCY	OUTPUT
op 1	1.00	99
op 2	3.00	87
op 3	5.00	79
op 4	7.00	75
op 5	9.00	72
op 6	11.00	71

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select algorithm 32, and set the operator values as shown above.
Voice Init does not affect the function controls. In later examples, we will be using the MOD. WHEEL to control index (don't worry about the word "index" —that will be explained in Chapter 4), therefore set the function controls on your instrument as follows; they will serve for further "X"-amples.
Poly/Mono — POLY
Portamento time — 0
Mod. Wheel Range — 99
Pitch — OFF
Amplitude — OFF
EG. BIAS — ON
Foot Control, Breath Control, After Touch — all OFF or 0.

How do we interpret this "visual" representation of a sound? Well, in the example shown (Fig. 1.2), the line on the left represents the fundamental or the first harmonic. To dispel confusion each of the components, including the fundamental, is most generally called a "partial". Collectively they form a spectrum. In the special case where the partials fall in the "natural" harmonic series- in other words they are all whole number (integer) multiples of the fundamental - they are called "harmonics", with the fundamental being harmonic number one. Sometimes the word "overtones" is used, and here beware of confusion because the first overtone (bearing in mind the implication of the word) will actually be the second harmonic. From here on, when we are looking at harmonic spectra, we shall refer to harmonics, not overtones. Partials which do not fall in the harmonic series will be specially referred to as inharmonic components of the spectrum. But, as we can see from the diagram (Fig. 1.2), these partials are in the harmonic series. The line to the right of the fundamental shows that there is the presence of the third harmonic, the next shows the fifth harmonic. We can notice by the relative height of the lines, which represent energy (or loudness), that the harmonics become progressively weaker. The note played from which this plot was made was A=880 cycles per second or 880 Hertz (Hz). Although it is the relationship between the harmonics which is of prime importance, nonetheless from this spectrum we can tell exactly the frequencies at which these harmonics occur. The first is at 880Hz (the fundamental in this case); the next is at 3 times 880 or 2640Hz; the next is at 5 times 880 or 4400Hz, and so on. This brings us to a quite important point, and that is that above about 15,000Hz, ears stop working! That is about as high as we can hear, whether we are listening for a fundamental or a high harmonic component in a "bright" sound. Therefore the spectral plot has this point marked on it.

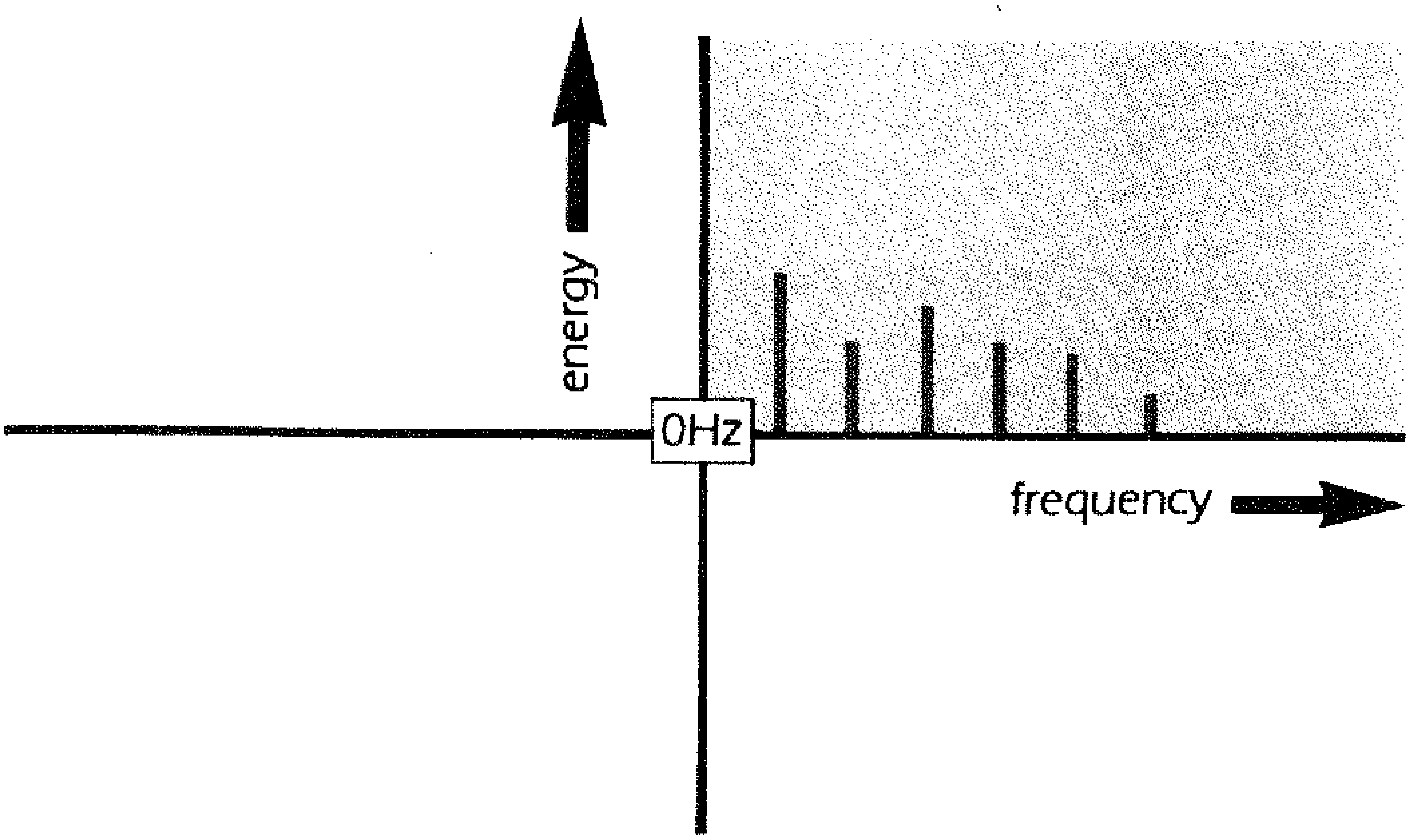


Fig. 1.3
A line spectrum.

Having introduced the idea of the spectrum, it must be said that there is more than one way of presenting it. In this book, there are basically three types of spectral diagram. The first is as in Fig.1.2, which is a print from a real-time spectrum analyser which shows a Fourier analysis of the output of a DX7. Sometimes, for precision and clarity, we will want to represent the spectrum in a second, more exact way – this is done with a line spectrum (see Fig.1.3). Notice that there is some space left for negative frequency and negative energy (concepts hard to imagine, but useful in understanding FM, as you will see in chapter 4).

There is one other type of spectral image which can be useful in helping to visualise sounds and that is the three-dimensional plot (Fig. 1.4). Sometimes, we will want to see how a spectrum changes with time (and certainly most interesting sounds do) and for this purpose a "3D plot" can be used. It is a series of spectra (or traces) shown one behind the other, and slightly offset in order to see them better, each one showing the change in the spectrum as the sound develops. The sound change in this diagram, for instance, is brought about by moving the DX7 mod. wheel from min. to max. in about

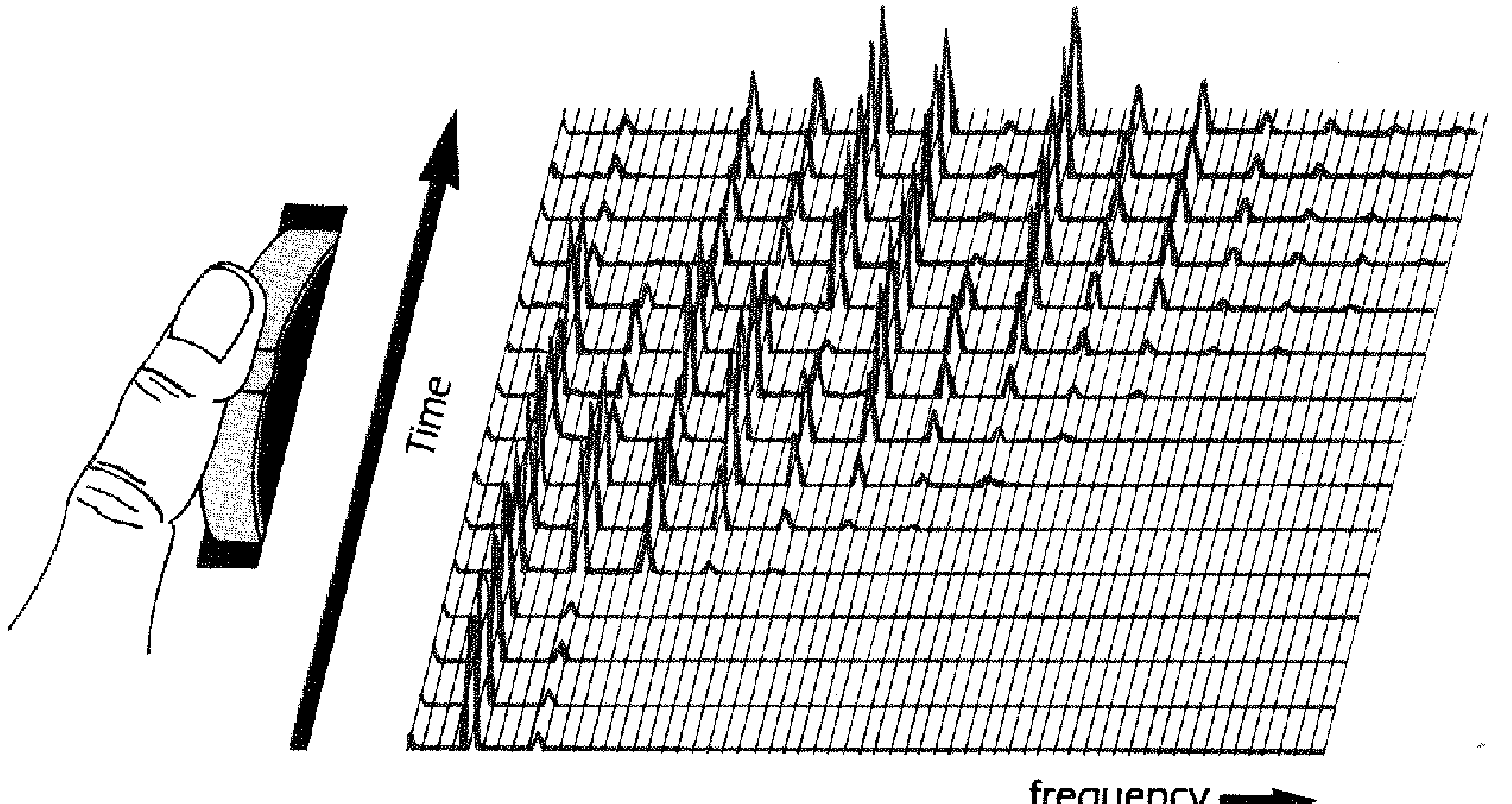
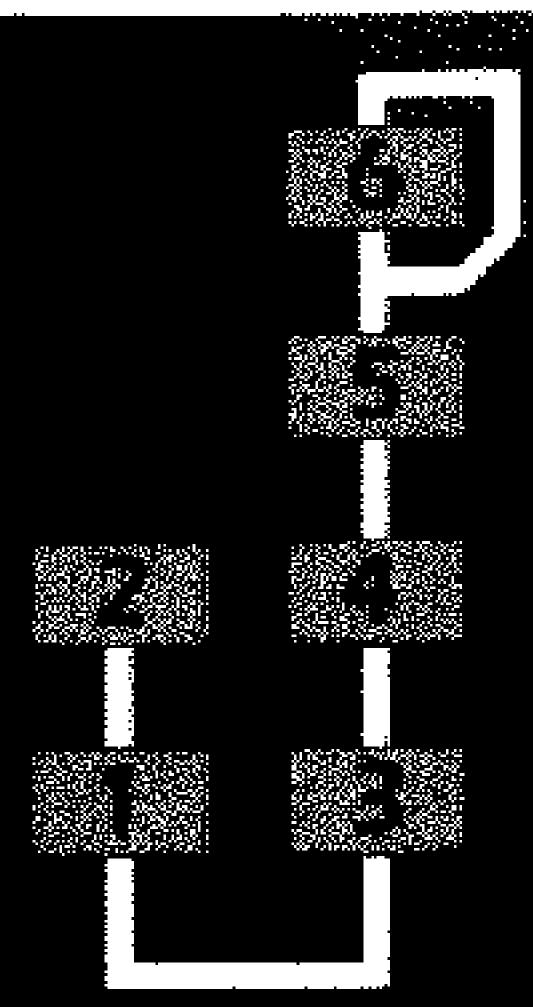


Fig. 1.4

A 3 dimensional plot showing 15 individual traces, each one showing how the sound has changed over a short period of time from the previous one. The total duration in this example is about 2 seconds, and the factor which causes the change of sound is the movement of the mod. wheel.

two seconds, and letting it control the output of operator two (that's changing the modulation index, but more of that later!). If you would like to hear the sound represented in this plot, then follow the instructions in "X" -ample 1. 2. The "how and why" all of these partials are produced with only two operators is, of course, the purpose of this book, and will be revealed in the following chapters.

"X"-ample 1.2



	FREQUENCY	OUTPUT
op 1	1.00	99
op 2	1.00	84
op 3	1.00	0
op 4	1.00	0
op 5	1.00	0
op 6	1.00	0

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select algorithm 1 and set the parameters as shown, but this time control the output of operator 2, with the modulation wheel. Make the AMS for operator 2, equal 3, and the function settings as in "X"-ample 1.1. A movement of the mod. wheel from min. to max. in about 2 seconds produces the sound from which the spectral plot in Fig. 1.4 was produced. (Ams is Amplitude Mod Sensitivity.)

So far, the very familiar word "waveform" has not appeared; yet, if you have had any previous experience working with synthesizers, it has probably been a basic part of "sound-talk" – rich waveforms, sawtooth waves, sine waves, square waves and so on. The problem with waveforms is that while they are very descriptive of the way air pressure or voltages change with time, they are not very descriptive of the actual timbre of a sound. Look at Figs. 1.5 and 1.6. If you are familiar with analogue synthesis you can probably imagine the sound which gives the first waveform (Fig. 1.5), but can you imagine the second (Fig. 1.6)?

Fig. 1.5

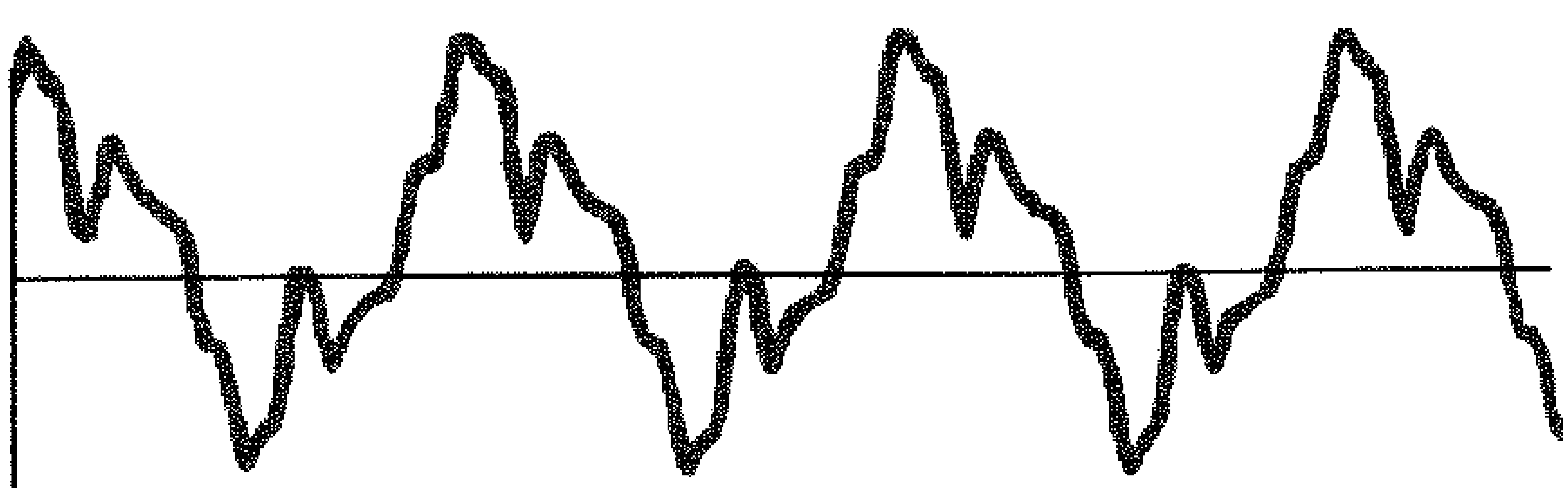


Fig. 1.6

Clearly the waveforms are both very different, but as a visual representation they really do not give us much immediate information about the nature of the sound which produced them. Actually, both of these waveforms are properly represented by the spectrum shown in Fig.1.2, in other words, taken from the same sound on the DX7. One waveform you recognise (as a square wave), but the other you do not. And that is why we do not normally use the waveform when describing a sound. The spectrum on the other hand, which is unique to any one sound, gives audible information about that sound, and is our most important visual aid or descriptive tool. An appreciation of what a waveform is, however, is important when beginning to read about the basic ideas covered in Chapter 2.

There remains just one other type of visual picture which is used in the text, and that is the "graph". Probably you are quite familiar with how a graph works, but in case you are not, read on .... A graph is merely a way of showing how two different things or variables change in relation to each other. Look at the diagram below, Fig.1.7.

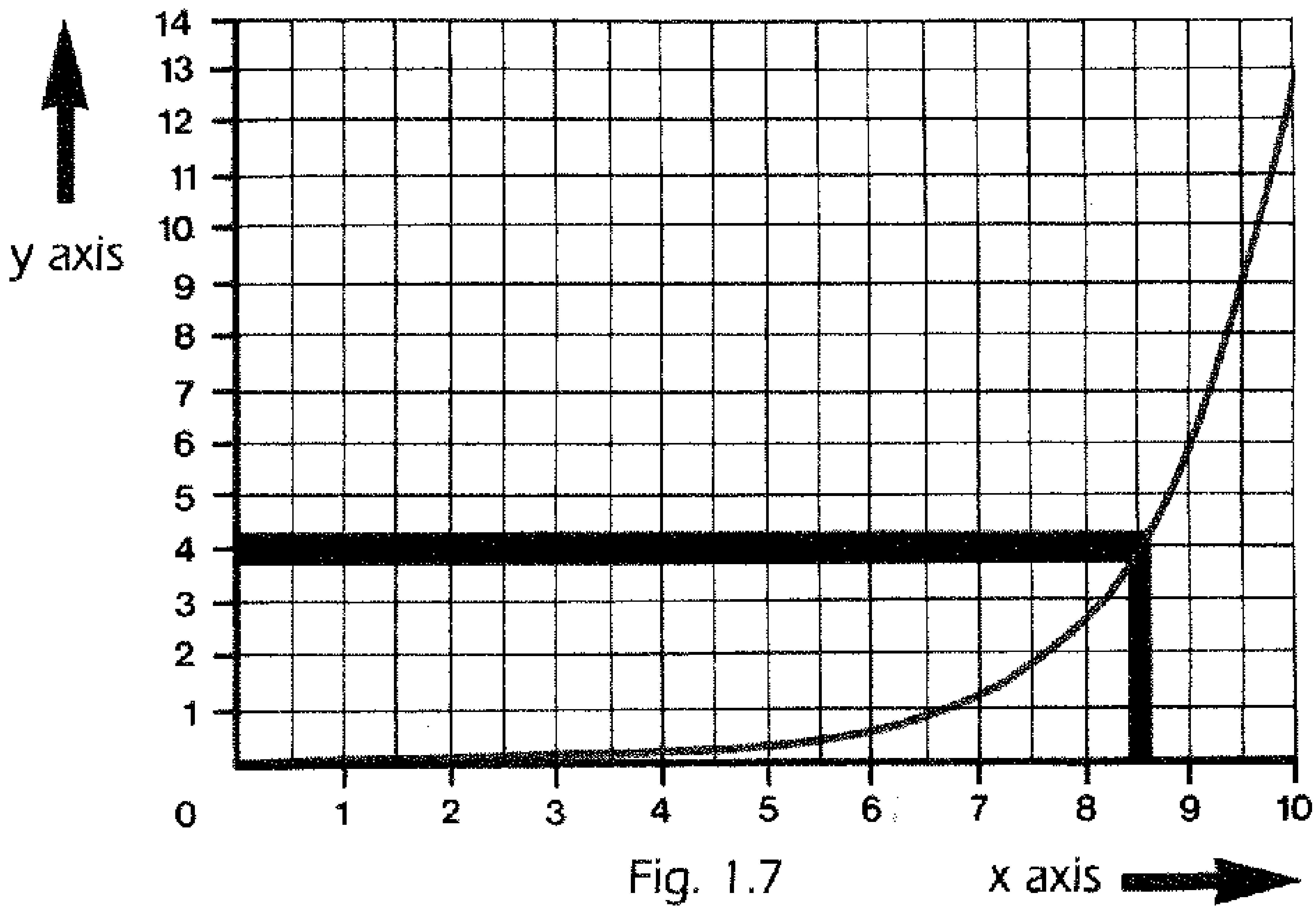


Fig. 1.7

A graph showing the relationship between two variables x and y. A value of 4 for y can be seen to be equivalent to a value of 8.5 for x.

By choosing a certain value on the vertical line, then following across horizontally until we meet the curve of the graph, we can then read down vertically to a corresponding value on the horizontal line, or vice versa. The horizontal line is commonly called the x-axis, and the vertical line, the y-axis. So by using a graph, for any given value of one variable we can determine a corresponding value of the other – for example, modulation index is a "function" of modulator output level on a synthesizer. That means given a graph of this function, we could look up output level on one axis and read modulation index off of the other. When studying FM, this will be a very useful function and you will find this graph in detail on page 58.

While the mathematics used in this book are actually very simple (we require only that you can add, subtract, multiply and divide), there are one or two terms which you may have forgotten from your schooldays . . . .

A "power" or "exponent" is the amount of times a number is multiplied by itself – that is to say,

$$2^3$$

or two to the power three, means $2 \times 2 \times 2$.

The words numerator or denominator refer to the parts of a fraction . . .

$$\frac{12}{7} \quad \begin{array}{l}\text{numerator}\\ \text{denominator}\end{array}$$

Sometimes, subscripts are used when one particular item from a class of items is referred to; this is rather like house numbers and street names. So the term J_2 means that in a class of objects which are collectively labelled by J, we want to consider object number, or the value of object number two. Subscripts are a convenient way of keeping and referring to lists, so that if, for example, we want to consider all the items in a class, or all the houses in a street, from zero right through to a number that we haven't decided yet (let's call that undecided number "n"), we can express this rather cumbersome sentence in the simple mathematical expression . . . .

$$J_{0, 1, 2, 3, 4 \ldots\ldots n}$$

There is one non-mathematical convention that we should establish here in the introduction, and that is "what C is middle C?". Well, in frequency terms, there is no problem – it is at 262 cycles per second. But notes are also given numbers, and conventionally on a grand piano middle C is C4 (that is the fourth C counting up from the bottom of the keyboard), whereas on our MIDI system "X" Series synthesizer C3 is found to be middle C (as this is more convenient with a shorter scale keyboard). As we are dealing with synthesizers in this book, we shall stick to the C3-middle C standard, but it is as well to be aware of the classical convention.

Occasionally in the text you will find "**rules**" in clearly marked boxes. These highlight some of the most useful and direct aspects of FM theory and should prove helpful guides to programming. Where these guides are perhaps less precise but nonetheless useful, they are called "Hints". Here is the first . . . .

Please read every page of this book carefully

CHAPTER 2

SOME BASIC IDEAS

The new digital technology allows an efficient implementation of a relatively new synthesis technique, Frequency Modulation Synthesis (FM). One of the advantages of FM synthesis is that with a few number of elemental units (oscillators or operators) an extraordinarily large number of different sounds can be produced. One might say that the timbral space is large while the computational space is small (all digital synthesizers can be considered as highly optimized computers for the computation of sound waveforms). But .... some knowledge in addition to musical knowledge is helpful in the effective use of FM. Some of this knowledge, such as acoustics and psychoacoustics, is interesting no matter what synthesis technique, or indeed instrument, is being used, and some is quite specific to FM. In this section we will present some basic acoustical ideas that are of interest in themselves and that will serve us well in gaining some insight into the workings of FM.

Acoustics is a discipline in itself and has a large body of knowledge which we cannot hope to explore here. It is concerned with a variety of auditory systems and receptors, vibrating mechanisms, reverberant spaces and transmission media. We will limit our discussion to the human ear and a loudspeaker in a room filled with air

Pressure Waves and Periodicity

In as much as our "tools" include an "X" Series synthesizer and an amplification system consisting of an amplifier and a loudspeaker, we can make some connections rather easily between some elementary concepts of acoustics, some trigonometry — a simple form of mathematics dating from antiquity — and some basic sounds from the synthesizer.

When sounding any tone of any of the presets of the synthesizer, we know that the loudspeaker cone is set in motion and that the nature of this motion has to do with the pitch, loudness and the quality of the sound which we hear. In fact, we can imagine slowing time to the point where we can trace this motion of the cone of the loudspeaker as it moves in and out around its rest point. We can think of this pattern of motion such that when it moves in an outward direction we can say it is positive and when in an inward direction it is negative. If we were able also to look at the pattern of motion of the ear drum in the presence of the loudspeaker, we would notice that it would be very similar except that where the cone moves "outward" the ear may move "inward" and the units of displacement would be very much smaller.

Knowing that the coupling of the cone and the ear is by means of air as the medium, we can infer with some certainty that the air in some way must transmit the same pattern, which of course it does. When the cone moves outward it causes a compression of air particles, and when it moves inward it causes a rarefaction, or a decompression, of particles. We can think of these particles as being elastic in that they tend to be spread equally throughout the room — always equidistant from one another. Therefore in a region that is compressed, the particles will move away from the point of greatest compression, whereas in a rarefied region they will move toward the point of greatest rarefaction. But since air particles have mass and

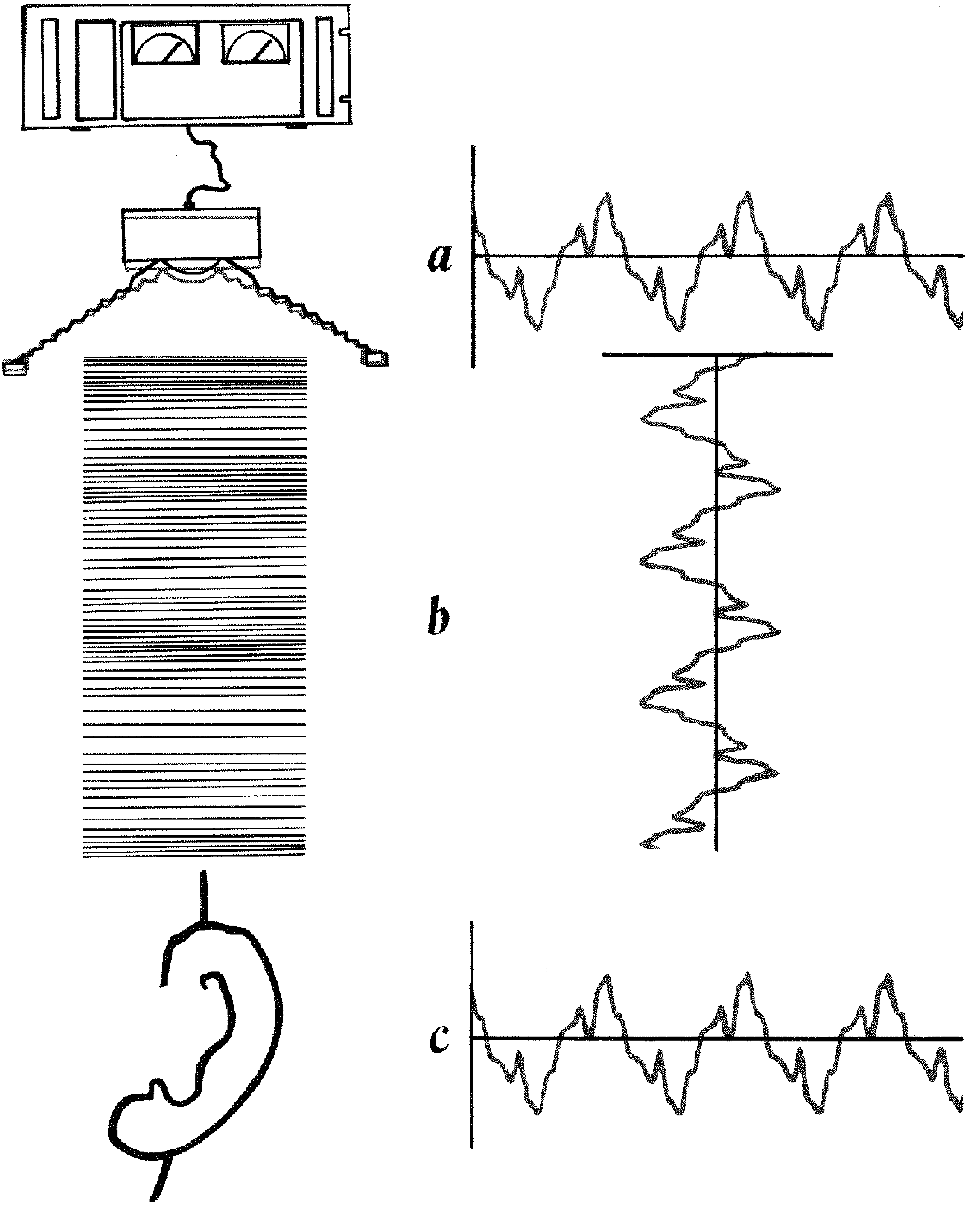


Fig. 2.1

The pattern of motion of a loudspeaker cone as it is displaced in time in proportion to a changing voltage applied to it. Let us say that the cone moves a maximum distance of 0.5mm in both a positive and negative direction. The motion is not perfectly smooth, however, as we can see in ***a***., where the direction of movement changes as the cone approaches the maximum and minimum. The motion of the cone causes compression and rarefaction of air particles (***b***.), which causes a pressure wave which travels in the direction of the ear, whose ear drum is set in motion in response to the pressure wave (***c***.).

therefore momentum, they tend to "bounce" rather like rubber balls – their motion does not stop at the instant they are equidistant from one another but rather continues on, creating other regions of compression and rarefaction in a general direction away from the speaker-cone, but always losing a bit of their energy until once again they are at rest. Because this wave action is a wave of instantaneous changes in pressure, it is called a **pressure wave.**

An oscilloscope attached at some point between the loudspeaker and the amplifier speaker terminals would show us the changing voltage which, when applied to the coil attached to the cone of the loudspeaker, induces a varying-strength magnetic field which causes the cone to be attracted and repelled, causing its motion relative to the speaker frame. The voltage, then, must be proportional to the motion. The pattern of change in air pressure is in turn proportional to the motion of the loudspeaker cone, and so must be the motion of the ear drum as it responds to this continuous change in pressure. These patterns of generation, transmission, and reception are shown in Fig. 2.1.

Were we to look at a large number of such acoustic pressure waves we would notice that there are, in general, two types which can be fairly easily discerned. One type consists of patterns of motion or voltage which are largely repetitive as in Fig.2.1,

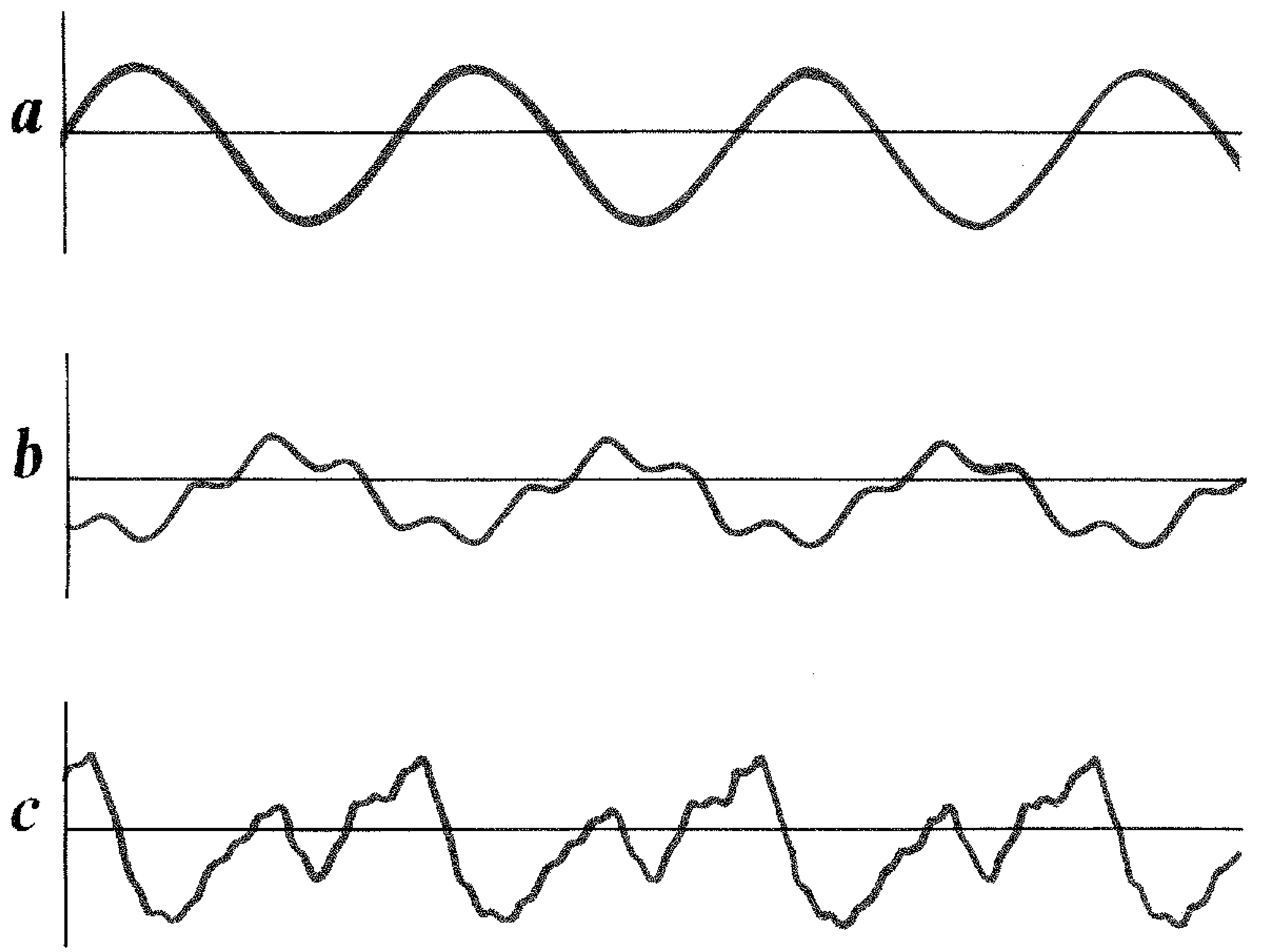


Fig. 2.2

Three periodic pressure waves where each has a different repeating pattern. Vertical axis represents amplitude of pressure; horizontal represents time.

and the other type consists of patterns where there are no apparent repetitions. The first is called periodic and the second aperiodic. Further examples of periodic and aperiodic pressure waves are shown in Fig. 2.2 and Fig. 2.3. Remember that these examples of waveforms are simply graphical representations – as you trace along the line of the graph you can see how pressure (in this case the vertical dimension) increases and decreases as time progresses. Time is represented in the horizontal axis.

There are two things we will note about these examples. First, those of Fig. 2.2 all produce a strong sense of pitch, while those of Fig. 2.3 are weak or ambiguous. Second, they are all plots of waves which were produced by an "X" Series synthesizer in which there is stored **only one wave form, a sine wave**. In fact, Fig. 2.2a is four periods or repetitions of a sine wave. The other five waveforms in Fig. 2.2 and Fig. 2.3 were all constructed from "mixes" of sine waves at differing periods and amounts.

How is it that such different shapes can be formed from a sine wave and what is it about a sine wave that makes it a fundamental unit of acoustics? In order to answer these questions, we must recall some observations about right-angled triangles first made by Pythagoras in the 6th century BC.

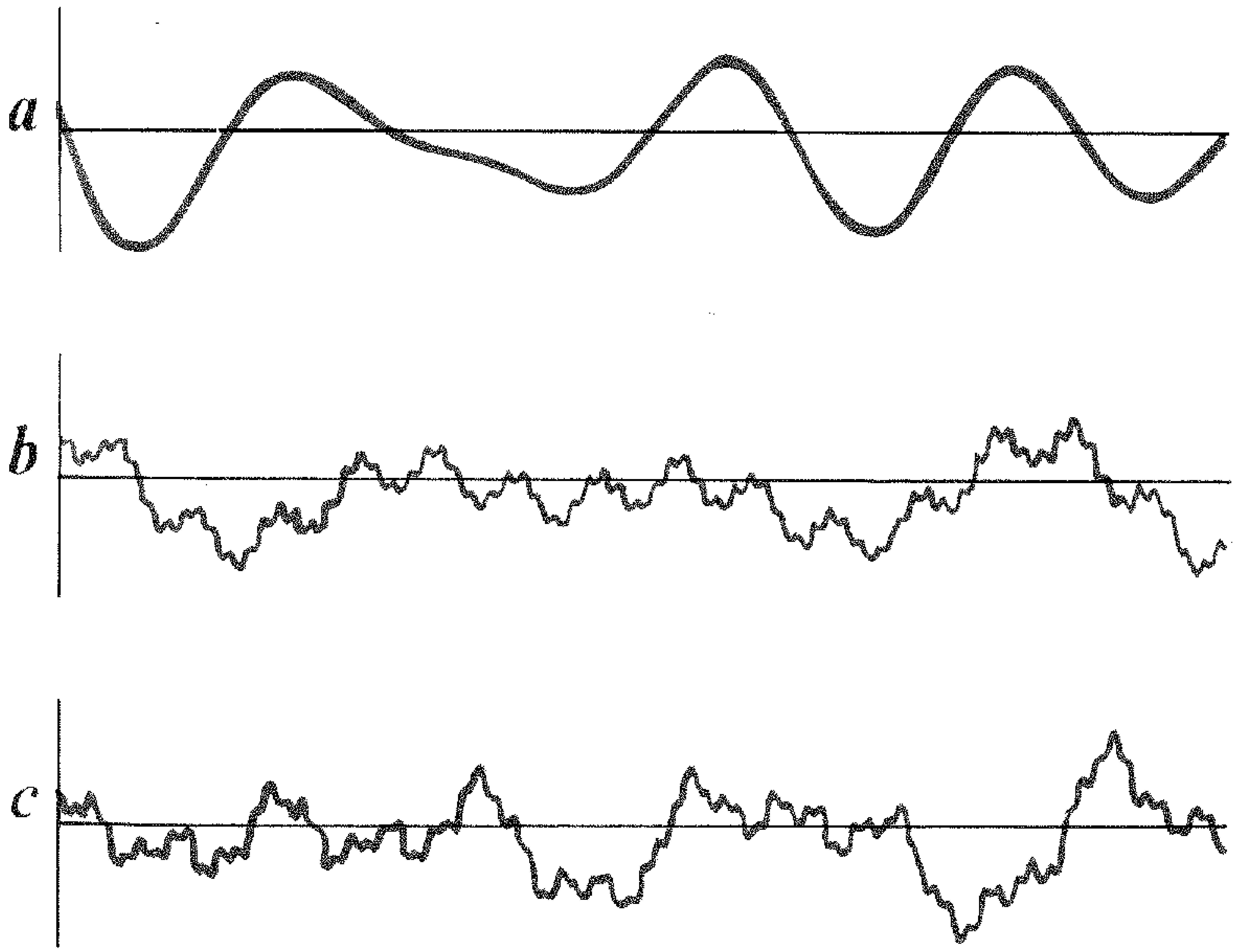


Fig. 2.3

Three aperiodic pressure waves where there is no discernible pattern Again, vertical axis is amplitude and horizontal is time.

Sine Wave

First we will see how a sine wave relates to a right-angled triangle, and then examine the properties of a sine wave. This may seem like an unnecessary excursion into mathematics, but in fact it is very simple and relates directly to our understanding of terms like "phase", and shows why a sine wave is such a powerful tool in both the analysis of complex sound and its synthesis.

Recall from trigonometry that a right-angled triangle has one angle which is 90° and that the side of the triangle opposite the right angle is, by convention, called the hypotenuse. The sine of either of the other two is defined to be the ratio of the length of the side opposite the angle and the length of the hypotenuse. As shown in Fig. 2.4 the sine of the angle theta (ϑ) is equal to side a, (the opposite) divided by side c (the hypotenuse) or:

$$\sin\vartheta = \frac{a}{c}$$

and the cosine of an angle is the ratio of the adjacent side and the hypotenuse or, again looking at Fig. 2.4, sides b and c or:-

$$\cos\vartheta = \frac{b}{c}$$

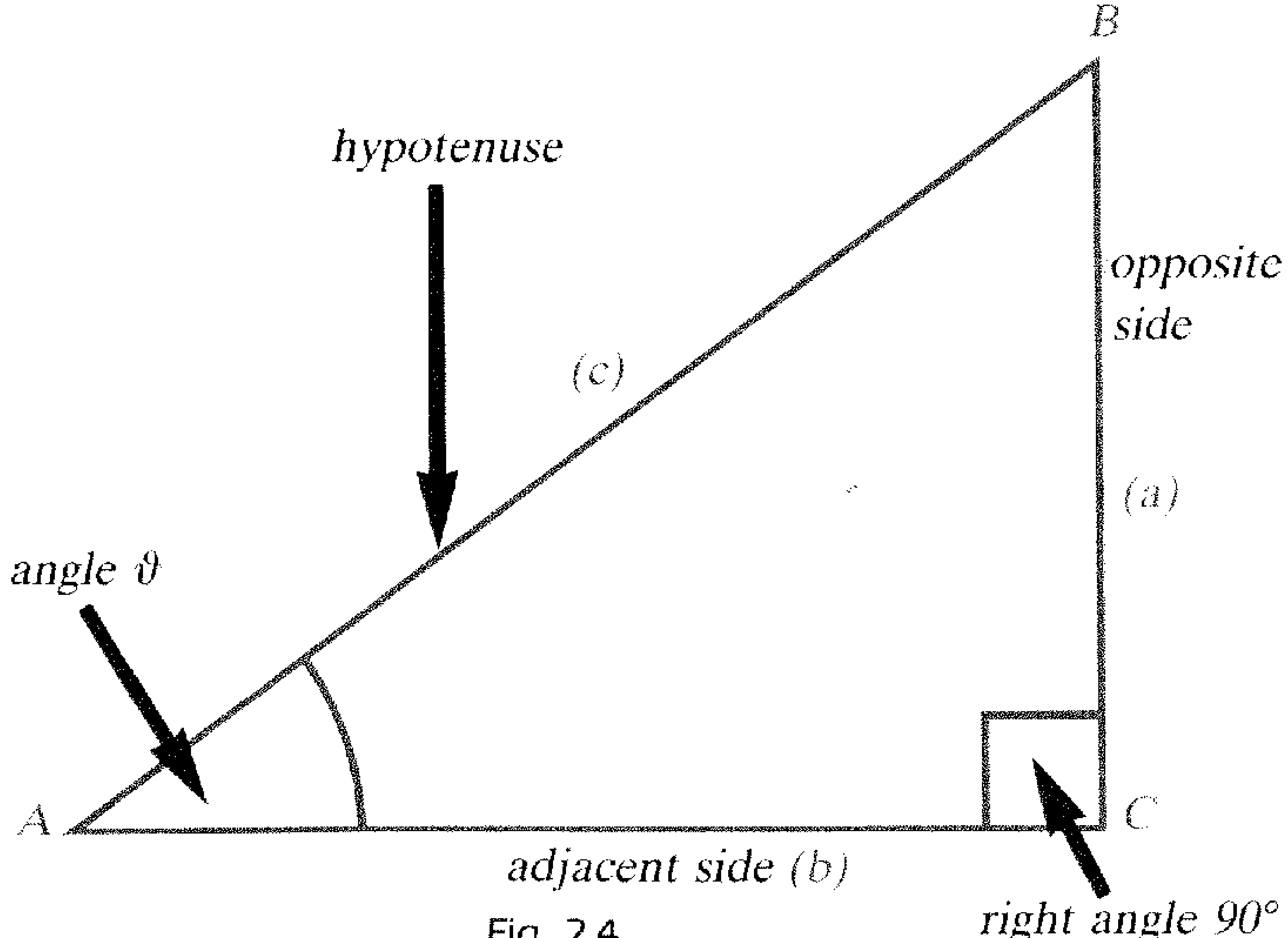


Fig. 2.4

A right-angled triangle.

These relationships between the lengths of the sides of a right-angled triangle are, of course, constant regardless of the actual size of the triangle. In other words, if by some means we had calculated that the value for sin 30° is 0.5, then we could solve the following non-musical problem!

For example, let's suppose that we are map makers and we need to know the precise distance between two points which we cannot measure directly (as shown in Fig. 2.5). We want to know the distance between points A and B but cannot measure directly because of an intervening body of water. We can, however, locate a point C from which we can see points A and B and that we can determine to be at an angle of 90° to B in relation to A. We can only measure directly the distance C to B, but we can see the other points. With only a protractor to estimate angles, a fairly reliable gait to step off the distance between C and B, and a table of values for the sine of angles from 0° to 90°, we could find an approximation of the distance AB.

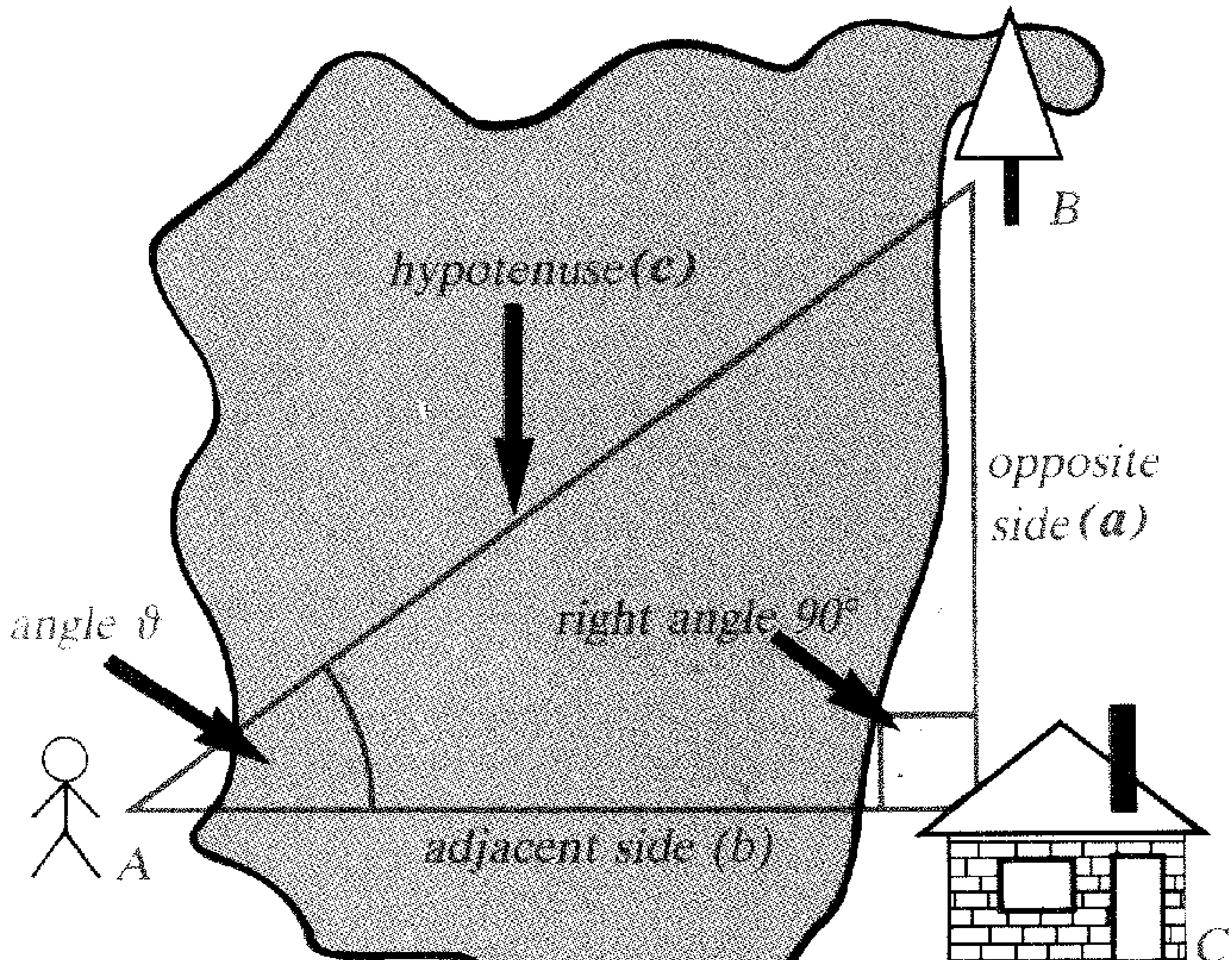


Fig. 2.5

If the distance BC has been paced off to be 100 metres and angle ϑ has been gauged to be 30°, then we can determine the distance AB if we know that sin 30° = 0.5. Since

$\sin\vartheta = \frac{a}{c}$ (see fig. 2.4) then $c = \frac{a}{\sin\vartheta}$ or, substituting our values, $c = \frac{100}{0.5} = 200$ **metres.**

But back to the original assumption — How *DO* we know that sin 30° is equal to 0.5? Let us figure out how to calculate a table of sine values. While the means we will use are a little unorthodox,it is not difficult and will lead us directly to the sine wave. And **that** is important, because we use the term frequently in our discussions of music and synthesis,perhaps without fully understanding exactly what it actually means. The tools we need are a protractor to measure angles and a ruler to measure the lengths of lines.

With our protractor we first draw a circle and divide it into four quadrants I, II, III, IV, the co-ordinate system as shown in Fig. 2.6. The radius of the circle is equal to 1 and we divide the y axis into units 0.1, positive above the origin where the y axis and x axis cross and negative below the origin. Now with the protractor, mark off the circle at 10° increments. We can now approximate the sine of these angles in the following way.

Knowing that:-

$$\sin\vartheta = \frac{\textit{opposite}}{\textit{hypotenuse}}$$ and in our unit circle

$$\textit{hypotenuse} = 1$$

$$\sin\vartheta = \frac{\textit{opposite}}{\textit{hypotenuse}} = \frac{a}{1} = a$$

We simply draw a perpendicular line from the angle in question to the x axis, which becomes our opposite side, ***a***. For example, in Fig. 2.6 if we draw a line from 40° to the x axis we can measure the length of that line to approximately 0.64, therefore:-

$$\sin 40^\circ = \frac{\textit{opposite}}{\textit{hypotenuse}} = \frac{0.64}{1} = 0.64$$

Because the hypotenuse (the radius of the circle) is equal to 1 for all of the right-angled triangles formed from the angles, we can approximate all of the sines by simply measuring the successive perpendiculars.

sin 10° = 0.17
sin 20° = 0.34
sin 30° = 0.50
sin 40° = 0.64
sin 50° = 0.77
sin 60° = 0.87
sin 70° = 0.94
sin 80° = 0.98
sin 90° = 1.00*

*Here we must accept some mathematical abstraction for, as we approach 90°, the perpendicular approaches the length of the hypotenuse (the radius). If we were calculating very small increments of angle, a right-angled triangle would exist even if the angle ϑ = 89.999°. But at ϑ = 90° our right triangle collapses to a line and we have two rather *abstract* angles both of which equal 90°.

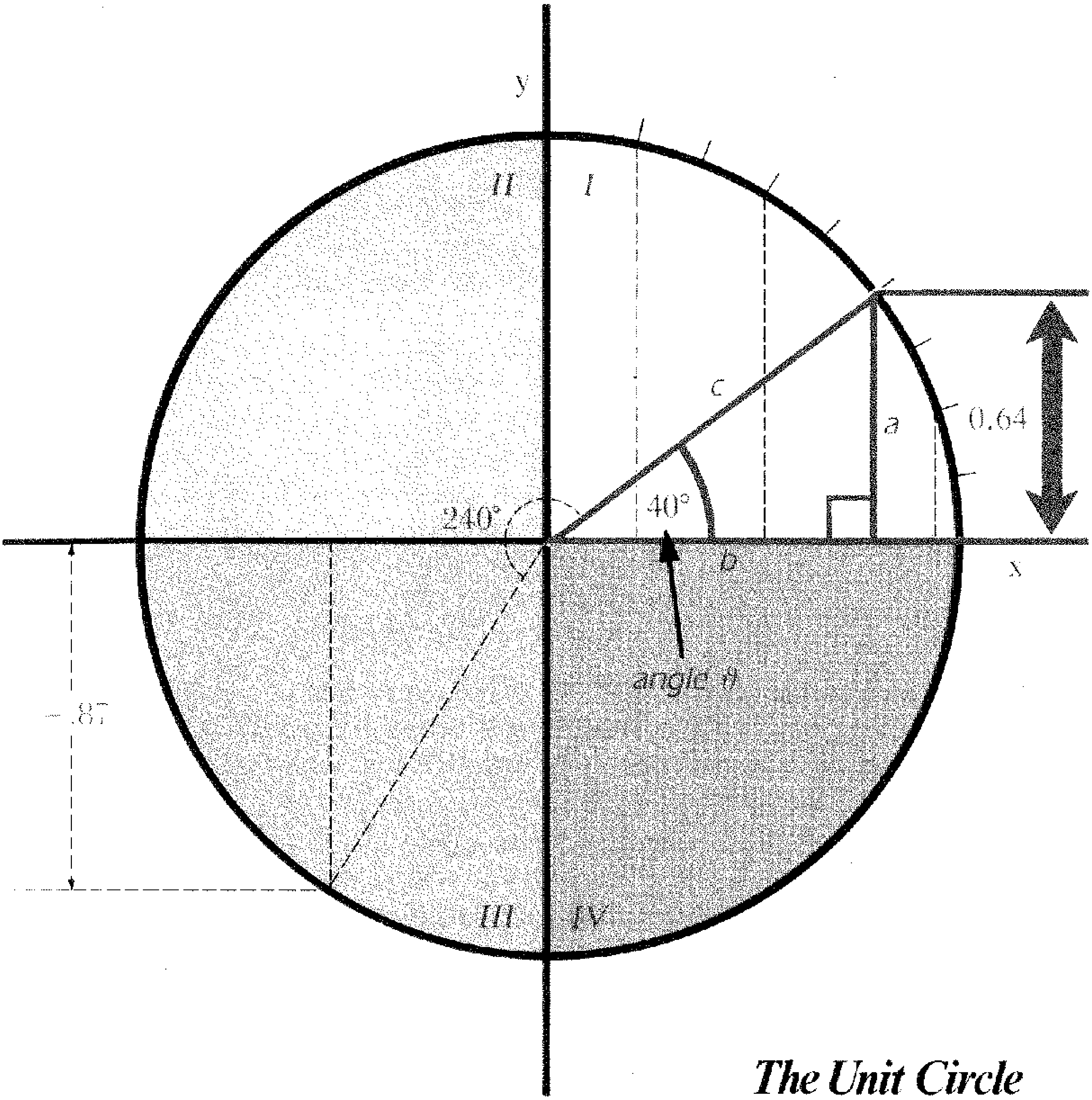


Fig. 2.6

A circle whose radius = 1 can be divided into a number of sections (in this case every 10°) from which a succession of right-angled triangles can be formed. For every triangle the radius of the circle is considered its hypotenuse thus allowing a straightforward visualization of the function sin ϑ which equals the height of the side a

$$\sin\vartheta = \frac{opposite}{hypotenuse} = \frac{a}{1}$$

With simple tools we have been able to approximate the sine function and therefore solve our map-making problem. We were able to do this through the use of one quadrant of a circle whose radius is equal to one. Such a circle is sometimes called a "unit circle" and we see in Fig. 2.6 that there are 270° which remain (90° in each of quadrants II, III, and IV). We can move on from our 90° angle then to 100°, 110°, 120°, etc., always incrementing the angle by 10°. To maintain our right-angled triangle, we let the adjacent side of the angle become ***negative x*** and draw the perpendicular as we did before. Obviously the height of line ***a*** measured on the ***y*** axis will decrease in the second quadrant with the same values of the first quadrant except in reverse order. Thus:-

sin 100° = 0.98
sin 110° = 0.94
sin 120° = 0.87
sin 130° = 0.77
sin 140° = 0.64
sin 150° = 0.50
sin 160° = 0.34
sin 170° = 0.17
sin 180° = 0.0*

Continuing beyond 180° we notice that the values for sin ϑ are the same as they were in quadrant I, except for the fact that the side ***a*** opposite the angle ϑ is now negative because below the origin, ***y*** is negative. For example (see Fig. 2. 6),

$$\sin 240^\circ = -.87$$

The continuation into quadrant IV will be the negative of quadrant II.

Now we will look at all of these values in the form of a graph, Fig. 2.7, where on the horizontal axis we mark the degrees for successive values of angle ϑ and on the vertical axis we plot sin ϑ, the values of which we have just determined from our unit circle. Thus we have sine of the angle theta as a function of theta.

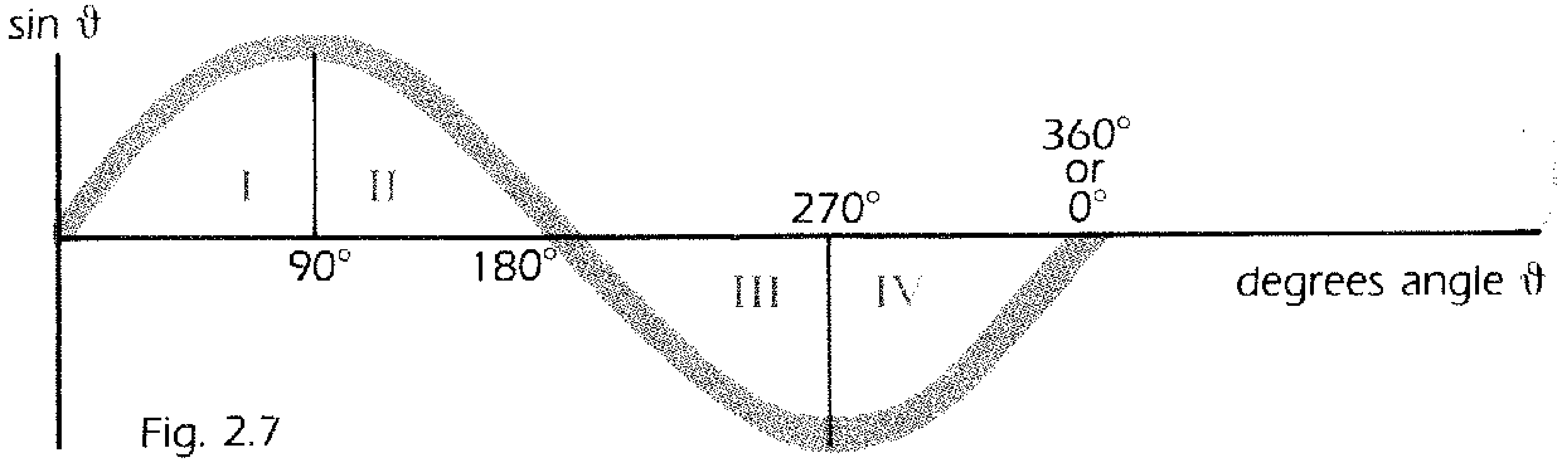


Fig. 2.7

A plot of sin ϑ as a function of ϑ reveals the familiar shape of a sine wave.

---

* The last of course is the reverse of the abstraction at 90°, only this time it is 0.

Starting at 0° again we could go through this entire process again and figure the values of **cosine** of theta rather than sine. As we said previously:-

$$\cos\vartheta = \frac{adjacent}{hypotenuse} = \frac{b}{c}$$ (see Fig. 2.4)

We can see in our unit circle, Fig. 2.6, that while sine of theta is 0, cosine of theta will be 1; that is, when theta is 0, then both the hypotenuse and the adjacent side ***c*** are equal to 1, as must be the ratio. (Remember we are looking at line ***b*** now).

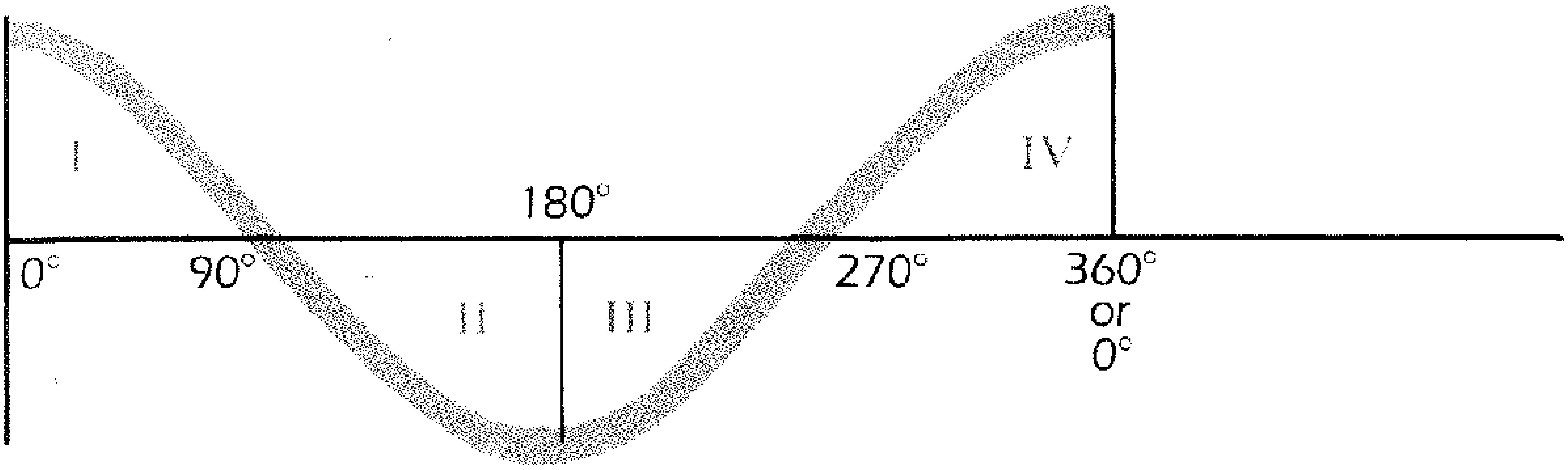


Fig. 2.8

A plot cos ϑ as a function of ϑ.

These two functions, sine and cosine, are related in another way. In Fig. 2.9 we see several periods of a sine function from which is extracted a shape that is exactly that of a cosine. That is, by plotting a sine function beginning at 90° and passing through 360° and ending at 90°, we will have plotted the same as the cosine of theta. Therefore:-

$$\cos\vartheta = \sin(\vartheta + 90°)$$

which leads us to the important concept of **Phase.**

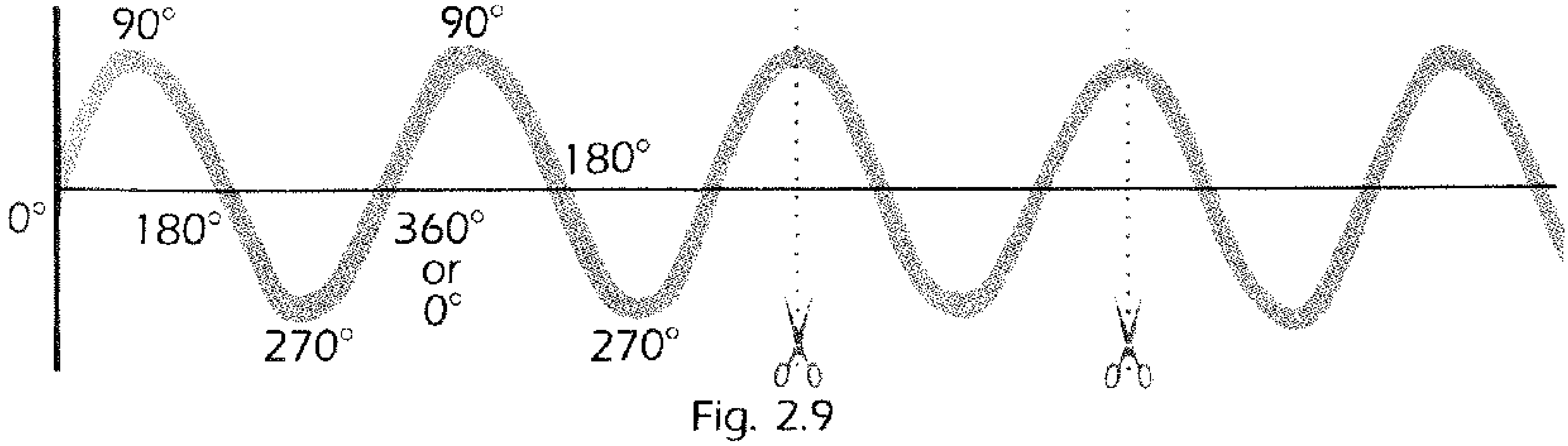


Fig. 2.9

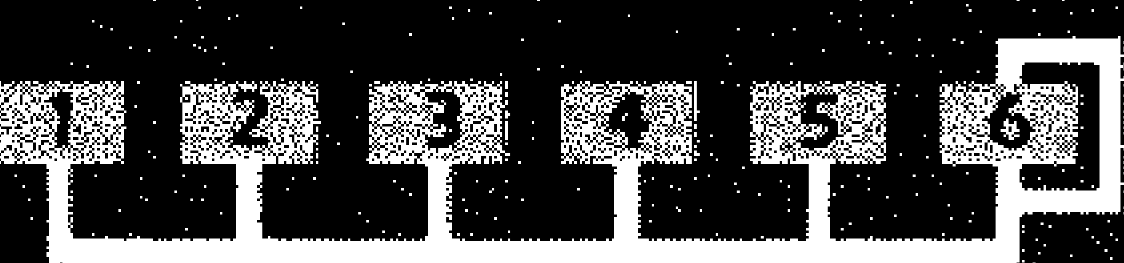
Several periods of a sine function, where one period beginning at a phase of 90° can be seen to be the same as a cosine function.

We can also make reference to the *phase* of a function derived from uniform circular motion, as we now know a sine function to be, where phase is a point to which the rotation (angle) has advanced relative to some reference point. Therefore, we can say that a cosine function is the same as a sine function beginning at a phase angle of 90°, where the reference point is assumed to be 0°.

What is it about these functions that are so special in their relation to acoustical theory? It has long been known that many natural sounds, including the steady state or sustain part of most musical instrument sounds, are more or less periodic in that the pattern of pressure variation is largely constant (see Fig. 2.2). Near the beginning of the 19th century a French physicist/mathematician, Joseph Fourier, realised that any periodic function could be resolved into mixtures of sine and/or cosine functions which may differ in amplitude and period but whose periods are related by whole number (integer) multiples. This applies to periodic sound waves as well.

For example, the square wave, which is a periodic function having a characteristic timbre familiar to all those who have acquaintance with analogue synthesizers, can be resolved into an infinite number of sine functions (this is sometimes called a Fourier transformation or analysis), whose frequencies are related by all being multiples of the odd Integers (1, 3, 5, 7, etc.), and whose amplitudes are related by the reciprocal of those integers ($^1/_1$, $^1/_3$, $^1/_5$, $^1/_7$, etc). With the "X" Series synthesizer we can add six sine functions together and approximate a square wave. Set up "X"-ample 2.1 (this is the "X"-ample we used in the introduction, when explaining the exercise format and the spectral diagram).

"X"-ample 2.1



	FREQUENCY	OUTPUT
op 1	1.00	99
op 2	3.00	87
op 3	5.00	79
op 4	7.00	75
op 5	9.00	72
op 6	11.00	71

INSTRUCTIONS: Starting from the VOICE INIT? position....

Select algorithm 32. Set the values of the operators to those shown above, then play any note to hear the familiar "clarinet" type sound characteristic of the square wave.

Of course, the wave that is produced by these six operators will not be exactly a square wave — for that we need an infinite number, not just six. But in this book we will be dealing with FM synthesis, not additive synthesis which is what we are experimenting with now at this learning stage. We used some new terms however, frequency and amplitude, which we need to define. Looking at Fig. 2.11 on the next page, we see six patterns, all of which seem to be sine waves but which differ in the relative number of periods on the **x** axis and in their heights on the **y** axis. When we measure the height of the wave at the point where it is at its greatest distance from 0, we are measuring its *amplitude* as in Fig. 2.11a.

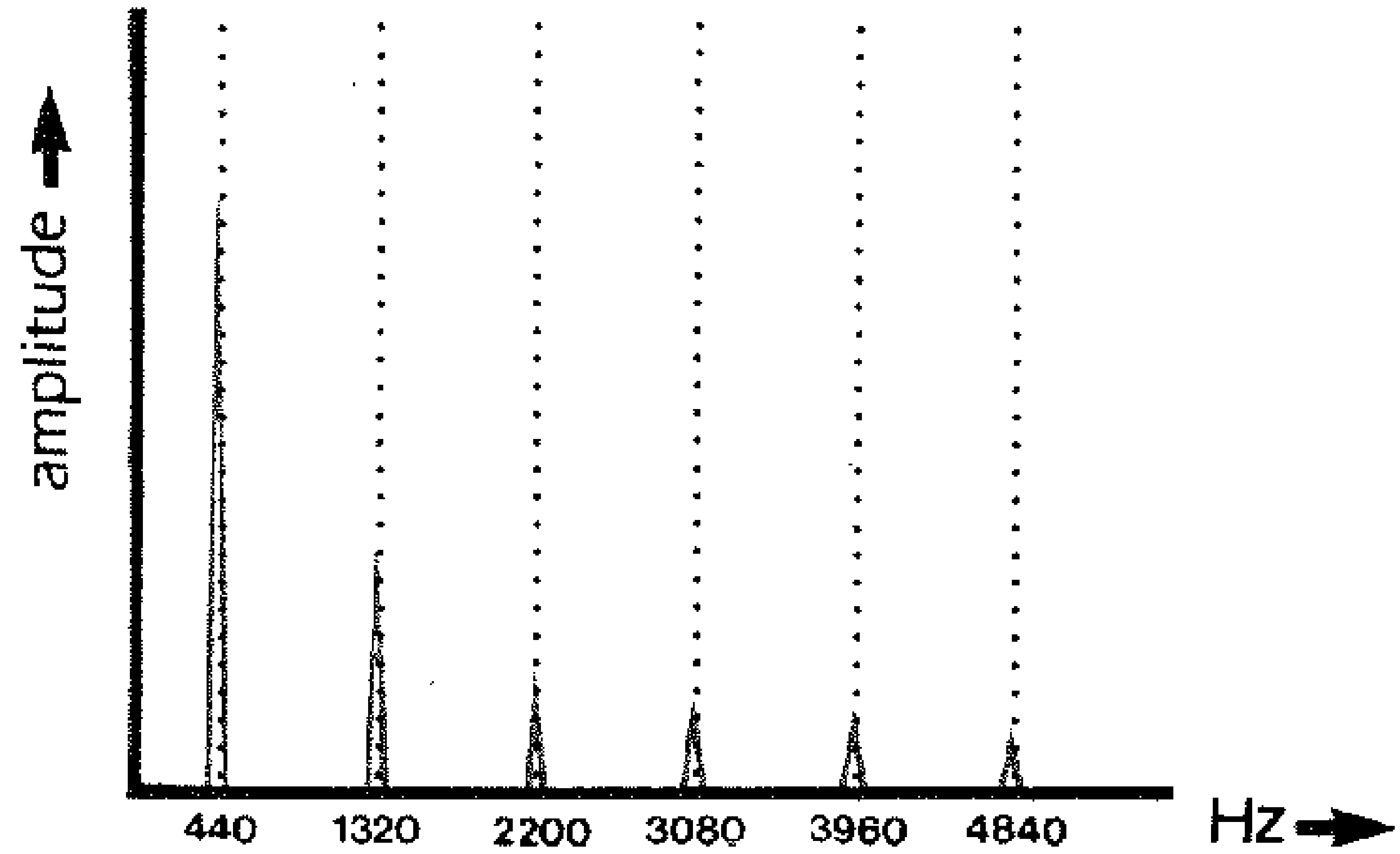


Fig. 2.10

Spectrum relating to "X"-ample 2.1. For note A440Hz (that's A just above middle C).

Reset the values in "X"-ample 2.1, turn off all of the operators except op. 1, and press the key A (=440). What you hear is a wave whose pressure varies in a manner as shown in Fig. 2.11a at a frequency of 440 periods, or cycles, per second. That is "cycles" because it is 440 cycles around our unit circle. Now look at the spectrum shown in Fig. 2.10, turn off op. 1 and turn on op. 2. You now are hearing the second component of the square wave at frequency three times that of the first — at 1320 cycles/sec (commonly called Hertz and abbreviated Hz) — and at an amplitude of ⅓ that of the first, as seen in Fig. 2.11b. Because the frequency is greater, the length of each period must be shorter. Frequency, then, is inversely related to period. Therefore:

$$\textbf{\textit{period}} = \frac{\textbf{\textit{1}}}{\textbf{\textit{frequency}}}$$

i.e. a wave whose frequency is 440Hz (cycles/sec) will have a period of $1/440$ sec.

In listening to first op. 1 and then 2, we notice that as frequency is greater, the sensation of pitch is higher, and that as amplitude is lower, loudness is less. The physical properties of frequency and amplitude are indeed linked to the perceptual properties of pitch and loudness, sometimes in rather complicated ways. It is not always correct to assume that those terms which we use to describe the physical world of sound can be applied directly to the perceptual world which happens inside our heads and which we call music.

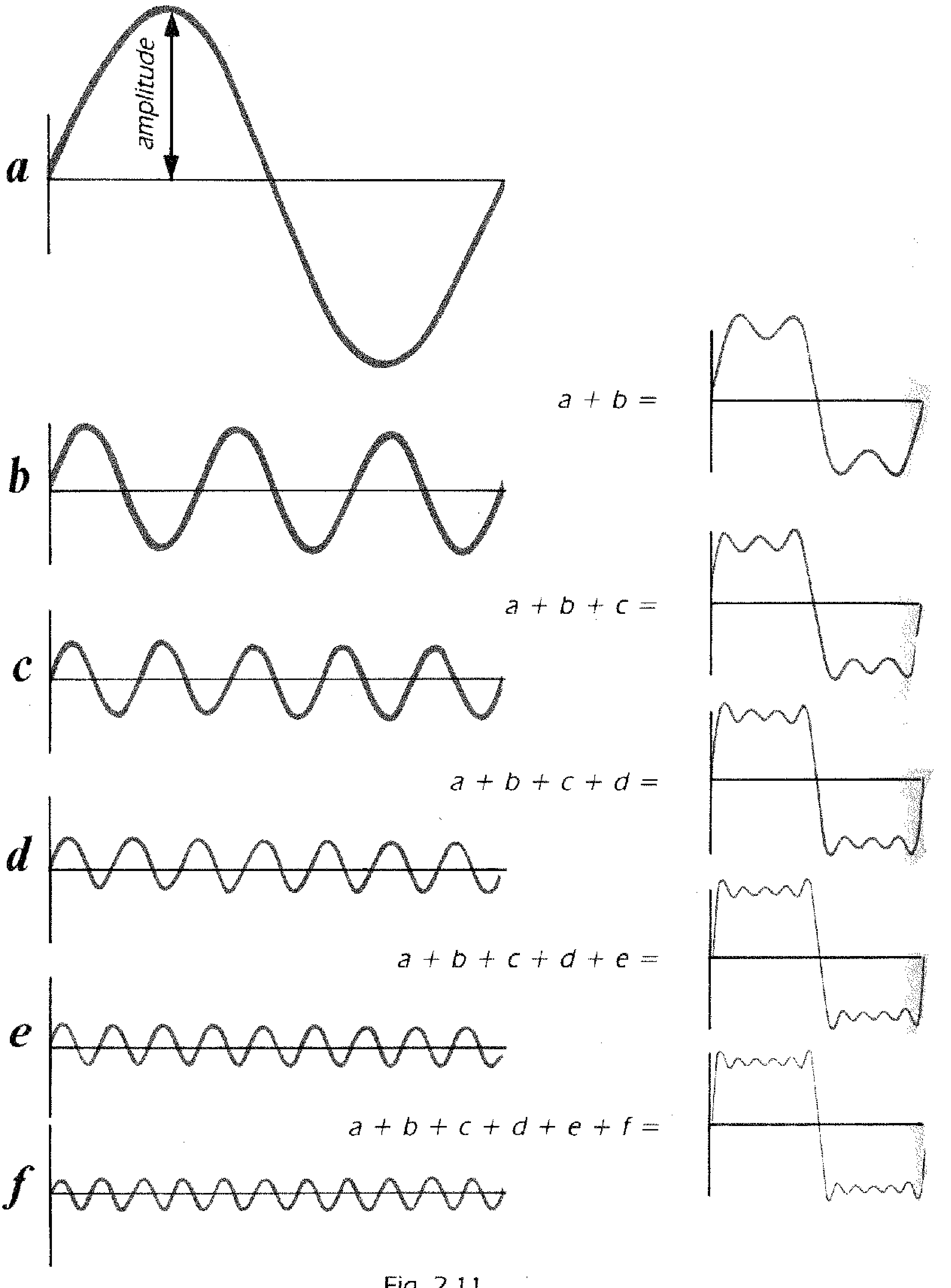


Fig. 2.11

The output of each of the six operators with the frequency and output values indicated. Added together they constitute the first six components of a square wave. On the right one can see that with the addition of each component the wave becomes more square-like.

While looking at Fig. 2.11 a-f and always pressing the key for A (440Hz), listen to each of the operators in turn, 1 through 6. With each operator the frequency is greater and the amplitude less. The frequency coarse parameter, then, specifies an integer which multiplies whatever frequency is associated with the key being pressed. (See appendices for a table of note frequencies.) The operator, then, produces a sine wave at that frequency. In this case op. 2, having a freq. coarse of three, produces a sine tone at 3 × 440 = 1320Hz ;op, 3 at 5 × 440 = 2200Hz, etc. (If we now press the key B, a whole step higher, the frequencies produced by each of the operators are integer multiples of 494Hz).

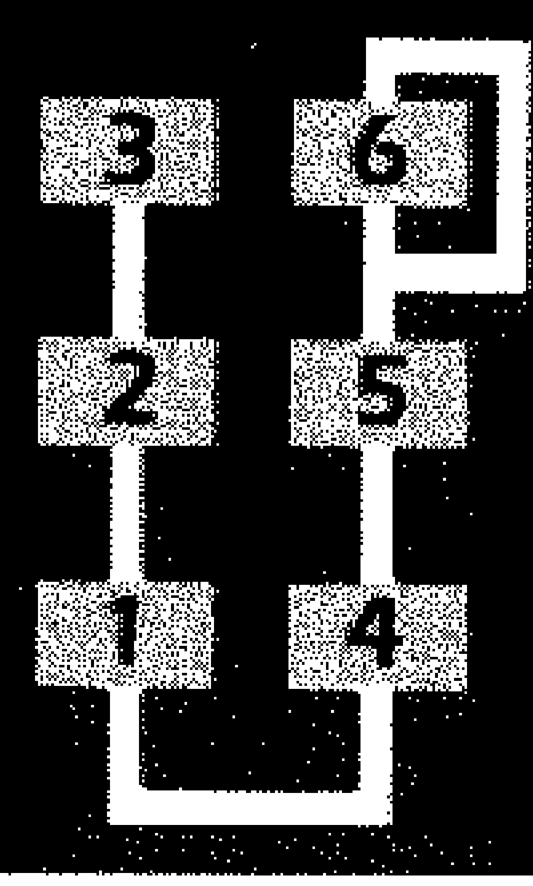
Return to A440 and listen to the sound as we add each operator, first op. 1, then op. 2 (you must repress the key every time to activate the additional operators), then op. 3, etc. The brightness of the tone increases as we add the higher frequency sine tones.

Frequencies which are related in this way by integers (that is to say they are products of *whole* numbers and not fractions) fall into what is called the *harmonic series*, and each frequency is called a harmonic. A harmonic is a sinusoidal vibration whose frequency is an integral (integer) multiple of a fundamental frequency. In the case above, for a fundamental of A (440Hz), we listened to the 1st, 3rd, 5th, 7th, 9th, and 11th harmonics. Whenever the term harmonic is used, then, it refers to a sinusoidal vibration or sine wave that is related to other sinusoidal vibrations by integral (whole number) multiples.

Before going on to our next subject, we can remind ourselves of the power of FM synthesis by trying another "X"-ample which produces the same sound as "X"-ample 2.1 but with the use of only three operators. Understanding how this is possible will become clear over the next chapters when we begin to study FM.

"X"-ample 2.2

	FREQUENCY	OUTPUT
op 1	1.00	99
op 2	2.00	69
op 3	4.00	64
op 4	1.00	00
op 5	1.00	00
op 6	1.00	00



INSTRUCTIONS: Starting from the VOICE INIT ? position.....

Select algorithm 3. Set the values of the operators to those shown above, then play any note to hear the familiar "clarinet" type sound characteristic of the square wave, this time produced by FM. Of course any algorithm which has a stack of three operators will serve for this example, as long as they have the values shown above. Store this sound and make a comparison with the sound made additively in "X"-ample 2.1

Pitch perception and frequency

Set up the following "X"-ample:

"X"-ample 2.3

	FREQUENCY	OUTPUT
op 1	1.00	99
op 2	2.00	99
op 3	3.00	99
op 4	4.00	99
op 5	5.00	99
op 6	6.00	99

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select algorithm 32. Set the values of the operators to those shown above, then turn off all operators and listen to each one successively.

Remember that the frequency coarse parameter multiplies the frequency associated with a key by an integer. Therefore what we have in this exercise, assuming that you play A (440Hz) just above middle C, is as follows:

	Freq. coarse parameter		freq. of key pressed		freq. of harmonic
op 1	1	×	440	=	440 (harmonic No. 1, called the fundamental)
op 2	2	×	440	=	880 (harmonic No. 2)
op 3	3	×	440	=	1320 (harmonic No. 3)
op 4	4	×	440	=	1760 (harmonic No. 4)
op 5	5	×	440	=	2200 (harmonic No. 5)
op 6	6	×	440	=	2640 (harmonic No. 6)

Perhaps you have noticed that the change in pitch between harmonic No. 1 and harmonic No. 2 is an octave, whereas the change in pitch between harmonic No. 2 and No. 3 is only a fifth, yet the difference in both cases is 440Hz. In fact, you can notice that the difference between each successive harmonic is a smaller *musical* interval than the preceding one although the *arithmetic* interval (the number of cycles per second) is the same (440Hz in this case). What we have noticed is of fundamental importance in the understanding of musical perception, i.e. the perception of constant musical intervals or pitch distance is not based upon constant differences in frequency. But what then?

Now we will make a modification to the previous "X"-ample and demonstrate to ourselves something important regarding the perception of pitch. We want to have an understanding of pitch and frequency, not only because it is interesting, but because we will learn about forms of graphic representation which will be helpful in our understanding of FM beginning in the next section. So, using **"X"-ample 2.3,** change only the freq. coarse values from 1,2,3,4,5,6 to 1,2,4,8,16, respectively. Never mind 32 for the time being — this experiment can be restricted to 5 operators.

Now turn all the operators off except op. 1 and listen while pressing the key an octave below A440; that's A220Hz just below middle C. Turn off op. 1 and turn on 2 and listen again. Always sounding the same key, listen to each of the remaining operators one by one. You may be surprised to hear a constant pitch distance of one octave as you progress through the operators. What, then, is the relationship of the frequencies that produced this constant pitch distance? The following paragraphs will help unravel the mystery which surrounds the idea of a "logarithmic" representation. Let's start by looking at the table of frequencies in this case.

Freq coarse parameter			freq of key pressed		freq of harmonic
op 1	1	×	220	=	220 (harmonic No. 1)
op 2	2	×	220	=	440 (harmonic No. 2)
op 3	4	×	220	=	880 (harmonic No. 4)
op 4	8	×	220	=	1760 (harmonic No. 8)
op 5	16	×	220	=	3520 (harmonic No. 16)

We can see that, while we hear a constant pitch distance of an octave from operator to operator, the frequency distance always doubles. Or we can say that to change the pitch by a constant perceptual distance, the frequency must change by a constant factor (in this case of an octave, by a factor of 2). In Fig. 2.12 we see a line representing frequency in Hertz. The line is divided into equal units of frequency on which are marked the points which we have heard to be successive octaves.

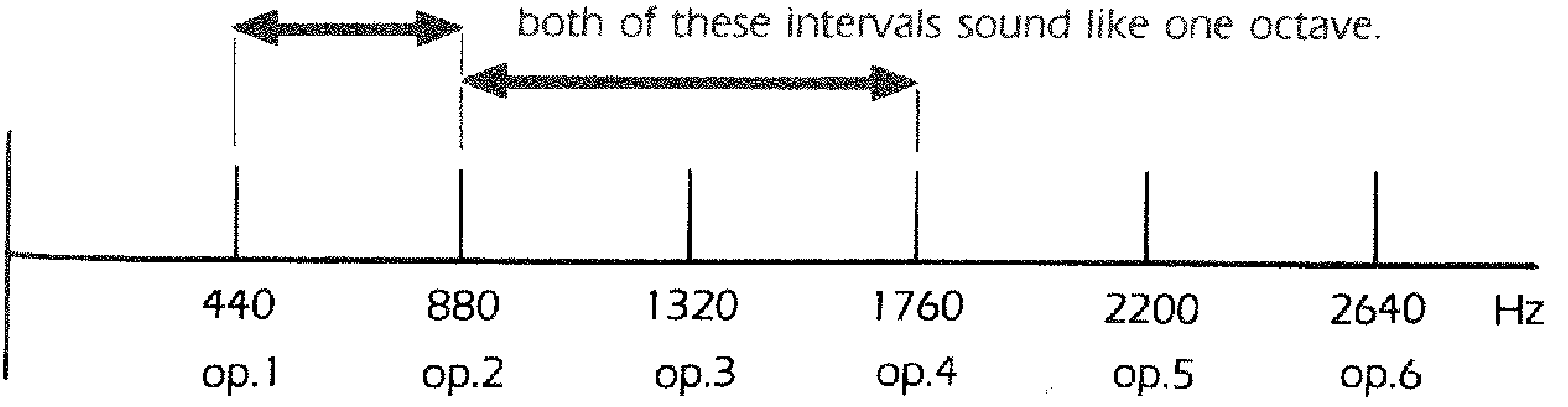


Fig. 2.12

With frequency plotted linearly along the horizontal axis, we can see clearly that equal octaves are not represented by equal distances, or frequency intervals.

Because this representation of frequency does not preserve visually the equally perceived pitch distance, we should be aware of this when looking at our linear representations of FM spectra in the following chapters. When we think of frequency as it is *perceived*, it would be helpful to have a representation that is closer to how the ear works, and that is just what a *logarithmic* scale for frequency does. Fig. 2.13 shows two identical spectra, one with a linear frequency scale and one with a log frequency scale – offering a visual representation which is more similar to our perception of frequencies.

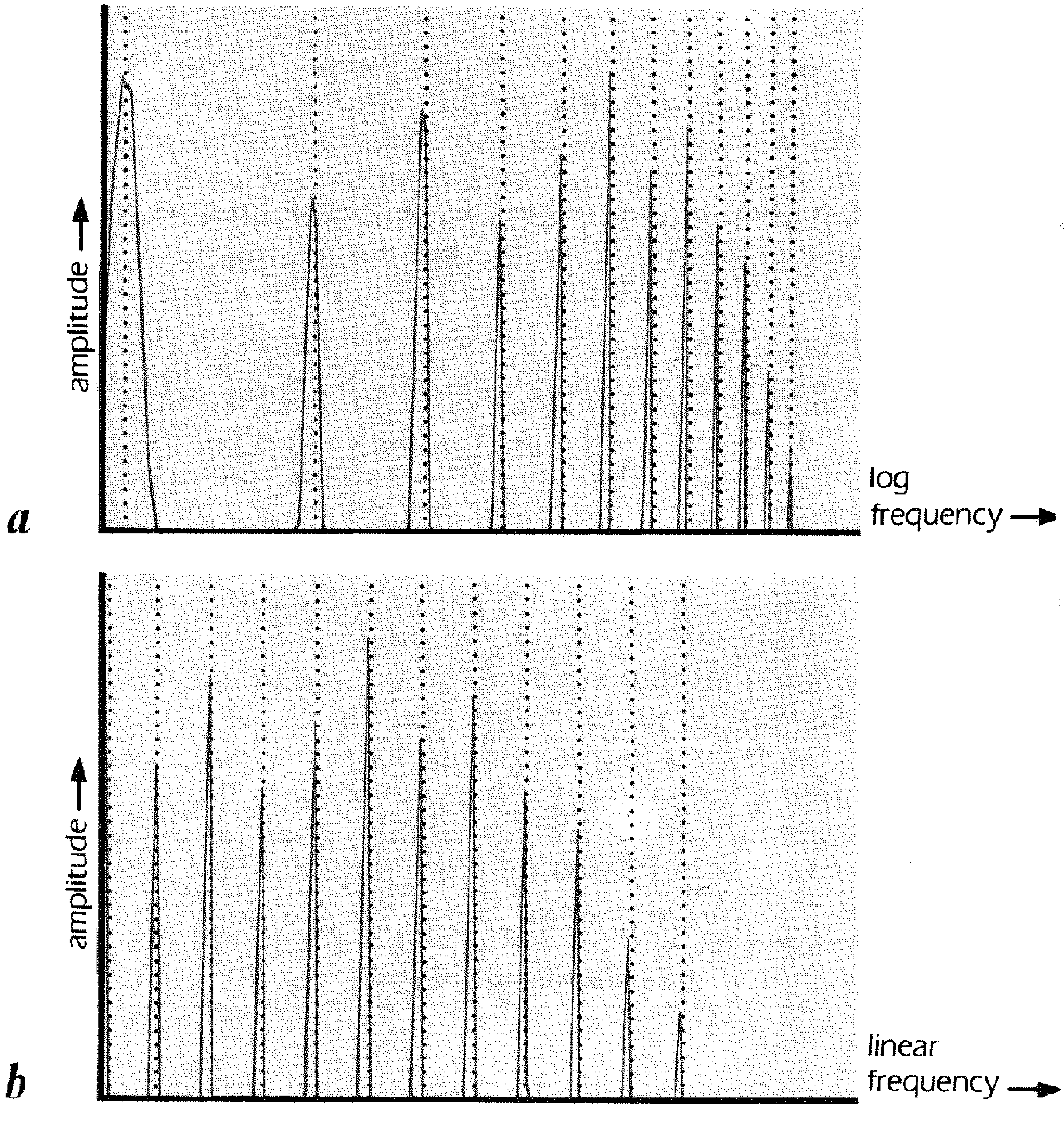


Fig. 2.13

Above are two representations of one typical DX7 sound. The first (a) represents the components close to a manner in which they are perceived; the second (b) maintains a constant frequency interval between the components and is perhaps more clear from a mathematical point of view.

Imagine stretching and squashing the graph in Fig. 2.12 as if it were printed on a rubber page so that the octaves were always represented by equal distances, and you have an idea of what a logarithmic representation is. But what exactly happens to the frequency scale if we do this, and how do we plot it? Well, it is not absolutely necessary to understand how to move from a linear representation to a logarithmic one, and indeed in this text for reasons of mathematical clarity, we shall be using linear frequency scales. However, it is an interesting and simple process which also reveals an understanding of frequencies in a musical scale, so a full explanation is given in appendix 1. There are two different ways, then, to plot frequency.

1) Logarithmically — which shows equal pitch intervals. 2) Linearly — which shows equal frequency intervals.

We have in Fig. 2.10 introduced another aspect of representation, one which has to do with loudness. We know intuitively that any frequency we hear has another quality besides pitch, and that is loudness; i.e., a single pitch can be heard at many different loudnesses, and conversely, a single loudness can be heard at many different pitches. In Fig. 2.10 we have shown on the vertical axis the output level of each of the operators of algorithm 32, such that we represent the six sinusoids' frequencies on one axis and their levels on the other. When we plot frequency (either log or linear) on the x axis, the representation is called the frequency domain, whereas when we plot time on the x axis, as in Fig. 2.11, the representation is in the time domain. What we are not able to see in the frequency domain representation is the phase relationships of the sinusoids (for example, all of the components in Fig. 2.11 have an initial phase relationship of $0°$, but in many cases, phase relationships between harmonics are only minimally perceptible, if at all).

The second and third periodic pressure waves of Fig. 2.2 are in fact made up of identical components in both frequency and output level. They differ in the initial phase relationships of the components. The waveforms are very different, yet they sound virtually identical. In this case the time domain representation does not necessarily reveal the odd harmonic or "square wave-ish" sound, while in the frequency domain, Fig. 2.10, we can see the relationship of odd harmonics independent of phase. There are other instances where the time domain is more revealing — amplitude envelopes, for example — and others where neither is an adequate representation. We are for the moment restricting the discussion to tones which are artificially simple, that is, tones which do not change in time. We want to understand these simple ideas before we consider the interesting but more complicated cases where the tones evolve in time. Frequency domain representations are often referred to as spectra. As with a prism we are able to see light broken up into a spectrum (singular of spectra) of light frequencies (colours), with sound we can see a tone broken up into a spectrum of audio frequencies (sinusoids). When we make reference to spectra, then, we mean frequency domain representations of sound, where frequencies are plotted on the horizontal axis and their levels (amplitude or intensity) on the vertical axis.

Loudness Perception and Intensity

As we have seen with pitch and frequency, our ears do not line right up with straightforward physical measures. Frequency components at a constant distance in Hertz are not perceived to have constant intervals in pitch, as we have demonstrated listening to the harmonic series. And so too with loudness. At the beginning of this section we noted that air particles, when excited by some vibrating source, alternate between states of compression and rarefaction. If the source continues its vibration at the same rate (frequency!) but over a greater distance (for example, if the loudspeaker cone's "in-out" excursion increases), then the maximum compression and rarefaction of the pressure wave will also increase. The measure of the pressure at the instant when it is greatest, or when the air particles are most compressed, is called the peak amplitude. As was the case with frequency, changes in amplitude or pressure must be transformed into another scale in order to approach the way that the ear senses loudness. A common and useful measure is sound pressure level (SPL), or intensity, the units of which are decibels (dB). A decibel is 20 times the log of the ratio of two pressures (two pressures since level is relational).

$$SPL = 20 \times \log_{10}\left(\frac{P_1}{P_2}\right)$$

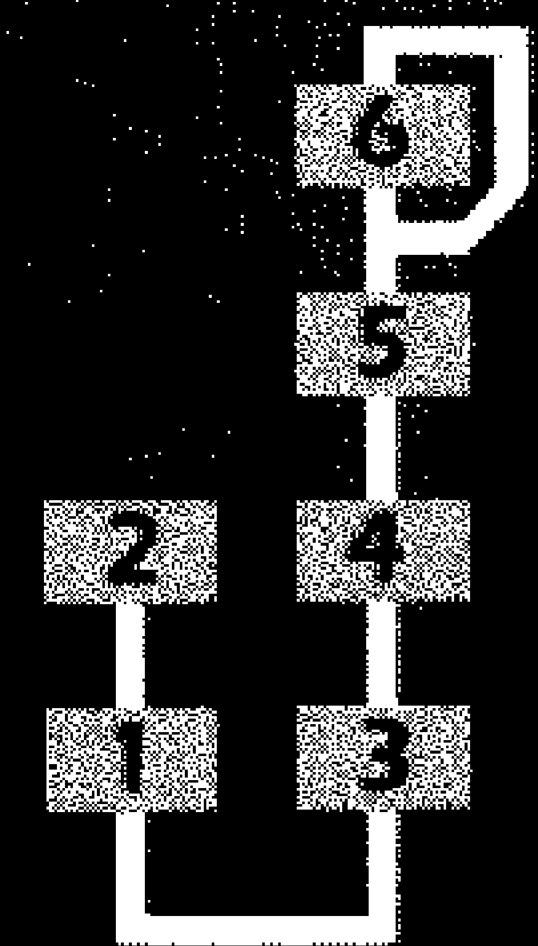
The output level of the "X" Series synthesizer is in a log scale, although not in decibels. The reason that it is not in a dB scale has to do with the way the arithmetic is most efficiently done in the hardware and the fact that of the three basic musical parameters, loudness, time and pitch, loudness is the least precisely perceived (doubling a note's time value or its frequency is much more perceptually apparent than doubling a note's loundness). The available computational power of the instrument was thought to be better used in other ways — the accurate determination of pitch, for example. In the explanation of FM which follows, we will especially rely upon the amplitude (which is a linear measurement) of frequency components rather than intensity (log).

Aliasing

We now want to demonstrate a rather surprising phenomenon which is utterly counter-intuitive, especially to those whose experience has been in the analogue domain. What will at first appear to be a deficiency of digital audio will finally be seen to be a useful attribute, especially in FM synthesis.

What "X"-ample 2.4 will demonstrate is a fundamental attribute of digital audio, whether synthesis, recording or sampling. Known as **aliasing** (described by the Nyquist Theorem), it states that no frequency can be reproduced that is greater than one half the sampling rate. The sampling rate is the number of samples (numbers or readings) per second used to form the sound wave. (The compact disc has a sampling rate of 44.1 kHz while the DX7's rate is just below 60kHz.) Furthermore, the theorem states that frequencies greater than one half the sampling rate will reflect about the half sampling rate. That is, if the sampling rate is 60kHz, the half sampling rate is 30kHz. So a frequency introduced at 31kHz cannot be produced, but will reflect and thus be heard at 29kHz. A frequency of 35.5kHz will reflect at 24.5kHz, and a frequency of 59.9kHz at 100Hz (0.1kHz) — the half sampling rate minus the amount by which the frequency exceeds it; 30,000Hz — (59,900Hz — 30,000Hz).

"X"-ample 2.4



	FREQUENCY	OUTPUT
op 1	1.00	99
op 2	5.00	99
op 3		
op 4		
op 5		
op 6		

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select algorithm 1 and set up the values shown above. Now, note by note, play an ascending chromatic scale starting from the lowest C on the keyboard. What do you hear? The tone itself is rather rich in harmonics, and as you would expect, it rises in pitch as you ascend the scale. Then at about B3, that is, B above middle C, there appears to be a slight change to the sound, a roughness that was not previously present. Continuing to ascend the scale, at E3 or E4 there is a very noticeable change, where not only is there a change in tone quality, but also there seems to be an additional pitch present. All of the remaining notes above this point have similar noticeable perturbations of the original sound and, what is more, in a seemingly random manner. Why should this be?

In digital recording, the signal to be recorded must first be (low-pass) filtered to ensure that there are no frequencies higher than the half sampling rate, which would reflect and distort the image. Remember, we are not only considering fundamental pitches, but also those high frequency components in very bright sounds. In the "X" -ample above, the highest harmonics began to reflect around the half sampling rate at B3 and as the pitch ascended, so more and more harmonics were reflected. (Appendix 7 shows a calculation which allows us to estimate roughly the sampling rate of the DX7 from this simple observation, but you will need to know something about FM before it is clear — so read on!) Where we require stability in the tone, this effect is undesirable and can be eliminated by progressively reducing the bandwidth as the pitch is raised by means of key scaling, as we shall see in Chapter 5. Of course we could also make use of aliasing for special effects, where this sort of randomness is desirable.

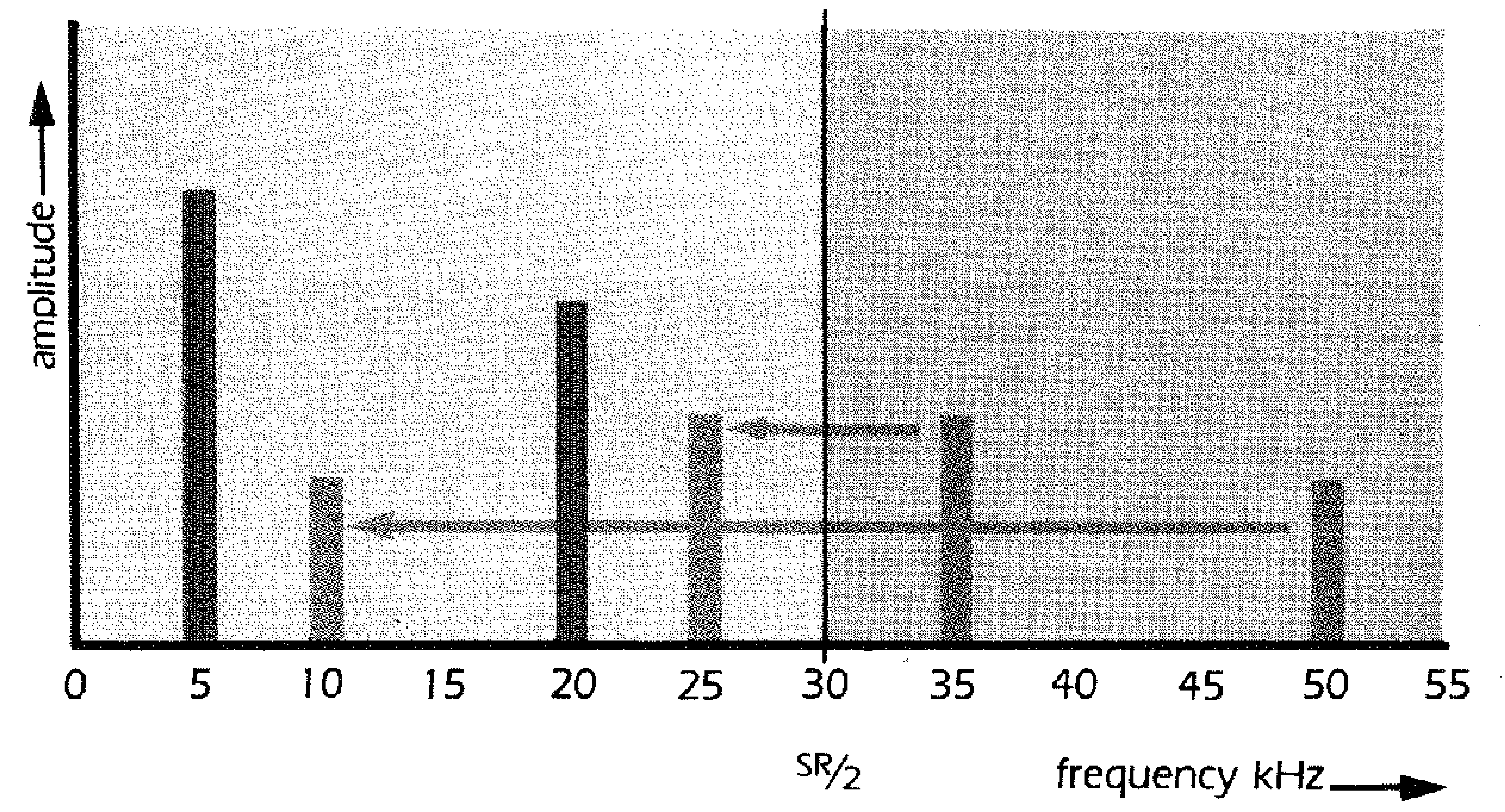


Fig. 2.14

In digital audio, no frequencies can be produced beyond the half sampling rate, SR/2. Any frequency above the limit will reflect, and be produced below it by the same amount, as shown in the two cases above. A spectrum is shown with some components above the half sampling rate — these are not produced, but reflections of these frequencies, indicated by the arrows, are produced instead.

Oscillators and Operators

There is one final topic that we should touch upon briefly before actually beginning some experiments with FM synthesis in the following chapter. For our purposes, it is sufficient to know that an Operator in an "X" Series digital synthesizer is equivalent to an oscillator in an analogue synthesizer. In the latter case, the oscillator produces a changing voltage according to some selected pattern such as sine, sawtooth, pulse, etc. An operator performs essentially the same tasks, but instead of a changing voltage it produces a series of changing numbers (samples) whose pattern is always that of a sine wave.

Knowing that most interesting sounds have many components, how is it that with only six operators we can generate and control considerably more than six harmonic or inharmonic components, when an operator alone can only produce a single pure tone? That is what FM synthesis is about, and we are about to discover how!

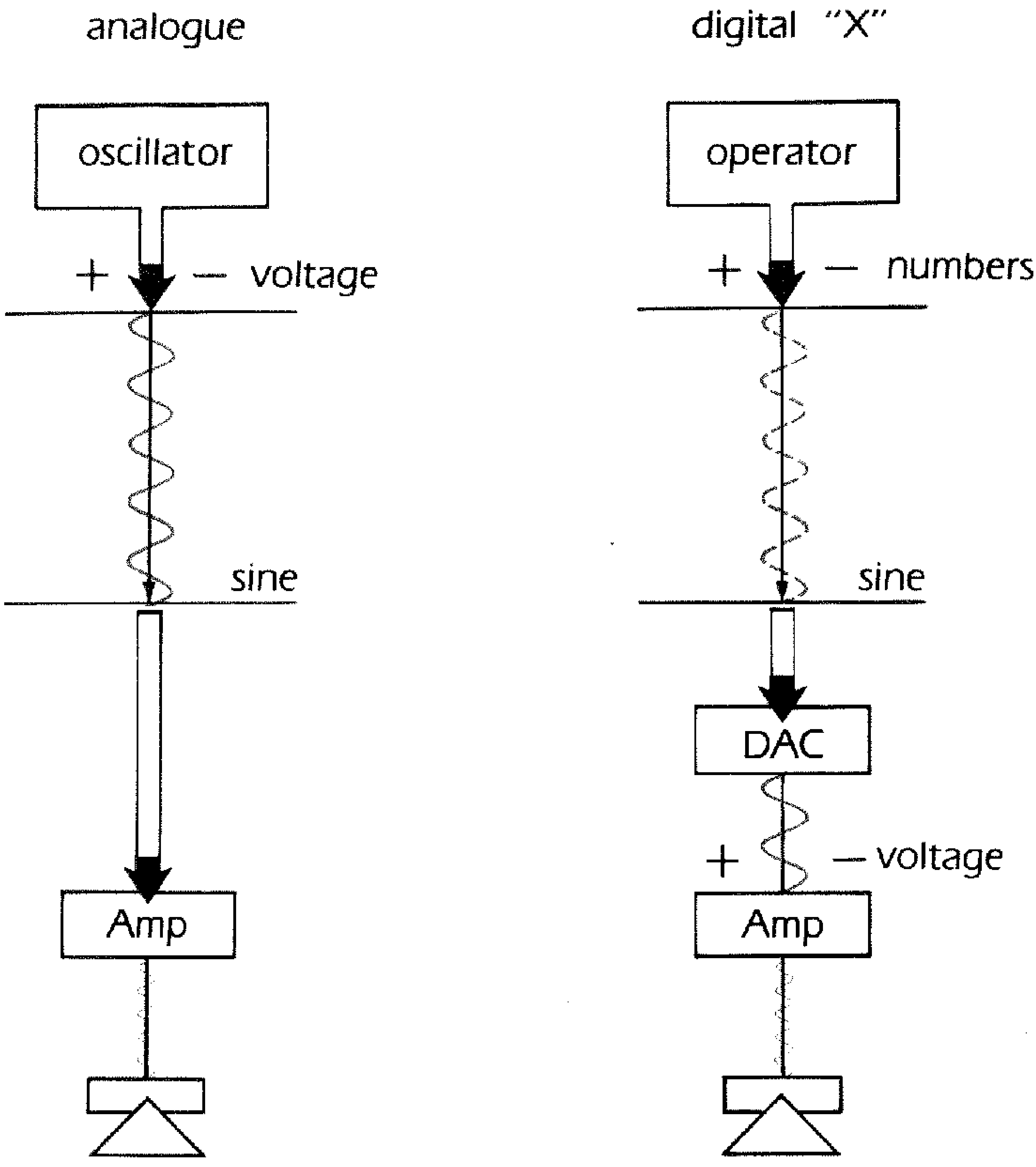


Fig. 2.15
In the digital case, the sine wave is stored as a series of numbers which are then changed to a form which we can eventually perceive – by the DAC (Digital to Analogue Converter).

CHAPTER 3

WHAT IS FM ?

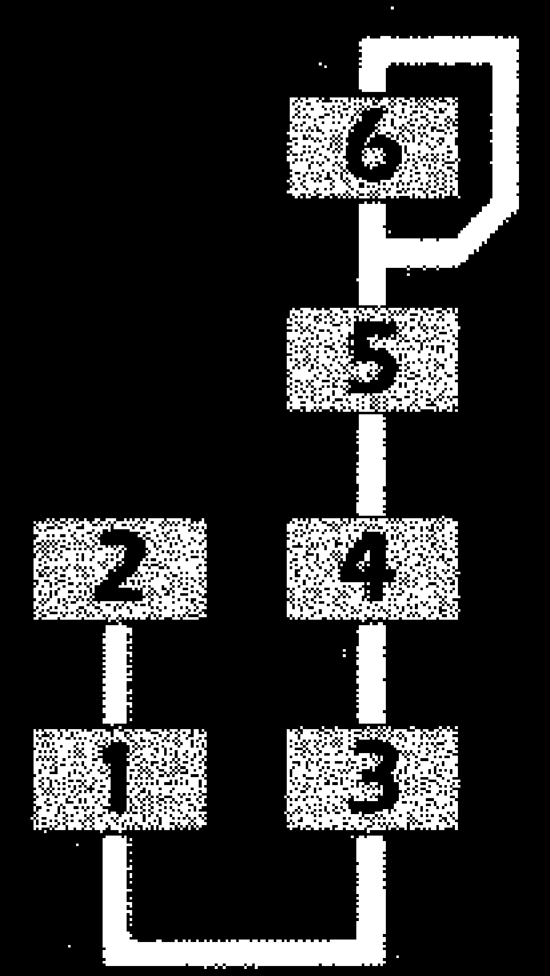
One form of frequency modulation which is familiar to all musicians is vibrato – that slight periodic fluctuation in frequency which we associate with many musical sounds. As it was through observation and experimentation with vibrato that lead to the original understanding and development of FM synthesis, you may find it helpful to follow the same path .

Vibrato and Frequency Modulation

In this chapter we will rely upon what we can hear to help us make some practical observations in regard to extreme vibrato. These observations will serve as the basis for a theoretical understanding of FM synthesis in the next chapter. The following example will serve as a basis for all the experiments in this chapter, so please note the following instructions. Only operators 1 and 2 are used; function controls (17-32) should be set with Mod. Wheel Range = 99; Pitch = OFF; Amplitude = OFF; EG Bias = ON; all others = 0 or OFF. Do this "X" -ample now.

"X"-ample 3.1a

	FREQUENCY	OUTPUT	AMS
op 1	97.72Hz*	99	0
op 2	1.00Hz	99	3
op 3			
op 4			
op 5			
op 6			



INSTRUCTIONS: Starting from the VOICE INIT ? position..

Select algorithm 1 and turn off operators 3-6 and set the above values. Select function control and turn Mod. Wheel EG Bias ON, Range = 99. (Remember that VOICE INIT does not initialize function controls, so be sure that they are set according to instructions). Now, set the modulation wheel to min. Press any key and slowly turn the modulation wheel from min. to max.
**You get 97.72 by selecting a) fixed freq. b) frequency coarse = 10.00Hz. c) Now select frequency fine and increase to 97.72 (maximum for this setting of frequency coarse).*

Important note: please refer to your owners' manual which will explain the practical details of how the output level of an operator can be controlled by the modulation wheel through the edit parameter Amplitude Mod. Sensitivity, and the function control EG BIAS. Also, if your synthesizer does not have a fixed frequency mode, don't worry, most of the "X"-amples are repeated in ratio mode.

Now we know what one operator does to another! We can hear that operator 2 causes a change in the frequency of operator 1 at a rate determined by the frequency of operator 2 (1 cps or Hz) and by an amount determined by the output level of operator 2 (from 0 to 99 according to the position of the modulation wheel). When describing vibrato of this sort,we normally speak of vibrato speed or rate, and vibrato depth, which correspond here to frequency and output level respectively of operator 2. In this "X"-ample the vibrato depth is changing as a function of the modulation wheel, while the vibrato rate is constant at 1 Hz. A plot of frequency vs. time showing the change is illustrated in Fig. 3.1a.

"X"-ample 3.1b

Start from "X"-ample 3.1a. Set the modulation wheel at mid-point between min. and max. and select F Fine (frequency fine) for operator 2, which should read 1 Hz. Press any key and now using the data entry slider, slowly increase the F Fine value from 1Hz to 9.772Hz.

In "X"-ample 3.1b the vibrato depth has been left constant while, with the data entry slider, we have changed the vibrato rate as shown in Fig. 3.1b. As the rate of the vibrato becomes faster, the effect is less and less like vibrato and more like pulsing. But even so, we still can hear the effect as a rapid change of frequency in time. We will now continue in the same manner to increase the vibrato rate to the extreme and observe what happens.

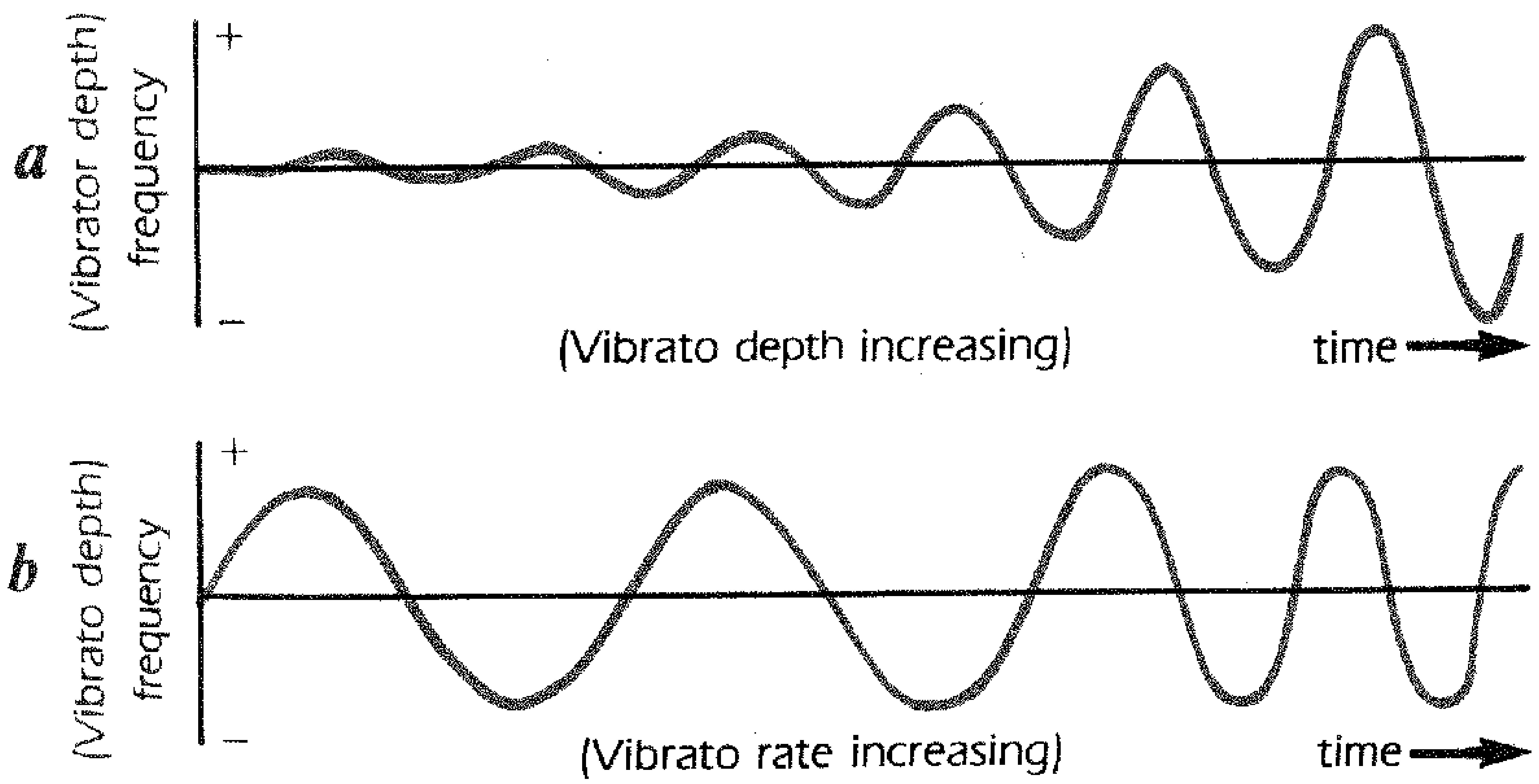
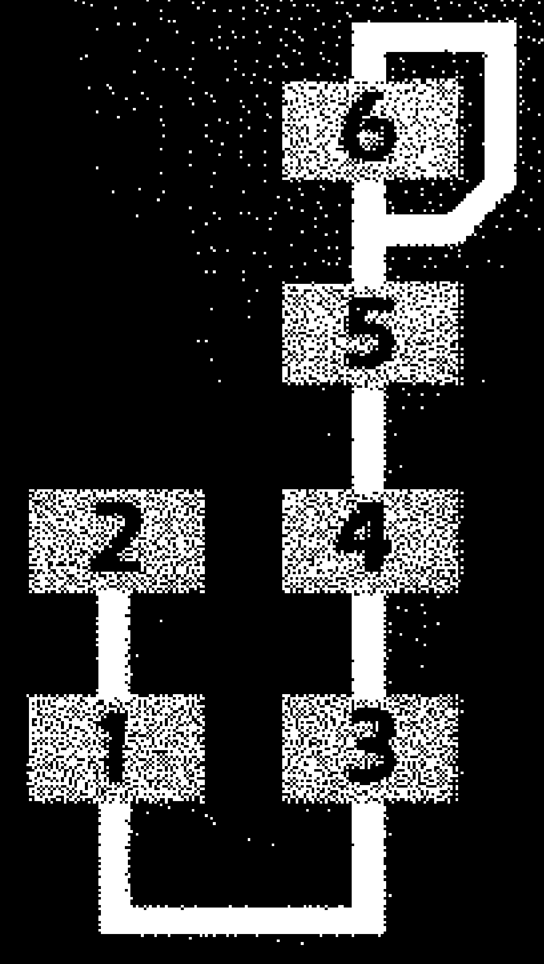


Fig. 3.1

Here we plot time on the *x*-axis and frequency on the *y*-axis. In Fig. 3.1a we show a constant vibrato rate while the vibrato depth changes around an average of 97.72Hz, as heard in "X"-ample 3.1a. Then in Fig. 3.1b the vibrato depth is constant while the vibrato rate increases, as heard in "X"-ample 3.1b.

"X"-ample 3.2a

	FREQUENCY	OUTPUT	AMS
op 1	97.72Hz	99	0
op 2	10.00Hz	99	3
op 3			
op 4			
op 5			
op 6			



INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select algorithm 1, turn off operators 3-6, and ensure that the function controls are set according to the instructions at the beginning of the chapter. Then set the parameter values shown above. Now set the modulation wheel at mid-point between min. and max. and select F Fine for operator 2, which should now read 10Hz. Press any key and use the data entry slider to very slowly increase the F Fine value from 10Hz to 97.72Hz. ***BUT NOT YET!!!!!*** *First let's think about what is happening.*

With a moderate amount of vibrato *depth* (set by the position of the modulation wheel or the output level of operator 2) we will slowly increase the vibrato *rate* (set by the position of the data entry slider as long as parameter Frequency Fine for operator 2 is selected) from 10Hz until it is the same frequency (97.72Hz) as the tone to which the vibrato is being applied. Imagine a string player moving his finger above and below some average position on the fingerboard at a faster and faster rate without changing the depth of the vibrato. While a real player could probably achieve a rate of 10Hz, a rate of 97.72Hz must be left to a player of the imagination — or alternatively, a synthesizer. *Now,* do the "X"-ample.

Astonishingly, at the maximum vibrato rate there is no longer the effect of vibrato, nor the pulsing apparent at the rate around 10Hz, *nor* do we hear a pure tone (a sine wave). We trust that you have not yet touched the modulation wheel, and that it remains at the mid-point between min. and max. What we do hear, however, is a tone having what seems like many, many harmonics, complex but periodic.

"X"-ample 3.2b

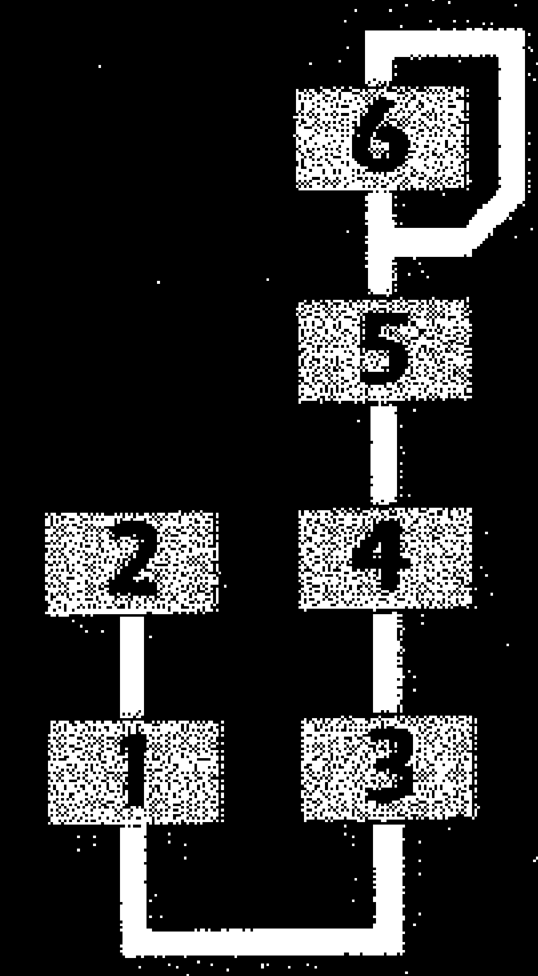
Now, while the vibrato rate (the frequency of operator 2) remains at 97.72Hz, set the modulation wheel at min., press any key, and very slowly move it toward max.

At min. position the sound is that of a pure tone since the output level of operator 2 is at 0. But gradually, as we move the modulation wheel, the tone becomes brighter, which tells us that harmonics are appearing in ever greater numbers. As we pass the mid-point the tone becomes even more complex until the output level of operator 2 is at max. position (output level 99). Notice that all this time there is no sense of vibrato and that the pitch of the pure tone does not seem to change. Try resetting the frequency fine parameter for op. 2 to 15.14Hz, then moving the mod. wheel from min. to max. – that's still pulsing. What about 61.66Hz? That's more like a change in tone, but the pitch changes. Somehow, then, a sine wave at a given frequency which has a sinusoidal vibrato applied to it at a rate that is equal to that frequency, results in a complex harmonic tone whose pitch is at the same frequency. The vibrato depth apparently affects the *number* of harmonics present, from one to many.

What of other frequencies? Can we assume that if both the carrier frequency and the modulating frequency (... oops, that's FM terminology and we are still talking about vibrato!) or rather, center frequency – the basic pitch – and the vibrato rate, are changed to 48.98Hz, the same sound is produced an octave lower? Let's try and see.

"X"-ample 3.3

	FREQUENCY	OUTPUT	AMS
op 1	48.98Hz	99	0
op 2	48.98Hz	99	3
op 3			
op 4			
op 5			
op 6			



INSTRUCTIONS: Starting from the VOICE INIT? position..

Select algorithm 1 and set the above parameters, and the Modulation Wheel to the mid-point, and play any key. The answer is YES! The same effect is produced as in example No. 3.2b. And an octave above 97.72Hz?

	FREQUENCY	OUTPUT	AMS
op 1	195Hz	99	0
op 2	195Hz	99	3

Again YES. Further examples would show that no matter what frequency is chosen, as long the two are equal and the position of the modulation wheel remains unchanged, the same sound will occur, but of course at different pitches.

Now we will interrupt these experiments briefly to re-define some ideas. The theory of FM synthesis could be developed around the terms we have used in relation to vibrato, but because there is an already existing vocabulary that describes FM theory as applied to radio broadcasting, and because this theory helps us to understand FM synthesis, we will avoid confusion by using these terms. Also, vibrato is a term that has a well-understood meaning in music, and to speak of vibrato rates and depths in *hundreds* of cycles is not truly consistent with that meaning. Therefore, rather than center frequency, vibrato rate and vibrato depth, we will from this point forward refer to (in the same order) **carrier frequency, modulating frequency** and **frequency deviation.** We can also establish symbols which will be a concise way of representing these concepts.

f_c = carrier frequency (center frequency)

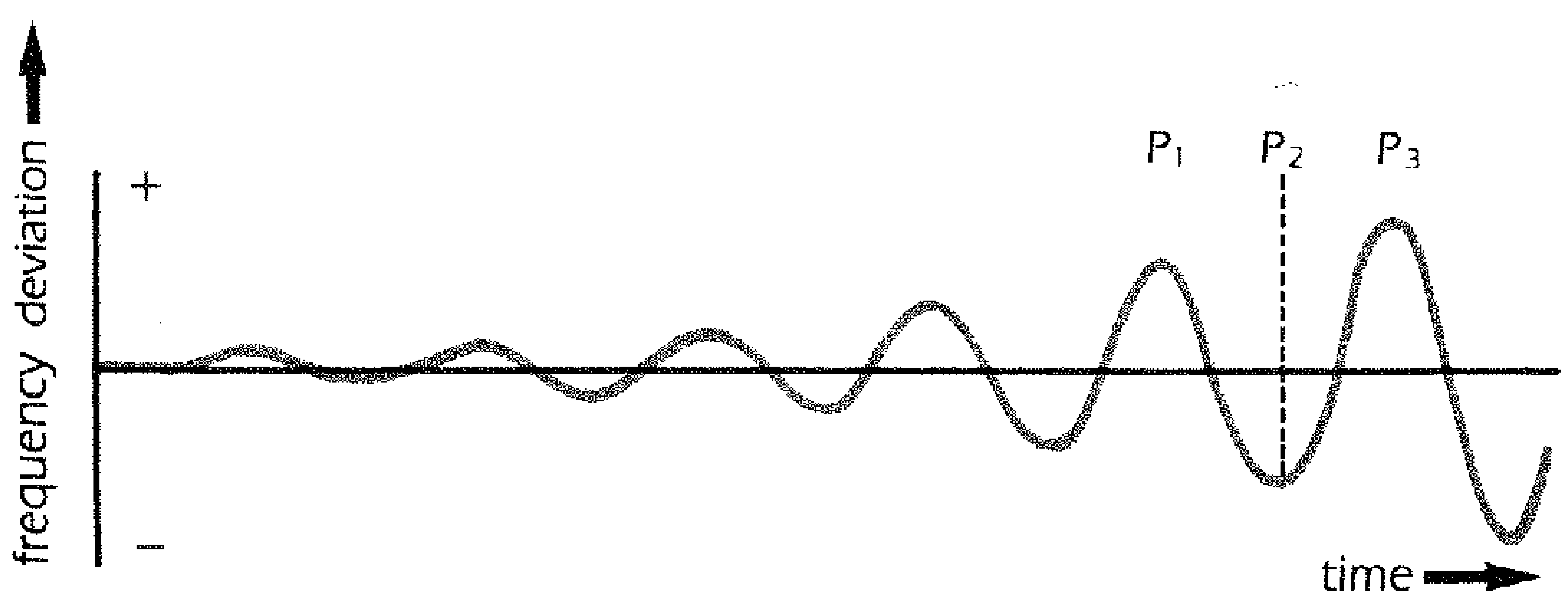
f_m = modulating frequency (vibrato rate)

Δf = frequency deviation (vibrato depth)

Because they are all frequencies, the letter f is used with the distinctions made by means of the additional characters; thus we would say f_c for the carrier frequency, f_m for the modulating frequency, and DELTA f for the frequency deviation. The Greek character delta (Δ) is used in mathematics as a symbol indicating differences, therefore in this case frequency deviation is the difference between the average (carrier frequency) and its maximum or peak deviation.

In the "X" Series synthesizers, operators that are carriers, whose frequencies are modulated by other operators, are always on the lowest line of the algorithm diagram. For example, in algorithm 1, operators 1 and 3 are carrier operators and operators 2 and 4 are modulating operators (as are operators 5 and 6 – but more about that in the chapter on complex FM). We already have a feeling for all of this from the previous experiment relative to vibrato. At this point it might be useful to do "X"-amples 3.1 and 3.2 again, remembering to think now of the new terms as shown in Fig. 3.2a below and 3.2b on the adjacent page.

a



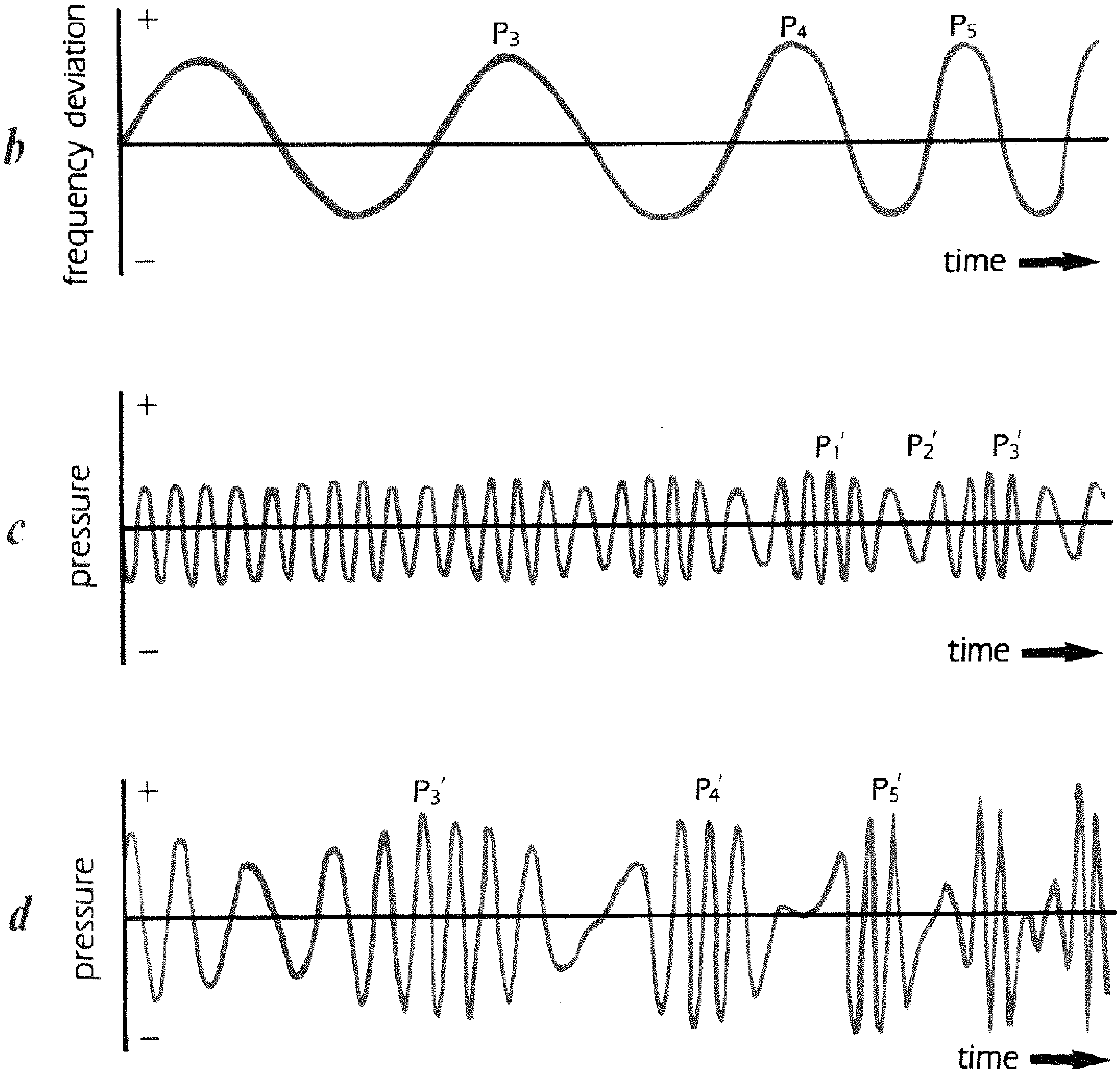


Fig. 3.2

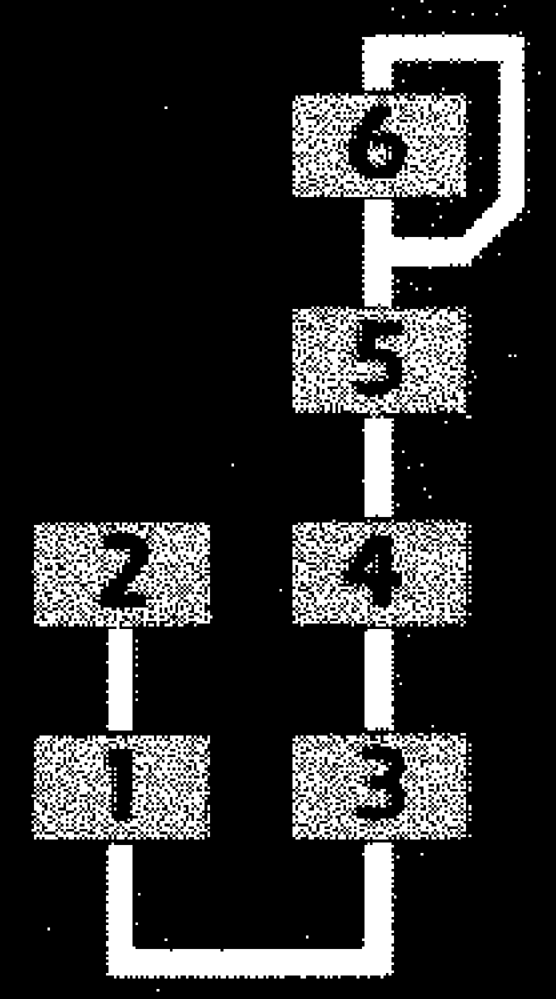
Here we show the plots from Fig. 3.1 but with the terms that are used in FM theory. It is important to remember that Figs. 3.2a and 3.2b are plots of frequency as a function of time, whereas in Figs. 3.2c and 3.2d we show their respective pressure wave versions. Notice that in "a", as the deviation increases from 0Hz while the modulating frequency remain$ constant, the period of the output wave in "c" begins to expand and contract in increasing amounts. That is, between points P1, P2 and P3, as the deviation (or instantaneous frequency) decreases and then increases again, the period of the output wave increases and then decreases accordingly as seen between P1', P2' and P3'. In Chapter 2 we pointed out that as the frequency increases the period decreases. In Fig. 3.2b, where the deviation is constant and the modulating frequency is increasing, points of maximum deviation P3, P4 and P5 are closer together, which is also seen in the pressure wave at P3', P4' and P5' where the shortest periods of the wave become closer together.

We now will continue with our empirical experiments, empirical because we are proceeding by observing with our ears what in the next chapter we will confirm by means of theory. These experiments are very important for gaining a practical understanding of FM.

In the next "X"-ample we will go through a number of carrier and modulating frequencies while maintaining a constant position of the Modulation Wheel. Set up the synthesizer as indicated below with operator 1 (the carrier operator) at $f_c = 100$ Hz and operator 2 (the modulating operator) at $f_m =$ 100Hz, then change the two as indicated. Sing the pitch of the very first (step 1) and ask yourself with every following example if the pitch has changed.

Remember that we have already determined in the previous example that if the carrier and modulating frequencies are the same, then the perceived pitch will be at that frequency as well. But in only the first step below are they the same. Do not

"X"-ample 3.4



OPERATOR	op 1	op 2	
OUTPUT	99	99	
AMS	0	3	
FREQUENCY	100Hz	100Hz	Step 1
	100Hz	199.5Hz	Step 2
	100Hz	302.0Hz	Step 3
	100Hz	398.1Hz	Step 4
	100Hz	501.2Hz	Step 5
	199.5Hz	100Hz	Step 6
	302.0Hz	100Hz	Step 7
	398.1Hz	100Hz	Step 8
	501.2Hz	100Hz	Step 9
	199.5Hz	302.0Hz	Step 10
	199.5Hz	501.2Hz	Step 11
	302.0Hz	199.5Hz	Step 12
	302.0Hz	398.1Hz	Step 13
	302.0Hz	501.2Hz	Step 14

INSTRUCTIONS: Starting from the VOICE INIT? position.....
Select algorithm 1 and set up the parameters as shown, with the first frequency values given, and the modulation wheel in the mid position. (Refer to the front of the chapter or previous examples for the function control settings.) Now go through the different frequency values by selecting Frequency Fine for each operator, and adjusting the data entry controls.

hold the key down as you make the change to the frequency (using the data entry slider), as you will soon forget the initial pitch. Also ignore the slow undulation or beating that occurs in some of the examples, as it will be explained in the next chapter.

Having done this exercise, the answer to the question "has the pitch changed for any of these pairs of frequencies?" is, quite evidently, "no". And while the frequencies change, the perceived pitch does not, even when neither the carrier nor the modulating frequency is at 100Hz as in the case of steps 10 — 14!

We might, however, want to qualify our answer by observing that the tone quality, or timbre, changes for some of the examples. That is an important observation and one we should bear in mind when we proceed into the next chapter.

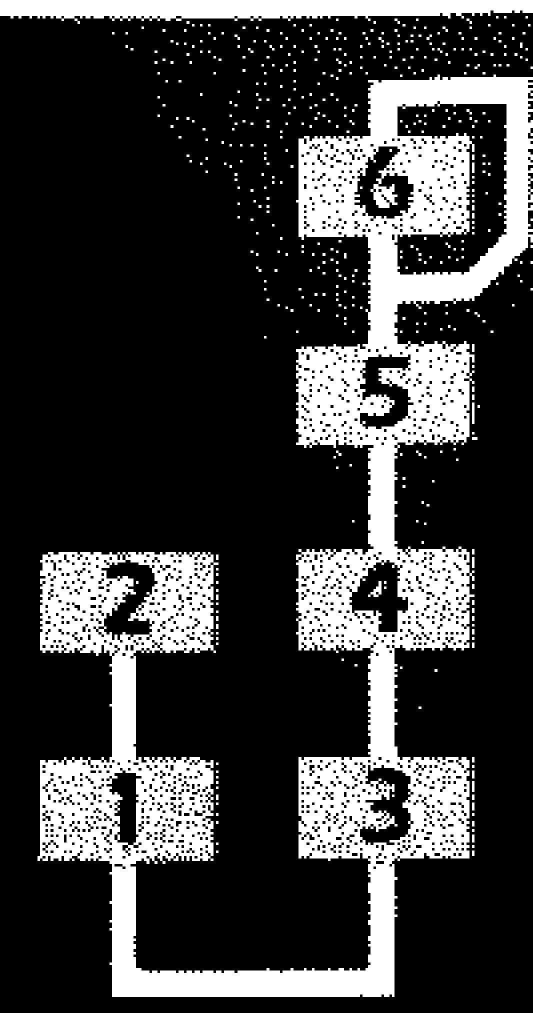
A little inspection of the frequencies chosen for "X"-ample 3.4 will reveal that they are not random but are as close as we can get (using the DX7 in this fixed frequency mode) to frequencies which are all integer multiples of 100Hz! In other words, we could form this list into a series of whole numbers which, when multiplied by 100Hz, produce exactly the required frequencies.

You will see the importance of this observation in the next chapter, where the term **frequency ratio** will be explained. But for now, let's stick to our empirical observations. Go through the process of "X"-ample 3.4 again, but this time in frequency ratio mode as shown in "X"-ample 3.5, rather than in the fixed frequency mode. (This exercise now becomes possible on "X" Series synthesizers which do not have the fixed frequency mode of the DX7).

The frequency produced by an operator is the result of the "pitch frequency" of the key being pressed multiplied by the value of the frequency ratio. Therefore, in Step 10 of "X"-ample 3.5, the frequency of operator 1 (the carrier) is 2 × 97.99 = 195.98Hz, and for operator 2 (the modulator) it is 3 × 97.99 = 293.97Hz, when key G1 is pressed. Because the ratio of the two is approximately the same as in Step 10 "X"-ample 3.4, the sound is nearly the same. Obviously, frequency ratio is a mode which is very useful on the "X" Series synthesizers for, now, the operators will track pitch as different keys are played, but will maintain the ratio between the frequencies of the two operators, thus keeping a consistent sound at different pitches.

Test yourself on recognising a similar tone or timbre at different pitches, and different tones at the same pitch, by repeating "X"-ample 3.5, but this time play a number of pitches and try different positions of the modulation wheel. Don't necessarily avoid the maximum position of the modulation wheel, but you *will* observe that it produces a very rich tone which may be difficult to evaluate aurally. Remember also, that when the wheel is at minimum, there is no output from the modulating operator and therefore the sound will be a sinusoid at a frequency determined by the carrier's ratio multipled by the frequency of the key being pressed. Of course if this ratio is not 1, then the pitch will not be that of the key pressed.

"X"-ample 3.5



OPERATOR	op 1	op 2	
OUTPUT	99	99	
AMS	0	3	
frequency ratio	1	1	(Step 1)
	1	2	(Step 2)
	1	3	(Step 3)
	1	4	(Step 4)
	1	5	(Step 5)
	2	1	(Step 6)
	3	1	(Step 7)
	4	1	(Step 8)
	5	1	(Step 9)
	2	3	(Step 10)
	2	5	(Step 11)
	3	2	(Step 12)
	3	4	(Step 13)
	3	5	(Step 14)

INSTRUCTIONS: Starting from the VOICE INIT? position.....

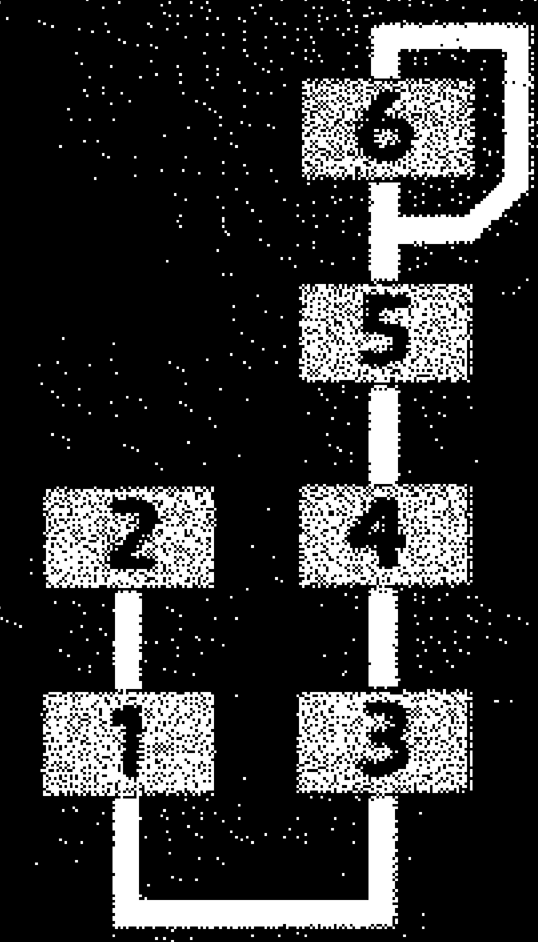
Select algorithm 1 and set up the values shown, then with the modulation wheel in the mid position, and starting from the first pair given, repeat the exercise of "X"-ample 3.4. But rather than press any key, press key G1 – the second G below middle C which has a frequency of 97.99Hz.

From the results of this exercise, you will probably have made some observations which relate to your analogue experience. The frequency pairs given in steps 2, 4, 12 and 13 all have the characteristic quality that is associated with a filtered square-wave. If we were told that a square wave is composed of odd numbered harmonics only, then we should rightly expect that the theory of FM would predict this in the case of the examples noted – otherwise, why worry about the theory when our ears allow us to make predictions and generalisations which complement our ability to hear inside the sound.

Before we move to the theory, however, let us continue with some experiments on the synthesizer from which we can gain useful additional information.

"X"-ample 3.6

OPERATOR	op 1	op 2
OUTPUT	99	99
AMS	0	3
FREQUENCY	100Hz	141.3Hz
	100Hz	162.2Hz
	100Hz	263.0Hz
	141.3Hz	100Hz
	162.2Hz	100Hz
	263.0Hz	100Hz



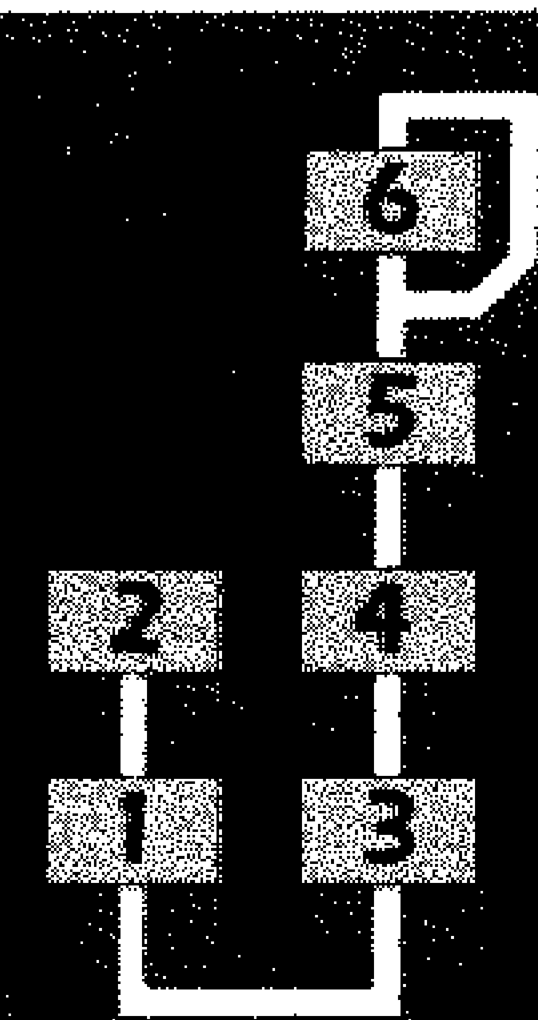
INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select algorithm 1 and set up the values shown, not forgetting to select Fixed Frequency mode, with EG BIAS = ON on the modulation wheel, as for all the other exercises in this chapter. Set the modulation wheel to the mid-point and listen to the tones produced by each of the pairs of frequencies.

You will notice that, whereas in "X"-ample 3.4 there was a strong sense of pitch for each of the pairs, in "X" - ample 3.6 the pitch is ambiguous — there seems to be a number of pitches in each tone, which don't fuse together. Repeat the exercise in the Frequency Ratio mode (non DX7 users can join in now) as set out in the following "X"-ample.

"X"-ample 3.7

OPERATOR	op 1	op 2
OUTPUT	99	99
AMS	0	3
FREQUENCY	1.00	1.41
	1.00	1.62
	1.00	2.64
	1.41	1.00
	1.62	1.00
	2.64	1.00



INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select algorithm 1 and set up the values shown, selecting Frequency Ratio mode, with EG BIAS = ON on the modulation wheel, as for all the other exercises in this chapter. Set the modulation wheel to the mid-point and, pressing key G1 (frequency = 97.99Hz), quickly note that the effect is the same as in "X"-ample 3.6.

Now, do the "X"-ample again but this time move the modulation wheel from minimum to mid-point for each pair of frequencies. Here, the effect is similar to that in X-ample 3.5 in that the complexity of richness of the tone increases, that is to say, the number of frequency components increase with the increasing output level of the modulating operator (operator 2), but in this case they seem not to be arranged in the harmonic series. That being so, the wave must be aperiodic.

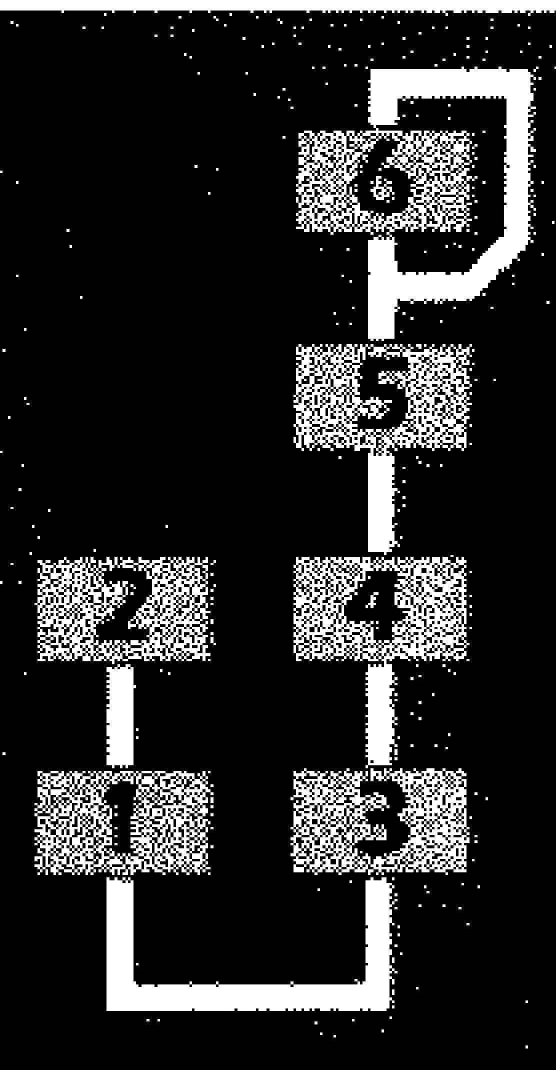
Repeat the "X"-ample a third time, but for each pair, instead of pressing G1, play a standard major scale in different keys. (Leave the modulation wheel at mid-point).

While we hear some features of the scale all right, the intervals do not have the same unambiguous quality that previous "X"-amples had, where the ratios between the frequency pairs were simpler.

While we are still doing experiments with the "X" Series synthesizer, let us just confirm one important point which you will have probably noticed from the previous "X"-amples. It is that the perceived fundamental pitch from a pair of operators, while it can be very unambiguous, is not necessarily the same as the carrier frequency. This of course takes us right away from the realm of vibrato, which is where we started this chapter.

"X"-ample 3.8

	FREQUENCY	OUTPUT	AMS
op 1	1000.0Hz	99	0
op 2	100.0Hz	99	3
op 3			
op 4			
op 5			
op 6			



INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select algorithm 1 and set up the values shown, then with Mod. Wheel EG BIAS = ON, move the wheel from min. to max. Clearly, the final pitch is ***not*** *that of the carrier frequency. (If you are not working with a DX7, use frequency ratio mode, op 1 = 10.00, op 2 = 1.00 and press key G1).*

What have we learned from the above experiments? Well, moving from the domain of vibrato into frequency modulation, we have learned that there are ratios of frequencies formed from simple integers (as in "X" -ample 3.5 and 3.8) which produce a periodic, rich tone, having in general a clear pitch. But, when the carrier and modulating operators have frequencies in complex, non-integer ratios, then an aperiodic tone is produced having ambiguous pitch. In both cases, the output level of the modulating operator, which relates to the frequency deviation (or vibrato depth, to refer to our original concept), determines the number of frequency components which are present in the sound.

As we have controlled this output by means of the modulation wheel, we have heard the richness or bandwidth of the spectrum change in time. An interesting idea presents itself when we consider that the output of an operator can be made to change automatically under the control of an envelope generator, rather than under the manual control of the modulation wheel. That is, the bandwidth of the spectrum can be made to change according to the slope of the envelope. We will get to that later, but let's remember that we can already see that possibility based only upon what we have done so far!

So, with an introduction to some mathematical jargon, a knowledge of some basic principles of acoustics and an aural concept of FM and its derivation from vibrato, we shall proceed to the theory.

CHAPTER 4

SIMPLE FM
The Theory

In this chapter, we shall be introducing some terms which will become our basic tools in predicting and working with FM spectra. Some of these you will be familiar with, but perhaps their real meaning and derivation are unclear. They are **I** for modulation index; and **c** and **m** for the frequency ratio of the carrier and modulator respectively —that is, the ratio of each operator to the frequency of the note played. A ratio of 2.00 therefore simply means that when A 440Hz is played, that operator has a frequency of 880Hz. You will be familiar with this from the exercises in the previous chapter. We shall also stop using one term from this point on, and that is frequency deviation or Δf, which, as you will see later, becomes incorporated automatically into the more familiar term "modulation index", — but more of that later. As you have learned, FM synthesis generates complex waveforms which have many frequency components — we are now going to study how, given certain information about modulation index, carrier frequency and modulator frequency, we can determine the exact position and amplitude of each of these components, when generated from a simple FM pair — in other words, how to calculate the spectrum.

In the last section we performed some experiments on the "X" Series synthesizer where we observed that, by maintaining a constant relationship between the carrier frequency, the modulating frequency, and the frequency deviation, we can move from frequency to frequency (and in this case from pitch to pitch) and maintain a constant spectrum which has more or less constant sound quality.

That is, if f_c = 440Hz (carrier frequency)

f_m = 110Hz (modulating frequency), and

Δf = 220Hz (frequency deviation)

as in "X"-ample 3.5 step 8 (playing note A440Hz), then a sine wave will change continuously in frequency at a rate of 110 cycles per second (modulating frequency), by an amount of 220 cycles per second (frequency deviation), above and below an average frequency of 440 cycles per second (carrier frequency). Should we want to make approximately the same sound an octave lower, we observed that by multiplying each of the three frequencies by ½, we could do so. Now, there is a special relationship between the modulating frequency f_m and the frequency deviation Δf, so special that the ratio between the two is given a special name — the **modulation index.** From this point on, then, we will designate the letter I to mean modulation index. Therefore:-

$$I = \frac{\Delta f}{f_m}$$

Equation 4.1

Reads — the modulation index, I, is equal to the frequency deviation, *delta f*, divided by the modulating frequency, f_m.

You can see that now, if we want to transpose the pitch of the above example down by one octave while maintaining a constant spectrum, this can be done by simply multiplying the carrier and modulator frequency by ½ and holding I constant. The frequency deviation, Δf, can be calculated from a simple transposition of Eq. 4.1. That is:-

$$\Delta f = I \times f_m$$

Equation 4.2

Reads – the frequency deviation, ***delta f***, is equal to the modulation index, ***I***, times the modulating frequency, ***f sub m***.

The perceived depth of the modulation is determined by the ratio of Δf (the frequency deviation above and below the carrier frequency) to f_m (the rate of modulation). This is analogous to what happens with vibrato, though not exactly the same – perceived vibrato depth is determined by the ratio of Δf to f_c (the center frequency), and f_m is heard as an independent quality: vibrato rate. For example, if we have a pitch at 100Hz having a vibrato depth of 5Hz (a little less than a semi-tone at this pitch) and want to transpose the same auditory effect three octaves higher to 800Hz, then the vibrato depth must also be multiplied by 8, to plus and minus 40Hz, to produce the same perceived semi-tone variation.

In FM synthesis, then, where we are beginning to see there is a special and useful link between the carrier and modulating frequencies, we will also link the deviation in the same way. Whenever we change the carrier and modulating frequencies by a given factor, the deviation must also change by the same factor in order to maintain a constant number of frequency components in the spectrum. The modulation index I, then, has to do with the number and amplitudes of frequency components in the spectrum for any given ratio of carrier to modulator – in other words, I determines the bandwidth, ("richness" or frequency space) occupied by the spectrum.

In the "X"-Series synthesizers we do not see this important term, modulation index, mentioned anywhere but that is because operators can be configured differently. An operator can act as a carrier in one algorithm and a modulator in another. And in order to make things simple by having one parameter for any operator whatever its position, Yamaha has very cleverly arranged things so that the output level of an operator, when used as a modulator, governs index. Modulation index as a function of output level is shown in the graph overleaf, Fig. 4.1.

Now we can explain something that you might have noticed in "X"-amples 3.1b and 3.2 of the previous chapter. In these experiments we changed the modulating frequency (vibrato rate) continuously while letting the frequency deviation (vibrato depth) remain constant. But was the depth constant? No, in fact, for as the modulating

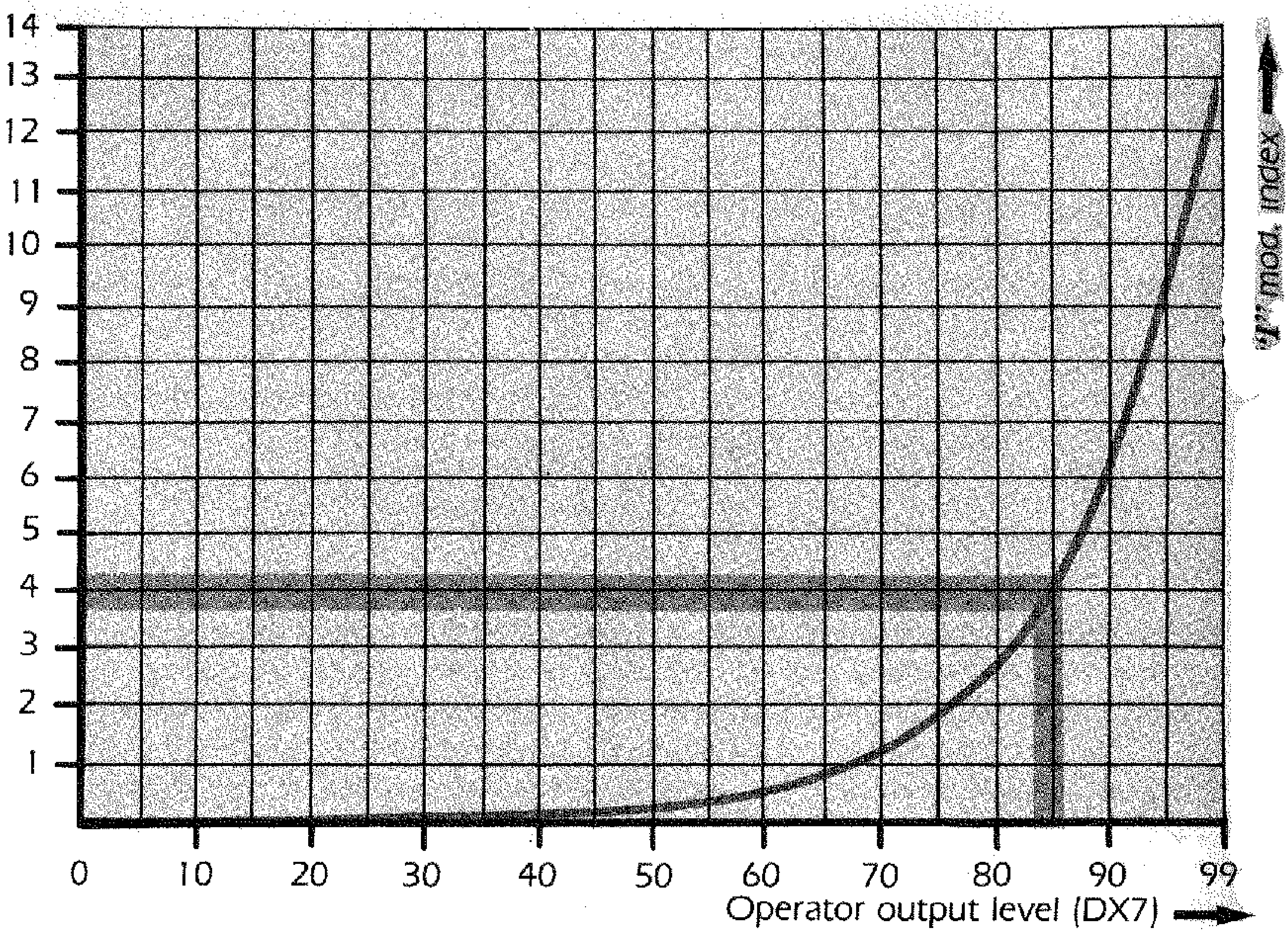


Fig. 4.1

Here we see a plot of modulation index *I* as a function of the output of an operator. When an operator is used as a modulator we can interpret its output as index according to the above graph. For example, a maximum output of 99 can be interpreted as an index of approximately 13, or an output of 85 is equal to an index of about 4, and an output of less than 50 is of negligible index (when considering our mathematical calculations). This does not necessarily mean that you will not be able to hear the effect under certain circumstances – do not under-estimate the sensitivity of your ears!

frequency increased so did the deviation. Listen to "X"-ample 3.1b again (because the range is much lower, it is easier to hear than in "X"-ample 3.2) and confirm for yourself that with the mod. wheel at about mid-point we hear a change in pitch of about one half step above and below 97.72Hz (G1) when the modulating frequency is 1.000Hz.

When the modulating frequency is increased to 9.772Hz, it is difficult to hear the lower limit, but the upper limit of the deviation is quite clearly a minor 6th. The reason for this change then (that the deviation is not constant), is that the *actual* output of an operator, when used as a modulator, changes as a function of its frequency, even though the LCD output level reading is constant. Output level, then, is related to modulation index.

In order to understand why we think in terms of frequency ratios rather than frequency in Hz, we will take another example. . . . . .

f_c = 220Hz

f_m = 440Hz, and

I = 4 or

$$\frac{\Delta f}{f_m} = \frac{1760}{440} \quad \text{or} \quad \frac{\text{frequency deviation}}{\text{modulating frequency}}$$

where we can change those frequencies by a common factor, for example, times 2 (up an octave), times ½ (down an octave), times 1.059 (up a semitone), etc. and maintain a similar quality — simply transposed!*

Let's look at these frequencies which result from multiplying by these factors.

	f_c	f_m	I
	220	440	4
(times 2.0)	440	880	4
(times 0.5)	110	220	4
(times 1.059)	232.98	465.96	4

Table 4.1

We observe that the ratio of the carrier to the modulating frequency is always 1 : 2 and that I is always 4 (**remember that the deviation is automatically computed for us from I and f_m so we need no longer concern ourselves with it**). What we hear is a sound which, at all of these frequencies, is "reedy". Therefore, we might guess that if we maintain a constant ratio of carrier to modulator and a constant "index", then we produce a result where the spectral form is exactly transposed with pitch. Indeed this is true. Rather than think in terms of *frequencies*, then, we could better think in terms of *ratios* of frequencies, which will allow us to avoid dealing with numbers whose ratios may be simple but not obvious.

---

* *In fact, the sound may not sound the same — exactly — because of the way that the ear responds to different frequencies. However, as we shall see, except for frequency the physical properties of the sound remain unchanged.*

While it is easy to see that 220 and 440 are in a ratio of 1 : 2, given 232.98 and 465.96, as in the previous example, it is not so obvious, and with other useful ratios such as 2 : 5 the frequency relationship would be even more obscure. So, it is with ratios of frequencies that we will develop our understanding of FM synthesis.*

To be clear, whenever we speak of a ratio of a carrier to a modulating frequency we will use ***c*** and ***m*** (which will often be integers, but not always). For example, in the case above we can simplify it greatly by expressing the relationships as:-

$$c{:}m = 1{:}2 \text{ and } I = 4$$

. . . . . which is all we need to describe the spectrum at any given pitch.

Now, we have discussed our basic terms, so it is time to start putting them to use. The following rule is fundamental to FM synthesis and should be memorised – it will tell us what are the potential**frequency components given a ratio of ***c to m.***

RULE 1

Frequency components resulting from values of *c* and *m* will follow a pattern according to:

$$c \pm km$$

for $k = 0, 1, 2, 3 \ldots n$

(**for** $k = 0$, **then** 1, **then** 2, **then** $3 \ldots n$)

This means that there will be a frequency component at ***c*** and components at ***c*** plus and minus whole number (integer) multiples of ***m***. As the amplitude of a component may be zero, or insignificant depending on the index, as will be shown later, this expression represents the frequency potential for simple FM. Let's take the case of ***c : m = 1 : 2*** and go through the expression, filling in some terms which describe the frequency components.

---

* *If you are mathematically minded and for any reason you need to work with frequencies in Hertz, then simply multiply the ratios for* ***c*** and ***m*** by the frequency of the note played. A table of frequency values is given in appendix 4.

** *The value of* ***I*** will decide whether or not they exist, or have enough energy to be heard or influence the timbre – more of that on the following pages!

step 1	*let k = 0*	
step 2	$1 \pm (0 \times 2) = 1$	$(c \pm (0 \times m)) = c$ the "carrier"*
step 3	*let k = 1*	
step 4	$1 + (1 \times 2) = 3$	$(c + (1 \times m))$ "1st order, upper side band"
step 5	$1 - (1 \times 2) = -1$	$(c - (1 \times m))$ "1st order, lower side band"
step 6	*let k = 2*	
step 7	$1 + (2 \times 2) = 5$	$(c + (2 \times m))$ "2nd order, upper side band"
step 8	$1 - (2 \times 2) = -3$	$(c - (2 \times m))$ "2nd order, lower side band"
	..... etc.	

Table 4.2

It is because the components fall on either "side" of the carrier they are called *upper* and *lower* side frequencies. The group of frequencies on either side of the carrier resulting from any one value of k are therefore referred to as the *upper and lower* side bands. We can refer to a specific side band component by what is called its *order*, which corresponds to the value of k, and"upper"or"lower"depending upon whether it is a result of $c + m$ (upper) or $c - m$ (lower). The "2nd order lower side frequency", for example (for the case above it would be –3), may seem a bit cumbersome, but because they are terms widely used in engineering we are likely to hear them in the context of FM synthesis. So we may just as well know what they mean.

In Fig. 4.2 on the next page we see a frequency line which has a positive and negative part around 0 Hz. On this horizontal line are vertical lines at the frequencies which we have just calculated. For now, we are only interested in finding out where the frequency components will fall, not their energy, so for the moment we make all of the amplitudes – represented by the height of the lines – the same.

In this particular Figure the lines representing the frequency components are rather exaggerated, but it is important at this stage that you have a clear image when we talk about "reflected" components. This is the first use of a line spectrum in our FM study. Notice that we have left space in all quadrants of the grafh, including the negative regions, and that the frequency scale is linear.

The reason for the regions of negative frequency and negative amplitude will gradually become clear. At this moment we are interested in where the frequency components fall according to our rule.

*Although there is a "±" from Rule 1, the carrier, c is only computed once, as will be seen in Table 4.2.

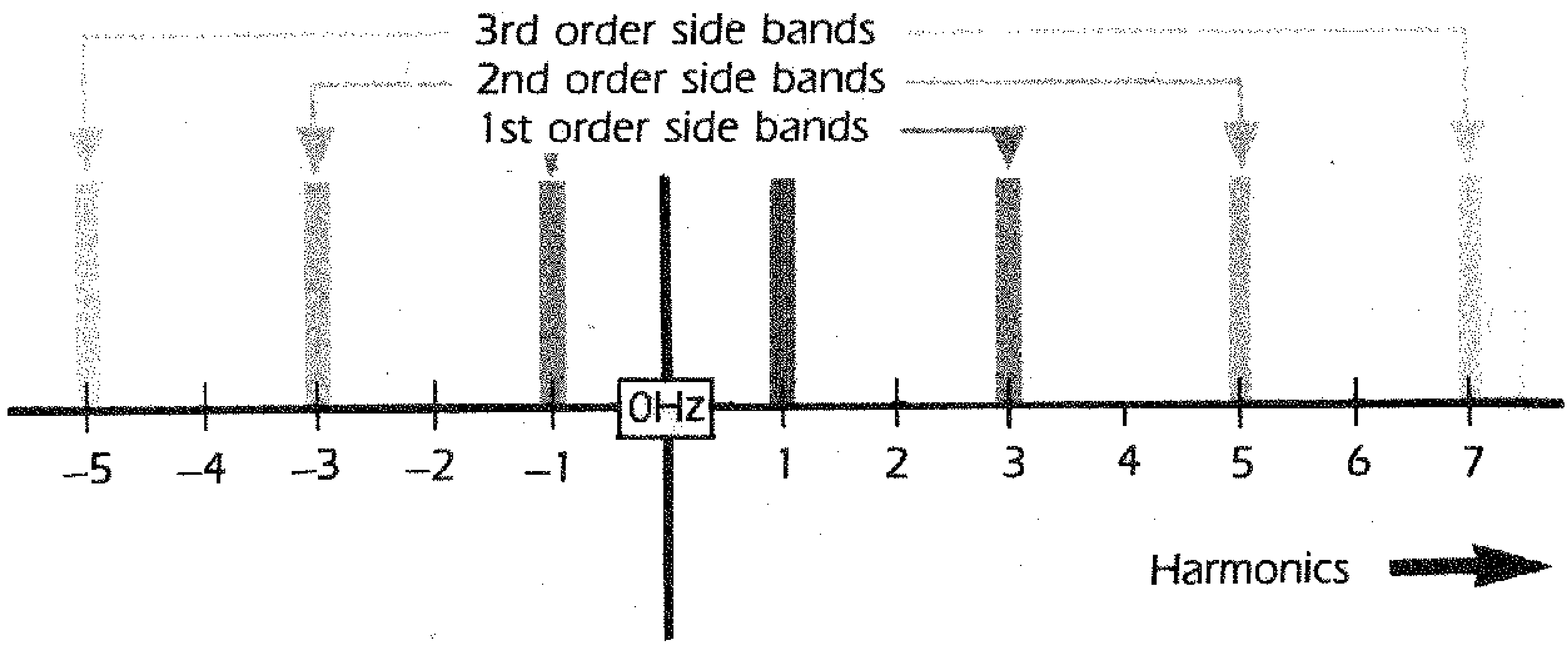


Fig. 4.2a

A plot of the frequency spectrum for $c : m = 1 : 2$ showing where the components fall on either side of 0Hz. For convenience we make all of their amplitudes equal, which, in fact, is never the case with FM. To the right of the carrier frequency (at 1) are the upper side band components and to the left are the lower side band components.

So what do we do about the components which have a negative sign (from steps 5 and 8 in Table 4.1, and there would be more if we were to continue with increasing values for k)? And how many components will there finally be, i.e. how large does k become, which relates to the bandwidth? For now, in answer to the first question, we will simply forget the negative sign and think of those components as positive. Therefore, in the case above there are two components at harmonics 1 and 3 – one original positive component and one "reflected" component which from the formula was negative but which we have, for the moment, considered as positive. Fig. 4.2b shows this imagined manoeuvre.

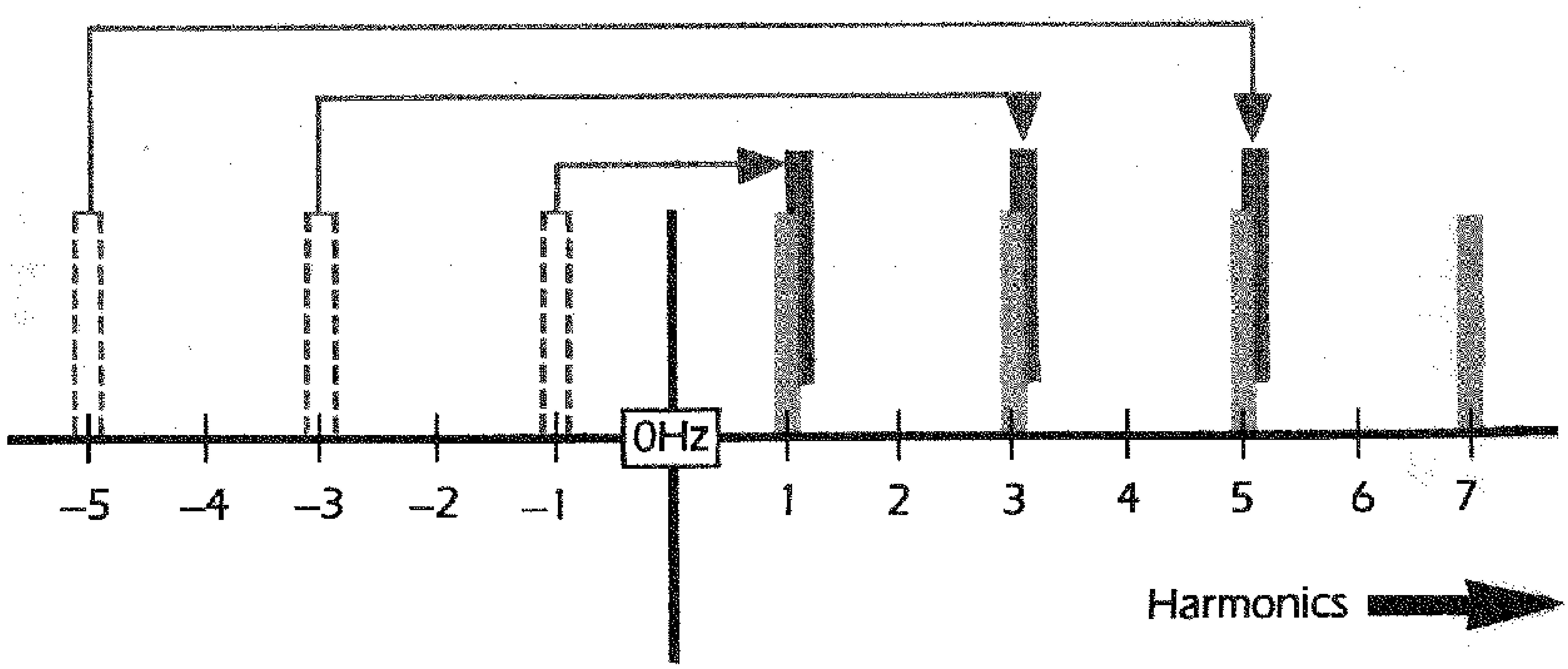


Fig. 4.2b

This shows how we have made negative frequency components positive by reflecting them around 0Hz.

Notice too, that they are all odd numbered harmonics. There is no component at 2, 4, 6 and so on, even when we consider the reflections, which is why we would hear a "hollow" sound rather like the filtered square wave we mentioned in chapter 3. The other question was: how large do we let k become? The answer to that is contained in our next rule.

RULE 2

To determine the number of side bands that should be calculated for an FM pair with a modulation index of I, add 2 to the value of the index I and that's about how big k should become!

So, for an index of I, $c \pm km$
for
$k = 0,1,2,3 \ldots n$
where
n approx. $= I + 2$

This tells us that, for example, with an index of 4, we need to calculate the positions of the first six pairs of side bands, and that the amplitudes of the seventh, eighth and so on are negligible. As we noted before, the bandwidth of the sound (or the frequency space which the sound occupies) is dependent upon the index, but now with the above we are able to be a bit more precise.

However, we still need to know what are the relative amplitudes of the components, i.e. are they all equal, or do they fall off 6dB per octave as they do in a square wave? No, they do not behave in either way and here we will explain a bit about some curious curves called Bessel functions which have been hiding in the background. They are curious because we do not have to know about them to build an FM synthesizer (midst all that is stored in an "X"-Series synthesizer, including the sine function, voice data, etc., Bessel functions are nowhere present), but we do have to know a bit about them in order to understand exactly why the synthesizer does what it does. One of the unique aspects of FM synthesis is that, while it is relatively simple to implement, it takes a bit of thinking to understand. Additive synthesis, on the other hand, is perfectly straightforward to understand but prohibitively expensive to implement in any useful form (it's not nearly as prone to "serendipitous* accidents" either).

** Just in case, here's a definition of serendipitous ... Characterised by serendipity, which is the gift of finding something valuable or agreeable not sought for (possessed by the heros of the fairy tale "the Three Princes of Serendip"). Often in voicing "X" Series synthesizers one finds the essence of some sound which is not the sound one is looking for. In fact some wonderful timbres have been discovered by simply taking the voice data intended for one algorithm and applying it to another . . . that is serendipitous accident. However, remember that recognising that such a sound is potentially good depends on your "good ears" and careful listening!*

So, below you can see the first six of an infinite number of Bessel functions.

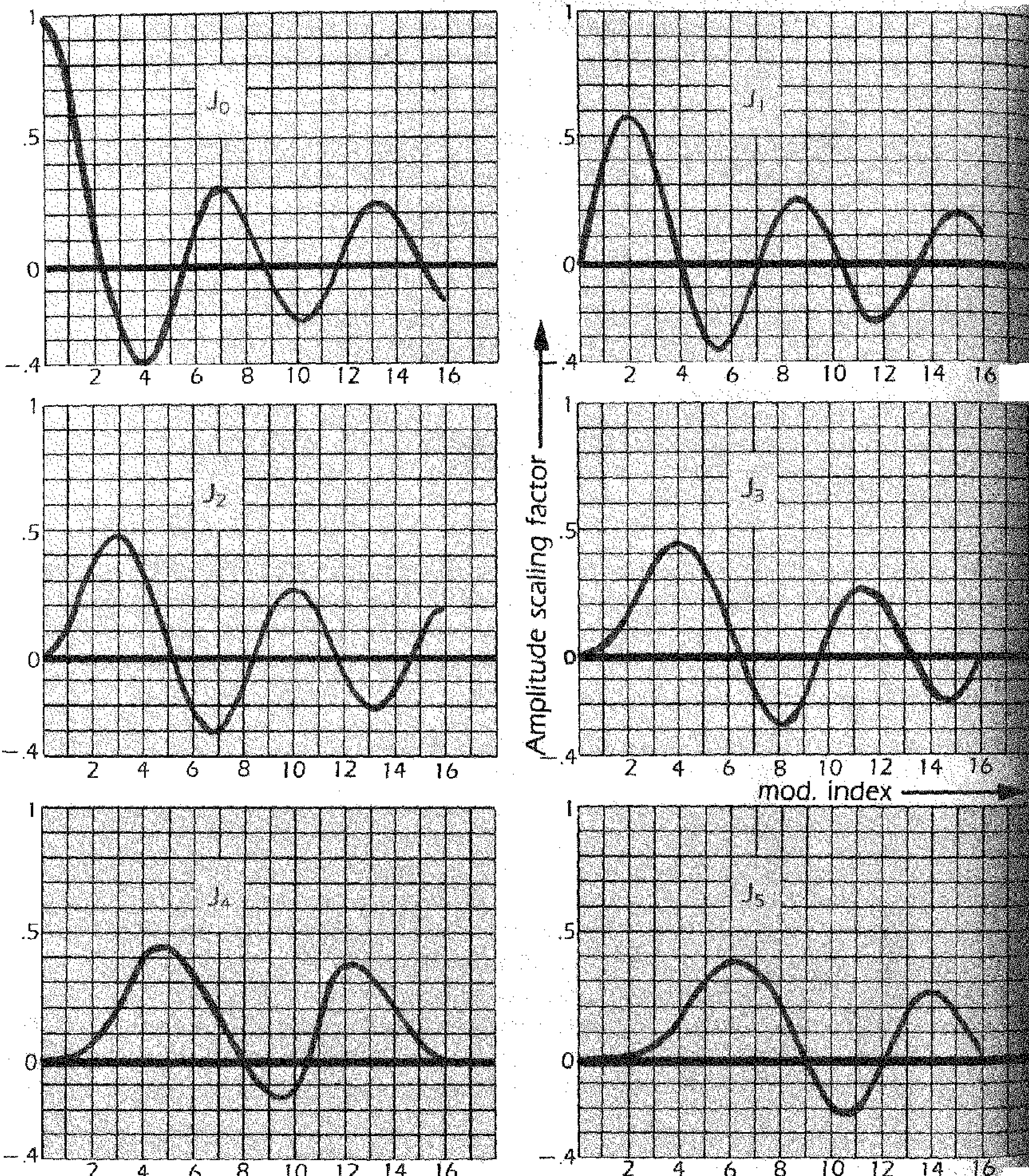


Fig. 4.3

The first six Bessel functions, J_0 through J_5. The modulation index is on the horizontal axis of each graph and the amplitude on the vertical axis. The intersection of the Bessel function with a value for the index yields the amplitude scaling factor for the appropriate side band components. Each J corresponds directly to a k order side band. (Accurate tables and graphs for the Bessel functions J_0 to J_{15} can be found in Appendix 3; use these for any calculations. The diagrams shown here are merely to represent the general form of the curves).

How do they relate to all of this? It is from the Bessel functions and modulation index that we can determine the relative amplitudes of all of the frequency components resulting from a given ratio of ***c : m***. Looking at these functions, we can see that along the horizontal axis is "index" and along the vertical axis is a scale ranging from –0.4 to + 1. This is a scaling factor for amplitude. The actual amplitude of the frequency components will of course be determined by how loud you set your amplifier — the scaling factor allows us to determine the relative amplitude of each component. We also note that each of the Bessel functions is named J_0, J_1, etc., each of these corresponding to our ***k*** values from $c \pm km$. For every order of upper and lower side frequency (that is, of order ***k***), there is a Bessel function J_k. It is referred to as order ***k*** to relate the amplitude scaling factor or coefficient for the upper and lower ***kth*** side frequencies to modulation index.

For example, if we are interested to know the amplitude of the second order side bands, whose position will be determined by $c \pm 2m$, then we must refer to Bessel function J_2, and read off the amplitude scaling factor according to the value of the modulation index. Or in another example, the 0th order Bessel function, J_0 (a glance at the original formula will show that when $k = 0$, *we are looking at the carrier frequency alone*), *with an index* $I \doteq 2.4$,* indicates that the carrier frequency will have an amplitude of 0. If, on the other hand, the index were $I \doteq 1$ then the carrier frequency would have an amplitude of 0.8. (This may sound a bit dense, but we will go through some examples which, with some re-readings of the above, will become clear.)

We add then to our previous formula:

$$c \pm km$$

the extra expression for amplitude, by consulting the appropriate Bessel function:

$J_k\ (I)$
for
$k = 0,1,2,3 \ldots n$
where
$n \doteq I + 2$

Now, as we advance ***k*** from ***0*** to ***n*** to provide the side band frequency ratios, we also use that value of ***k*** to determine the Bessel function from which we get the amplitudes. But before we can get on and try some calculations, we have to live with one small mathematical complication and that is the provision that **odd order lower side frequencies have a negative sign.** Therefore, when ***k*** is odd ***(1,3,5 ...)*** the lower amplitude scaling factor or "Bessel coefficient" is multiplied by ***–1***.**

---

* *The symbol* $\doteq$ *is read " is approximately equal to."*

** *Here is a reminder of some elementary arithmetic. When two negative signs are multiplied together the result is positive, i.e.,)* $-1 \times -1 = +1$. *Two like signs make a positive; two unlike signs make a negative.*

Noting that the Bessel functions themselves are sometimes positive and sometimes negative, depending on the value of index I (the coefficient for J_1 when $I = 4.1$ is a little less than −0.1, for example), what does it mean to have a negative frequency having a negative Bessel coefficient which is, in addition, negative because it is of odd order? First, we will form the previous relation as a table, which makes the problem a little easier to visualise.

Spectral Components for Simple FM			
AMPLITUDE	FREQUENCY		
Amplitude Coefficients	Side Frequencies		
(scaling)	odd order	Lower	Upper
$J_0\ (I)$		c(carrier)	
$J_1\ (I)$	$\times\ (-1)$	$c-m$	$c+m$
$J_2\ (I)$		$c-2m$	$c+2m$
$J_3\ (I)$	$\times\ (-1)$	$c-3m$	$c+3m$
$J_4\ (I)$		$c-4m$	$c+4m$
...	...	...	...
$J_k\ (I)$		$c-km$	$c+km$

Table 4.3

The above table, together with the Bessel functions, is all that we need in order to plot the spectrum of any ratio of $c:m$ at any value of index, I. First, however, we must resolve the question regarding negative frequencies and negative amplitudes, because finally in order to be able to interpret and understand the spectrum, we want to see all of the frequency components with their amplitudes represented in the positive domain. So:

Make the negative frequency component positive and change the sign of its amplitude. That is, if the ratio of $c:m = 1:2$ and $I = 4$, then the first-order, lower side band will fall at a frequency of −1 (the result of $c-m$ or $1-2$) with an amplitude of approximately −0.07 from the Bessel function $J_1\ (4) = -0.07$ (where the modulation index $I = 4$ intersects with the 1st order Bessel function), as shown in Fig. 4.4a on the next page. However, because it is odd order and a lower side frequency, it is multiplied by −1, changing it to +0.07, as shown in Fig. 4.4b, below. Following the rule, then, this component is placed in the positive domain with a change of sign as shown in Fig. 4.4c. (Finally, we will want all components to have a positive amplitude sign in addition to a positive frequency, but one step at a time!).

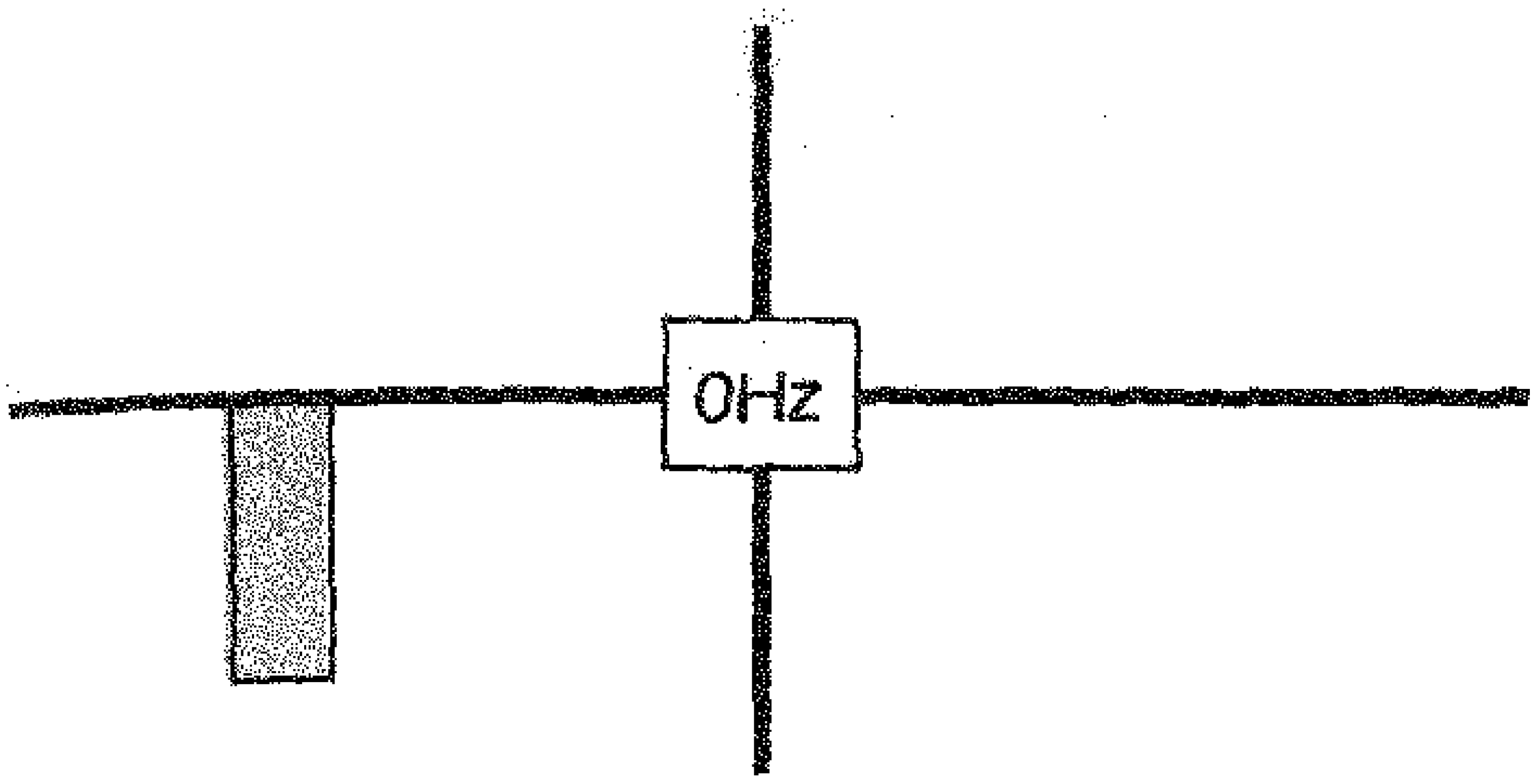


Fig. 4.4a

For $c : m = 1 : 2$ and $I = 4$, $c - m$ is at a frequency of -1 with an amplitude of $J_1(4) = -0.07$

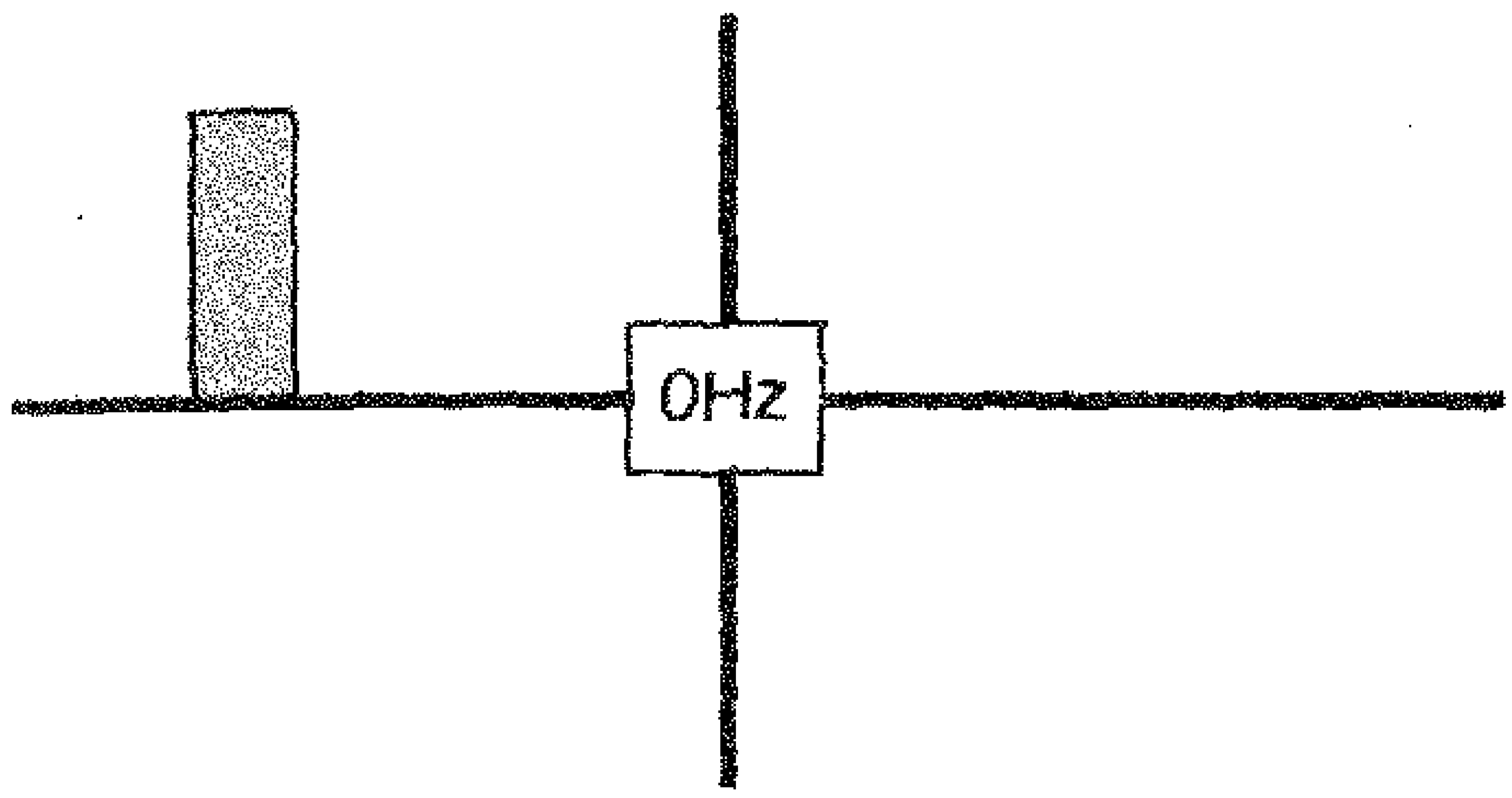


Fig. 4.4b

Because it is an odd order and lower side frequency it is multiplied by -1. (change its sign)

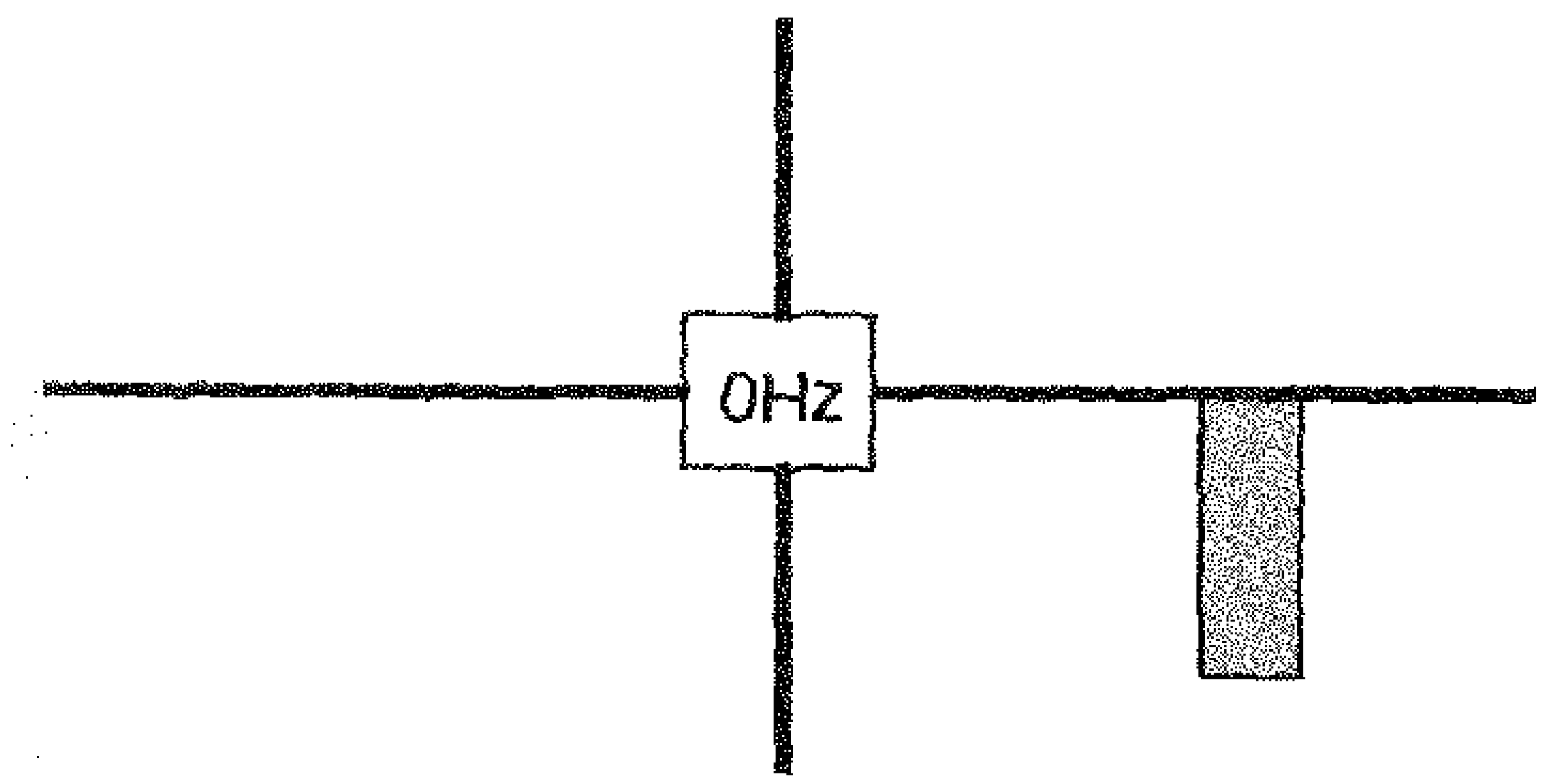


Fig. 4.4c

The component is placed in the positive frequency domain with yet another change of sign

The order in which we apply these operations is not important. For example, the odd order lower rule could have been applied to the result of the $c - m$ operation, resulting in a frequency in the positive domain at the outset. For reasons of consistency, however, we will first plot all negative frequencies in the negative frequency domain.

We could simply stop here and accept the mathematical rules given. However, we can quite simply understand why we make a negative component positive and change its sign because of a property of waves and phase, which we have already discussed in Chapter 2.

There is only one phase relationship with which we need be concerned here, and that is of two sinusoids whose phase differs by 180 degrees (180°). Fig. 4.5 shows a sinusoid having an amplitude of 1. By looking at one period of this sine wave at two different points (phases) in its cycle we get two different shapes, both sine but one with a phase difference of 180°.

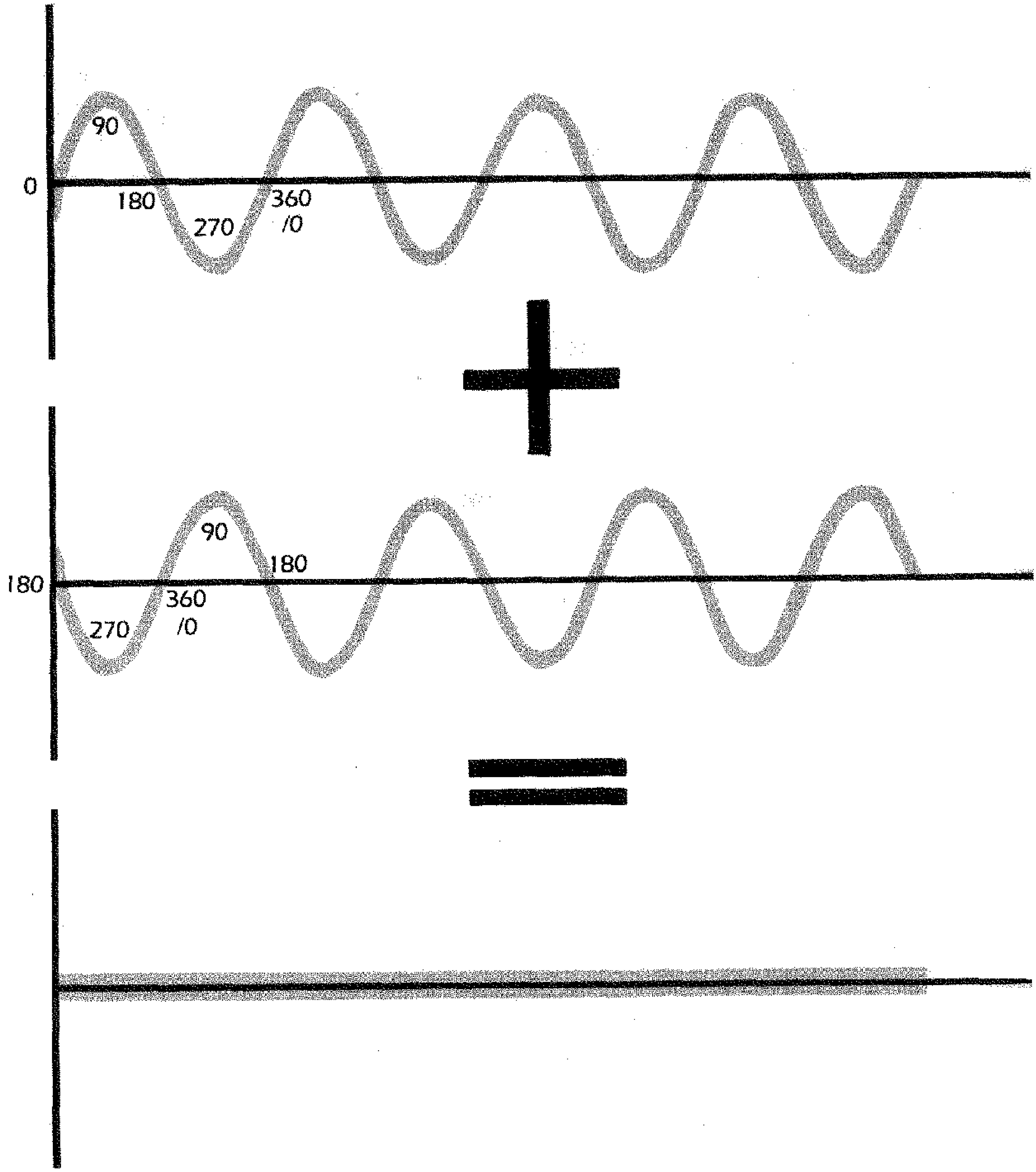


Fig. 4.5

From a sinusoidal wave two different shapes are extracted, one a sine wave, and the other a sine wave + 180°. If these two were added, they would cancel.

We can express this relationship in another way however, as −sine, i.e. if a sine wave is multiplied by −1 the result is the same as sine + 180°, as shown in Fig. 4.6.

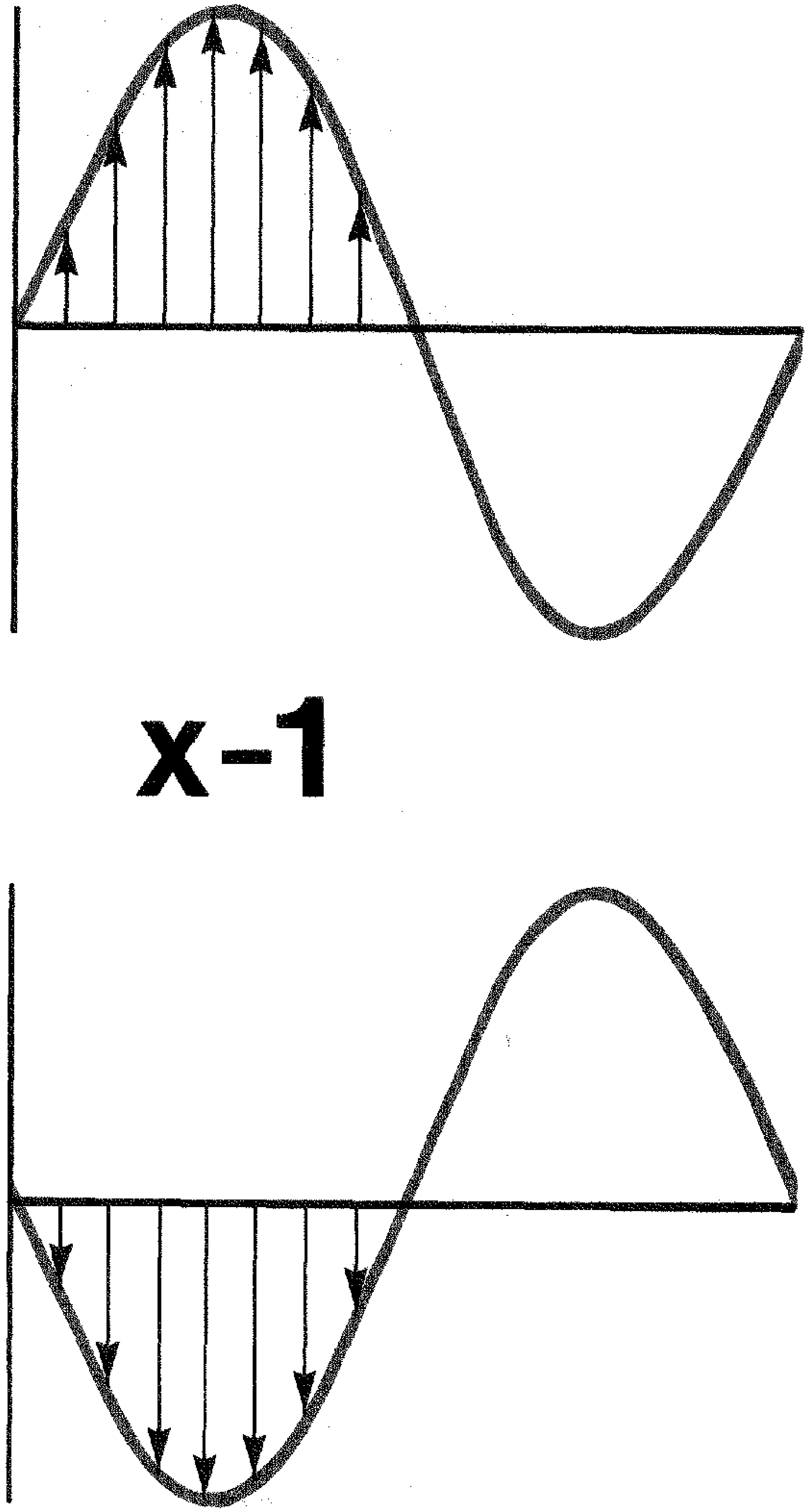


Fig. 4.6.

If every point of a sine wave is multiplied by –1 it is the same as sine ± 180°.

And finally there is yet one more way to express this relationship. This way is important because it allows us to see, very simply, what is meant by a "negative frequency". We will imagine a wheel which turns freely counter-clockwise on an axle, Fig. 4.7a. The wheel turns at a constant rate of 100 cycles every second, or at a frequency of 100 cps (Hz). Now, there is a nail in the rim of the wheel whose height above and below the axle, as the wheel turns through 360°, is shown in Fig. 4.7b. (You will notice that this looks like a sine wave, which in fact it is, because our wheel

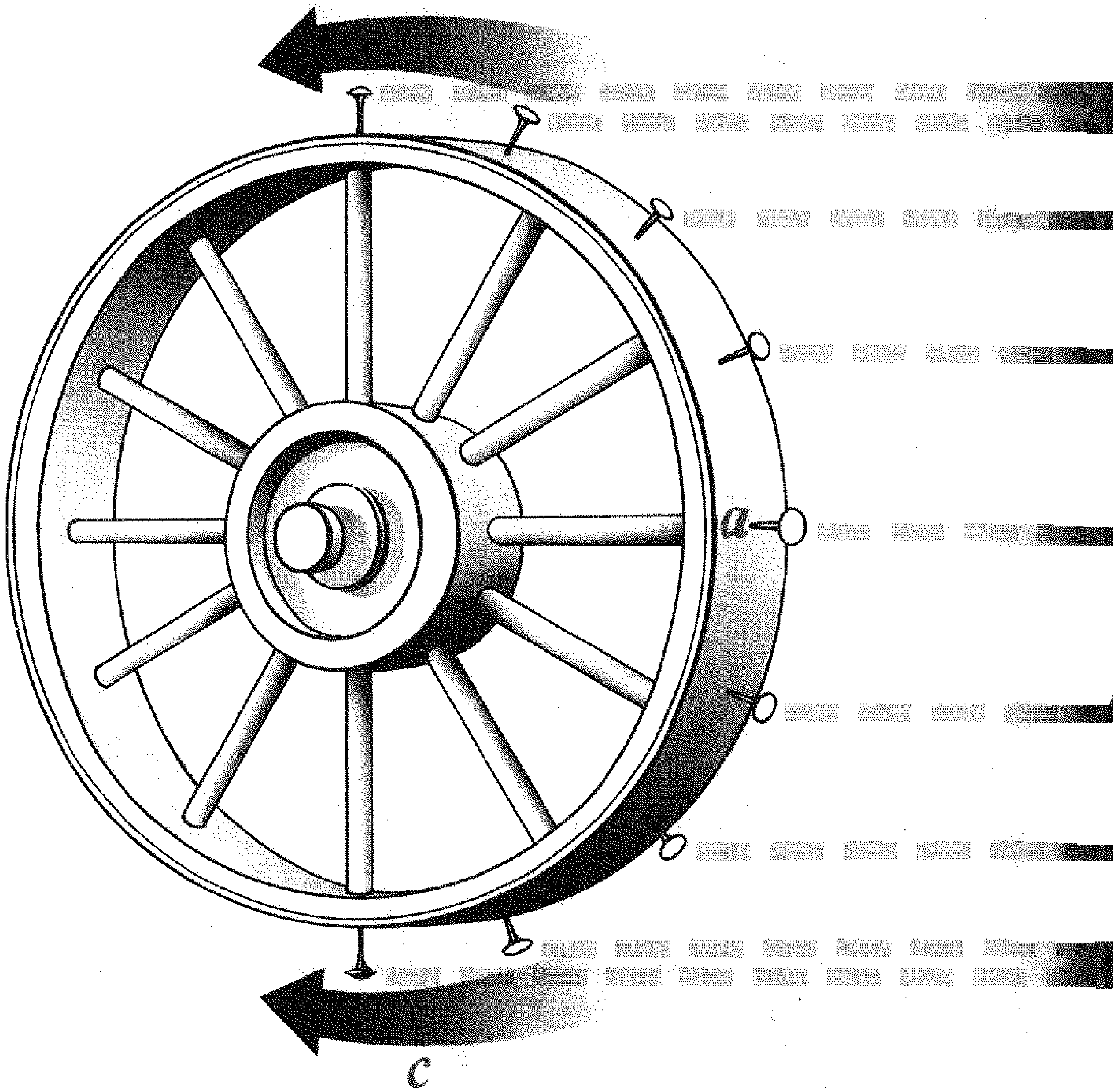


Fig. 4.7a

A wheel rotates at 100 cps in the direction shown by the top arrow. There is a mark on the rim at 0°.

Fig. 4.7b

Graph of height of mark above and below the axle as the wheel turns through 360° in $^1/_{100}$ sec.

is the "unit circle" of chapter 2.) The duration of one period of the wave is 1/100 sec. If we decrease the frequency of rotation to 50 cps then we would lengthen the period. If we decrease the frequency still more to 0 cps then we would obviously have no change in height of the mark relative to the axle. And now the point . . . . what happens if we have a frequency of −100 cps? The wheel rotates in the opposite direction with the resulting change in the phase of the plot, as seen in Figs. 4.7c-d.

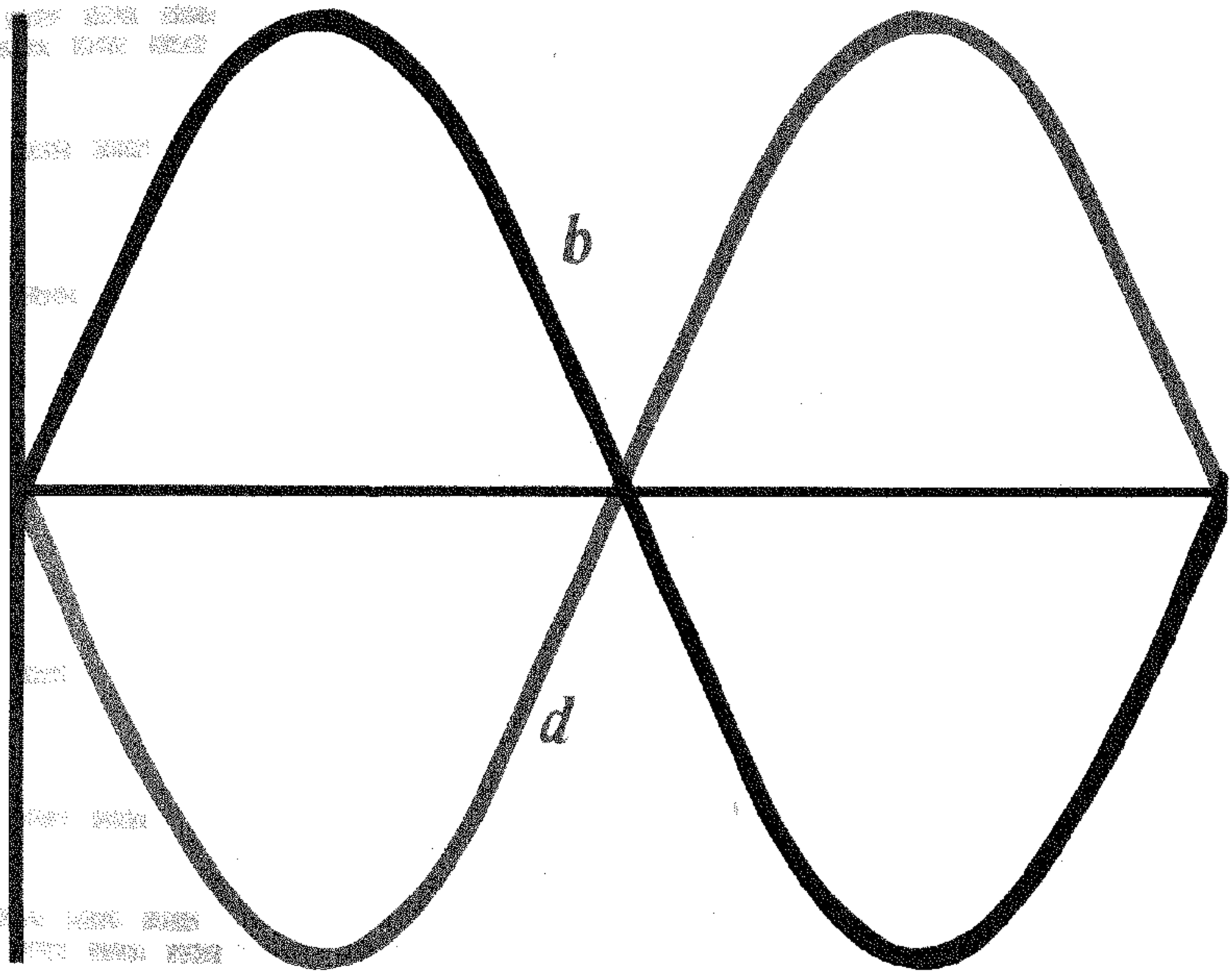


Fig. 4.7c

The wheel rotates at −100 cps (in the opposite direction, thus at a negative frequency.

Fig. 4.7d

Graph of height of mark relative to the axle as the wheel rotates in opposite direction showing a change of phase.

This, then, is negative frequency. We can think of negative frequency in terms of a change in rotational direction which is equivalent to a change in phase of $\pm 180°$ *or* a sine wave times -1; they both amount to the same thing.

Finally, we must be sure that we understand how to deal with the "negative" amplitudes. In fact it is very simple – we just make a normal algebraic addition of the values given, taking account of their signs. In Fig. 4.8 we can see the addition of four different pairs of sine waves represented graphically.

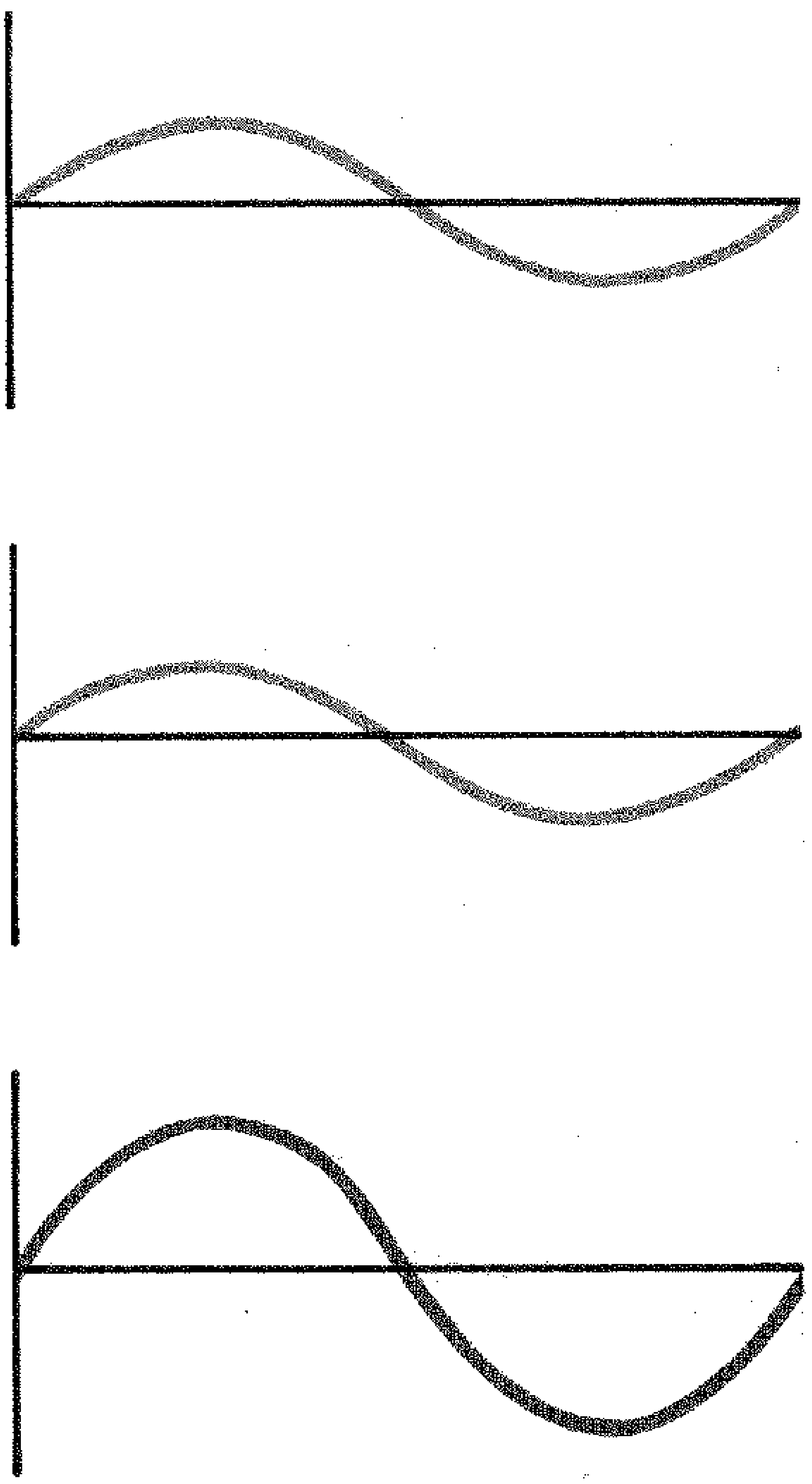


Fig. 4.8a

Two waves having like signs add. In this case amplitudes of 0.5 sum to 1.

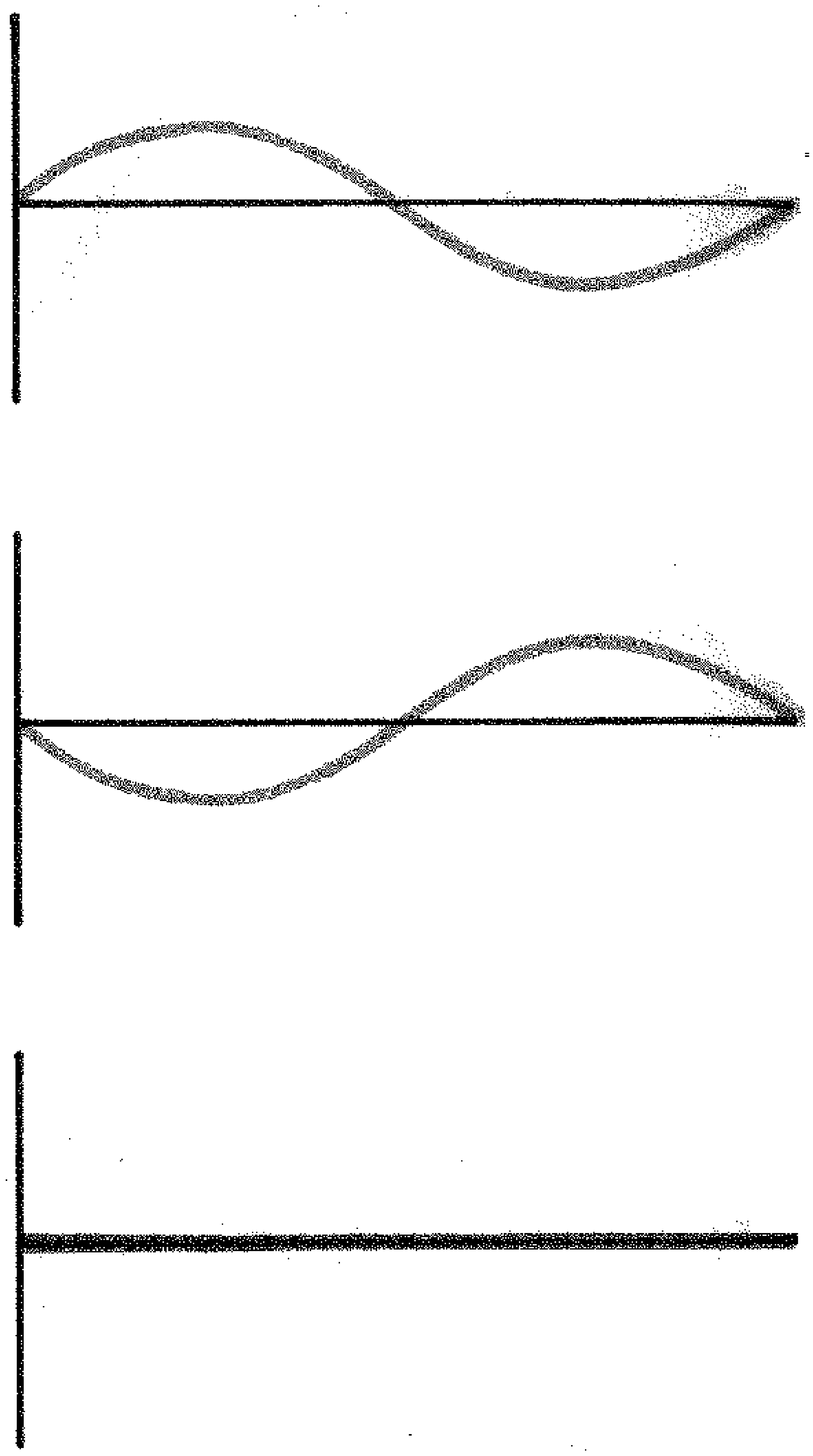


Fig. 4.8b

Now, having opposite signs (or a phase difference of 180°) but the same amplitude, they exactly cancel.

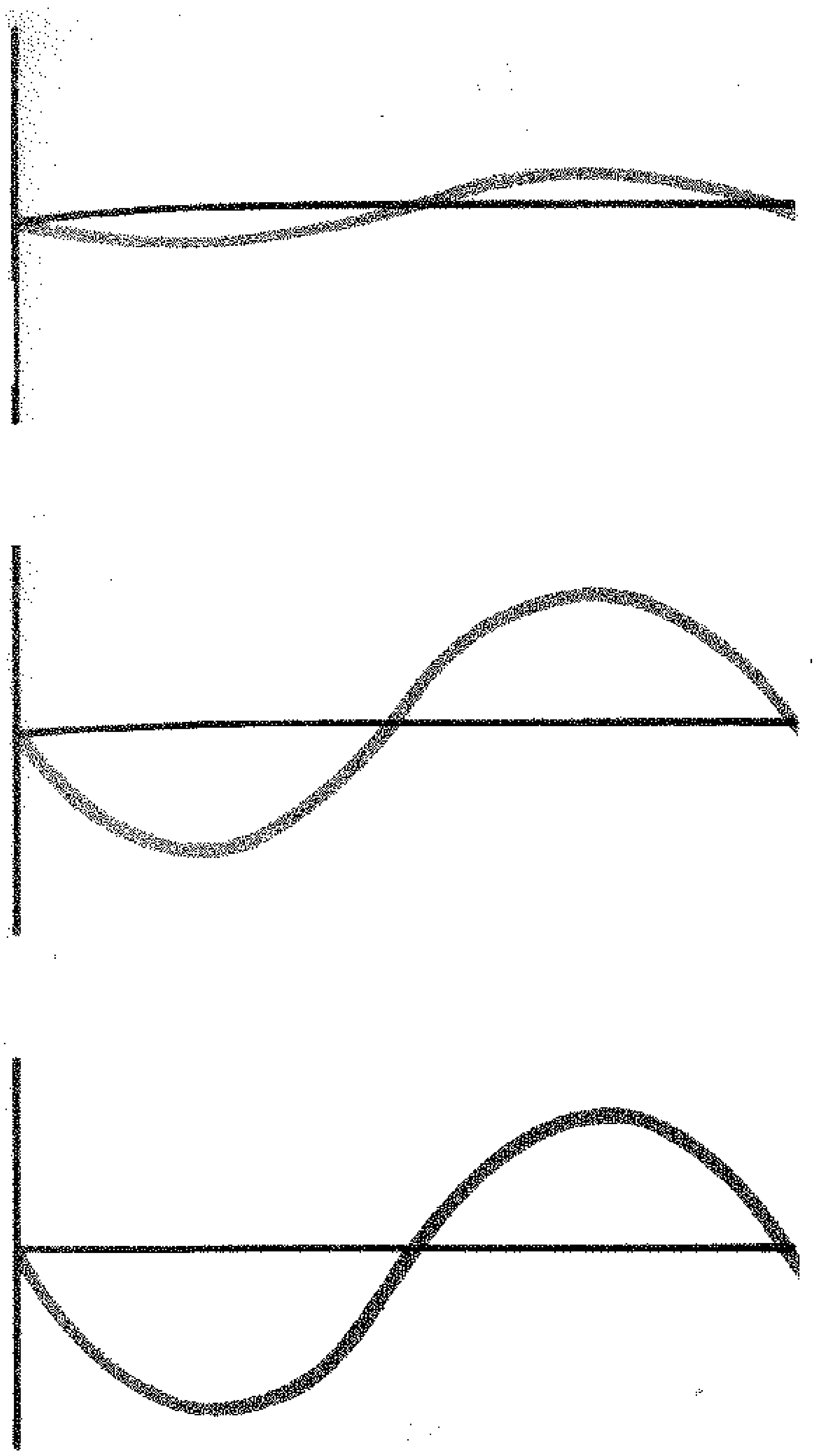


Fig. 4.8c

Again like signs, but this time negative, but with different amplitudes again. They add.

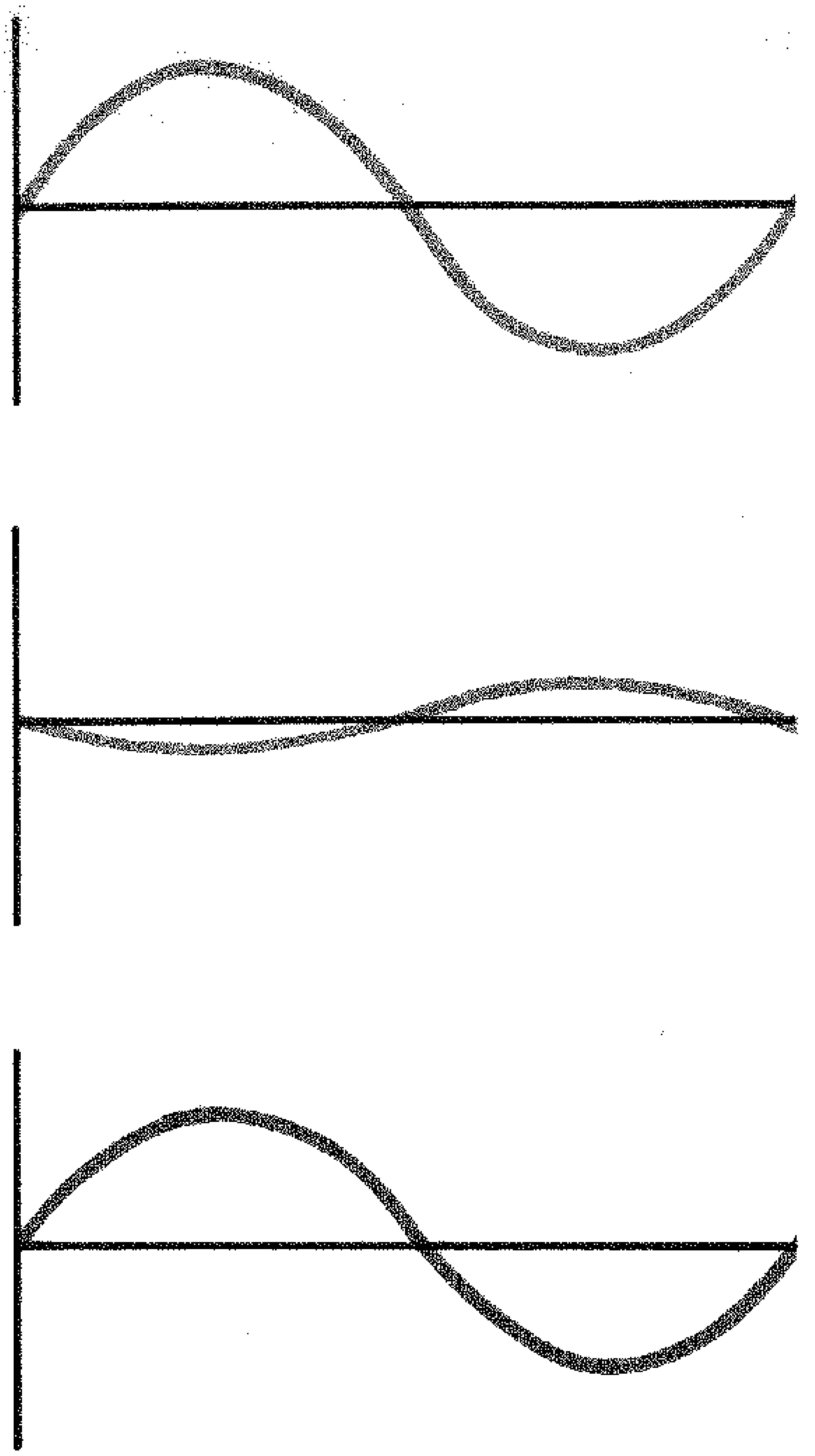


Fig. 4.8d

And finally, opposite signs with different amplitudes. They do not cancel but rather one subtracts from the other.

Our own physiological/psychological response to sound has the final word on this subject, however. Because of the ear's insensitivity to phase difference between different components in a spectrum, any frequency component in the positive frequency domain, which after having calculated its exact amplitude is finally found to be negative, can now be considered positive, allowing the final spectrum to appear neatly, all in one quadrant.

First, we will substitute the actual values in the expression and form a table.

$J_k\ (I),\ (c \pm k_m)$
for
$k = 0, 1, 2, 3 \ldots n$
where
$n \text{ approx} = I + 2 = 6$

To determine the Bessel coefficients for the components we look at each order successively and take the approximate value of the function on the vertical axis at the point where it intersects with the modulation index at a value of 4 on the horizontal axis.

Frequency Components for Simple FM where

$$c{:}m = 1{:}2 \text{ and } I = 4$$

AMPLITUDE	FREQUENCY		
Amplitude Coefficients	Side Frequencies		
(scaling)	odd order	Lower	Upper
$J_0\ (4) = -0.39$		$c = 1$	
$J_1\ (4) = 0.07$	(–1)	$1 - 2 = -1$	$1 + 2 = 3$
$J_2\ (4) = 0.36$		$1 - 4 = -3$	$1 + 4 = 5$
$J_3\ (4) = 0.43$	(–1)	$1 - 6 = -5$	$1 + 6 = 7$
$J_4\ (4) = 0.28$		$1 - 8 = -7$	$1 + 8 = 9$
$J_5\ (4) = 0.14$	(–1)	$1 - 10 = -9$	$1 + 10 = 11$
$J_6\ (4) = 0.05$		$1 - 12 = -11$	$1 + 12 = 13$

Table 4.4

The value of 0.05 for the sixth order Bessel coefficient is only an approximation. By looking at Appendix 4 (the table of Bessel functions) we can estimate what the value might be. In any case, the significance of any difference is minimal.

We can infer that the Bessel functions of higher orders will be insignificant in amplitude.

Now, we plot the spectrum exactly according to our values in the table.

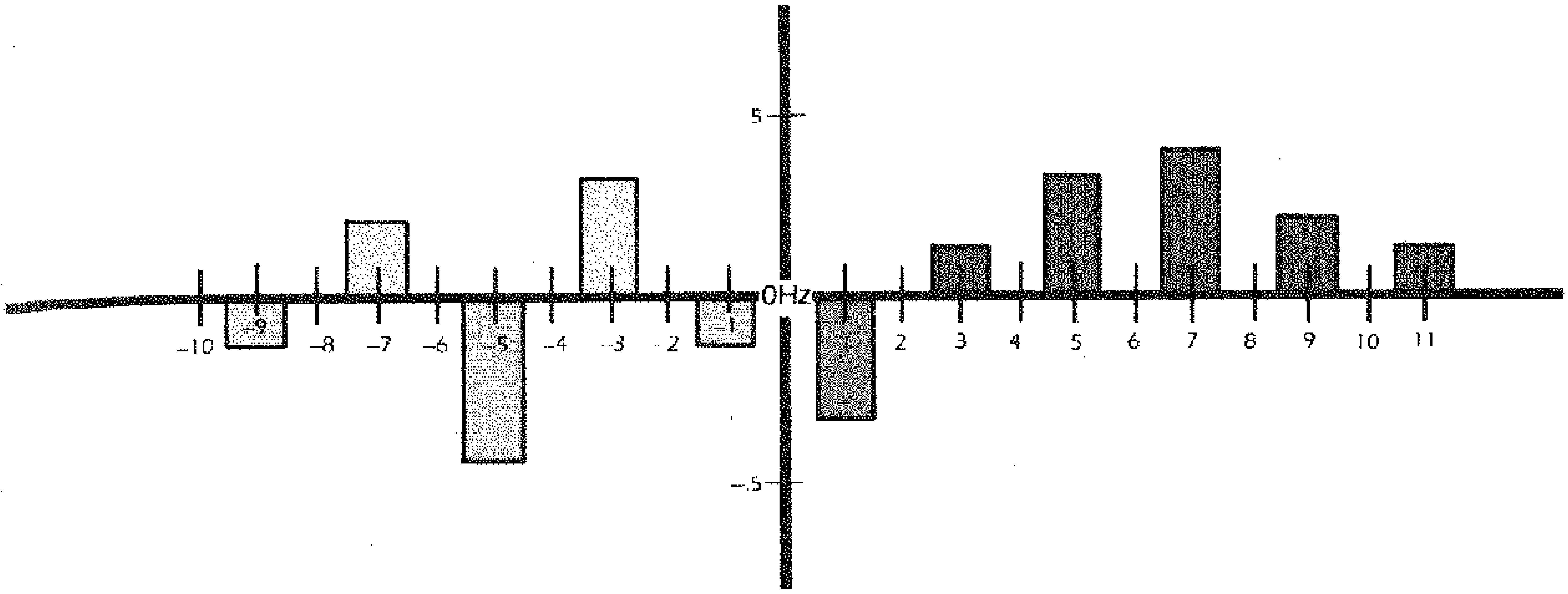


Fig. 4.9

A plot of the frequency spectrum for $c : m = 1 : 2$ and $I = 4$, showing where the components fall on either side of 0Hz with their amplitudes as determined by the Bessel coefficients. Note that we have included a second step here and that the amplitudes of the odd order lower frequency components have changed signs according to the rule, eg. the component at −1 has an amplitude of −0.09 instead of +0.09, the component of −5 has an amplitude of −0.43, etc.

Now, remembering that in making a negative frequency positive we must simply change its sign, Fig. 4.10 shows the "reflection" of the negative components around 0Hz.

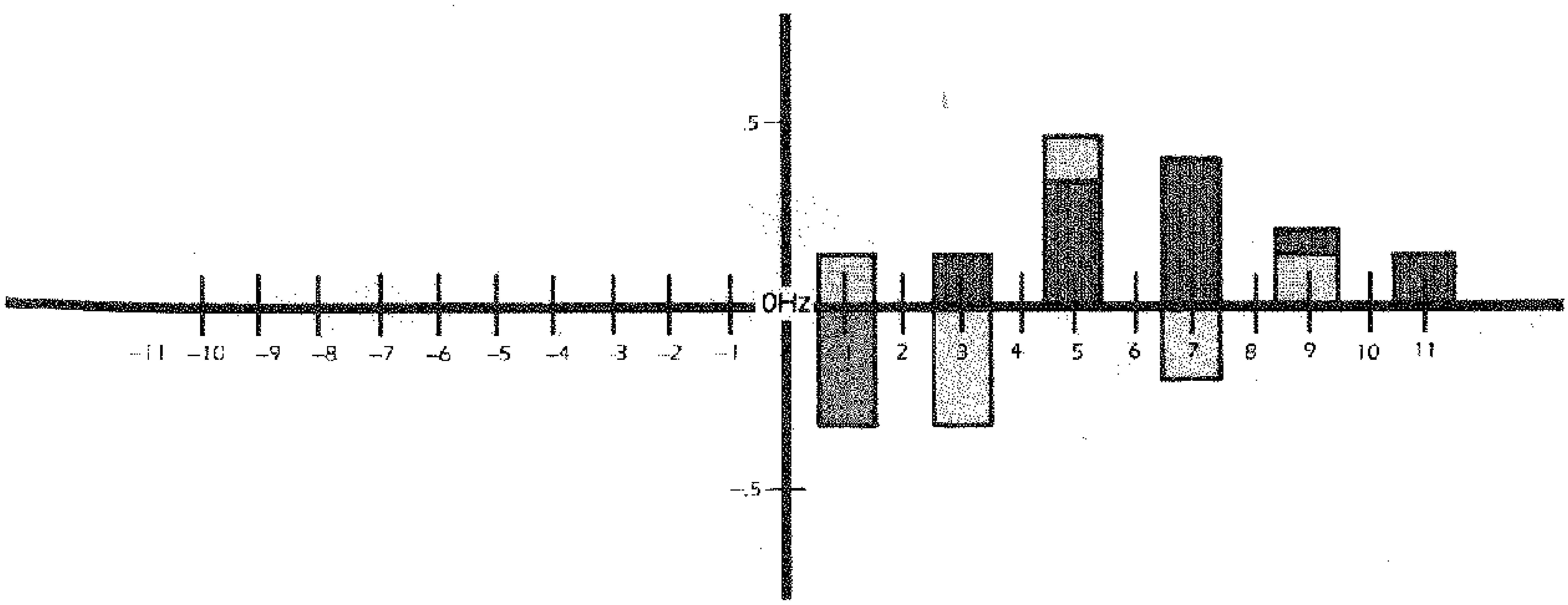


Fig. 4.10

Following the rule that negative components are made positive with a change of sign, they are reflected around 0Hz and fall in their corresponding positive position. For example, the component at −3 having an amplitude of +0.36 falls at +3 with an amplitude of −0.36. Components at 5 and 9 are overlaid here, not added.

We can now add or subtract the components at each frequency according to like or unlike signs, as shown in Fig. 4.11a, and then make all components having negative amplitude positive, as shown in Fig. 4.11b. This last operation is possible because the ear is largely insensitive to constant differences in phase (remember from chapters 2 and 3), and we can better see the relative amplitudes of the components if they are all drawn in the same direction.

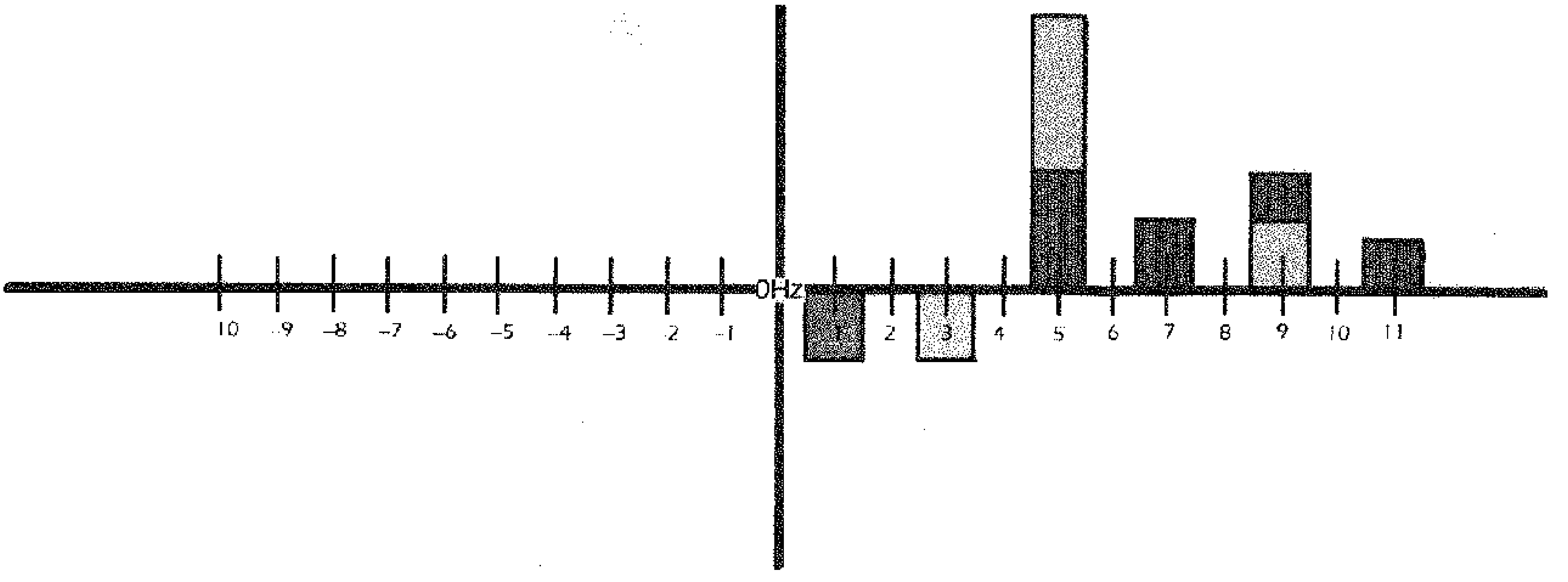


Fig. 4.11a
Where reflected frequency components fall on components in the positive domain, as in Fig. 4.10, the sum is accomplished by subtracting unlike signs and adding like signs. This spectrum represents the relationship of the frequencies contained in the wave as it would be produced in synthesis.

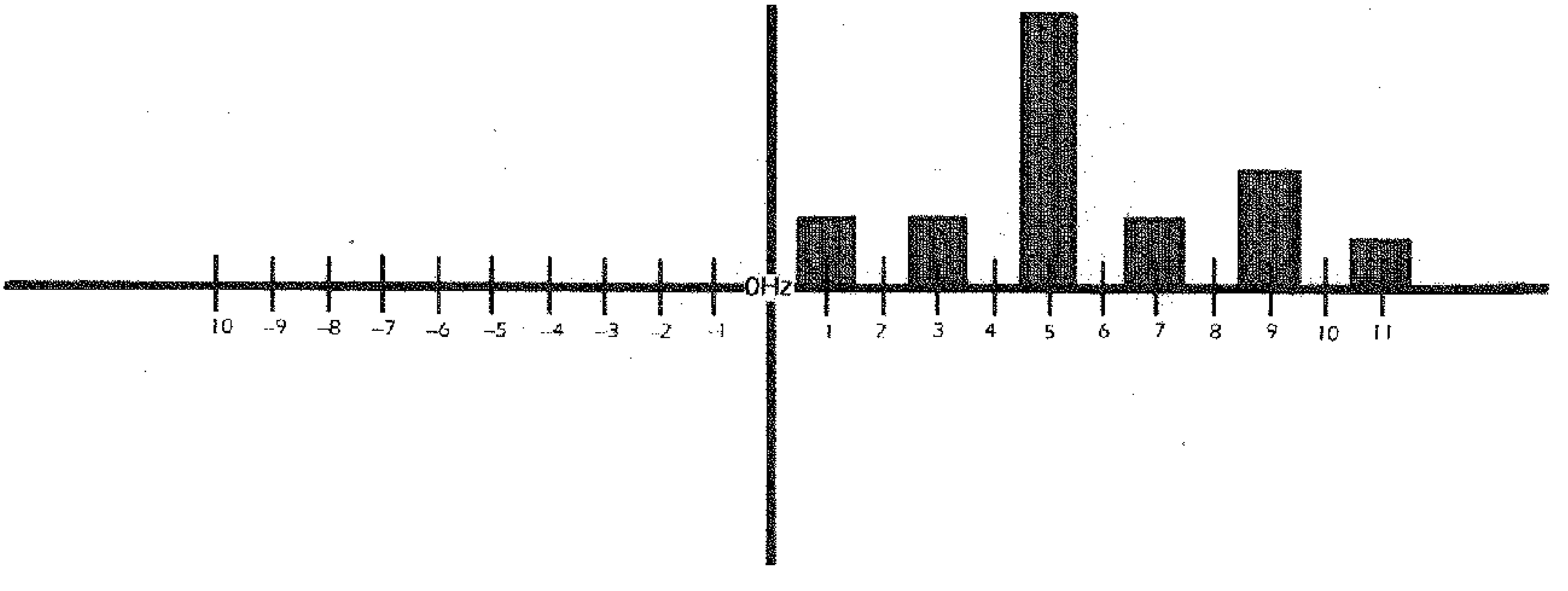


Fig. 4.11b
But, because the ear is largely insensitive to the differences of phase, we make all of the components positive in order more easily to see the amplitude relationships. This is now our "normalised" spectrum for the ratio $c : m = 1 : 2, I = 4$.

What have we learned? **RULE 1** tells us how the frequency components will fall in relation to one another, or the pattern of the components. We have just seen, for example, that a ratio of $c{:}m = 1{:}2$ results in a pattern where only odd numbered harmonics are present. What of a ratio of $c{:}m = 1{:}3$? We apply the rule and quickly see that every third harmonic will be missing. That is $c \pm 0m = 1$, $c \pm 1m = 4$ and -2, $c \pm 2m = 7$ and -5, etc. What of a ratio of $c{:}m = 2{:}3$ or any other ratio of simple integers? Since the relation of frequency components that form the spectrum is important perceptually, we might just form a table of components that result from ratios of simple integers. What else have we learned?

	Ratio of *c* : *m*								
	1:1		1:2		1:3		1:4		
$J_0(I)$	1		1		1		1		0th order (carrier)
$J_1(I)$	0	2	−1	3	−2	4	−3	5	1st order side frequencies
$J_2(I)$	−1	3	−3	5	−5	7	−7	9	2nd order side frequencies
$J_3(I)$	−2	4	−5	7	−8	10	−11	13	3rd order side frequencies
$J_4(I)$	−3	5	−7	9	−11	13	−15	17	4th order side frequencies
etc.									

	2:1		2:3		2:5		2:7		
$J_0(I)$	2		2		2		2		0th order (carrier)
$J_1(I)$	1	3	−1	5	−3	7	−5	9	1st order side frequencies
$J_2(I)$	0	4	−4	8	−8	12	−12	16	2nd order side frequencies
$J_3(I)$	−1	5	−7	11	−13	17	−19	23	3rd order side frequencies
$J_4(I)$	−2	6	−10	14	−18	22	−26	30	4th order side frequencies
etc.									

Table 4.5

	Ratio of $c : m$								
	3:1		3:2		3:4		3:5		
$J_0(I)$	3		3		3		3		0th order (carrier)
$J_1(I)$	2	4	1	5	−1	7	−2	8	1st order side frequencies
$J_2(I)$	1	5	−1	7	−5	11	−7	13	2nd order side frequencies
$J_3(I)$	0	6	−3	9	−9	15	−12	18	3rd order side frequencies
$J_4(I)$	−1	7	−5	11	−13	19	−17	23	4th order side frequencies
etc.									

	4:1		4:3		4:5		4:7		
$J_0(I)$	4		4		4		4		0th order (carrier)
$J_1(I)$	3	5	1	7	−1	9	−3	11	1st order side frequencies
$J_2(I)$	2	6	−2	10	−6	14	−10	18	2nd order side frequencies
$J_3(I)$	1	7	−5	13	−11	19	−17	25	3rd order side frequencies
$J_4(I)$	0	8	−8	16	−16	24	−24	32	4th order side frequencies
etc.									

	5:1		5:2		5:3		5:4		
$J_0(I)$	5		5		5		5		0th order (carrier)
$J_1(I)$	4	6	3	7	2	8	1	9	1st order side frequencies
$J_2(I)$	3	7	1	9	−1	11	−3	13	2nd order side frequencies
$J_3(I)$	2	8	−1	11	−4	14	−7	17	3rd order side frequencies
$J_4(I)$	1	9	−3	13	−7	17	−11	21	4th order side frequencies
etc.									

Table 4.5 (sequel)

RULE 2 tells us that the output level of the modulating operator (modulation index) determines the number of components whose amplitudes will be perceptually significant. Therefore, small output – few components, large output – many components. We have also learned that the amplitudes of the components are determined by Bessel functions.

In Table 4.5, then, we see the components that result for each ratio, one at the carrier and then two for each higher order, the lower and the upper side frequencies. We know that lower side frequencies that are less than 0Hz (those with minus signs in the table) can be considered as components in the positive domain with a change of phase. And if these components which reflect around 0Hz fall upon components already there then they either add energy or take away energy depending upon like or unlike signs.

By inspecting Table 4.5 we can see that all of the ratios are irreducible fractions and that all of the resulting components are integers, therefore members of the harmonic series. Note that in every case there are odd numbered harmonics and in some cases both odd and even, but that in no case are there only even numbered harmonics, – hence the condition of ratios forming irreducible fractions. (ratio of ***c : m*** = ***4 : 6*** is the same as ***2 : 3*** an octave higher).

At some point it would be instructive to listen to all of these ratios. What you will notice is that the strength of the pitch perception is less when the components are far apart. Compare 4:7 and 4:3.

What is the result of ratios of ***c : m*** where they are not integers; for example a ratio of ***1 : 1.4***? Let us again form a table and look at a few.

	Ratio of ***c : m***				
	1:1.4		*0.5:1.6*		
$J_0(I)$	*1*		*0.5*		0th order (carrier)
$J_1(I)$	*–0.4*	*2.4*	*–1.1*	*2.1*	1st order side frequencies
$J_2(I)$	*–1.8*	*3.8*	*–2.7*	*3.7*	2nd order side frequencies
$J_3(I)$	*–3.2*	*5.2*	*–4.3*	*5.3*	3rd order side frequencies
$J_4(I)$	*–4.6*	*6.6*	*–5.9*	*6.9*	4th order side frequencies

Table 4.6

Here the components do not fall in a pattern that is obviously related to the harmonic series nor do they sound like they are periodic, i.e. pitch is ambiguous indeed. Such spectra we call *inharmonic* and we will see in the next sections how they are important in themselves as the basis of many sounds — such as percussive sounds — and how they are important secondary ingredients of many sounds based on harmonic spectra.

PROBLEM

What sort of Spectrum will this give us?

2.00

1.00

Mod. Output level = 85

To give FREQUENCY Components

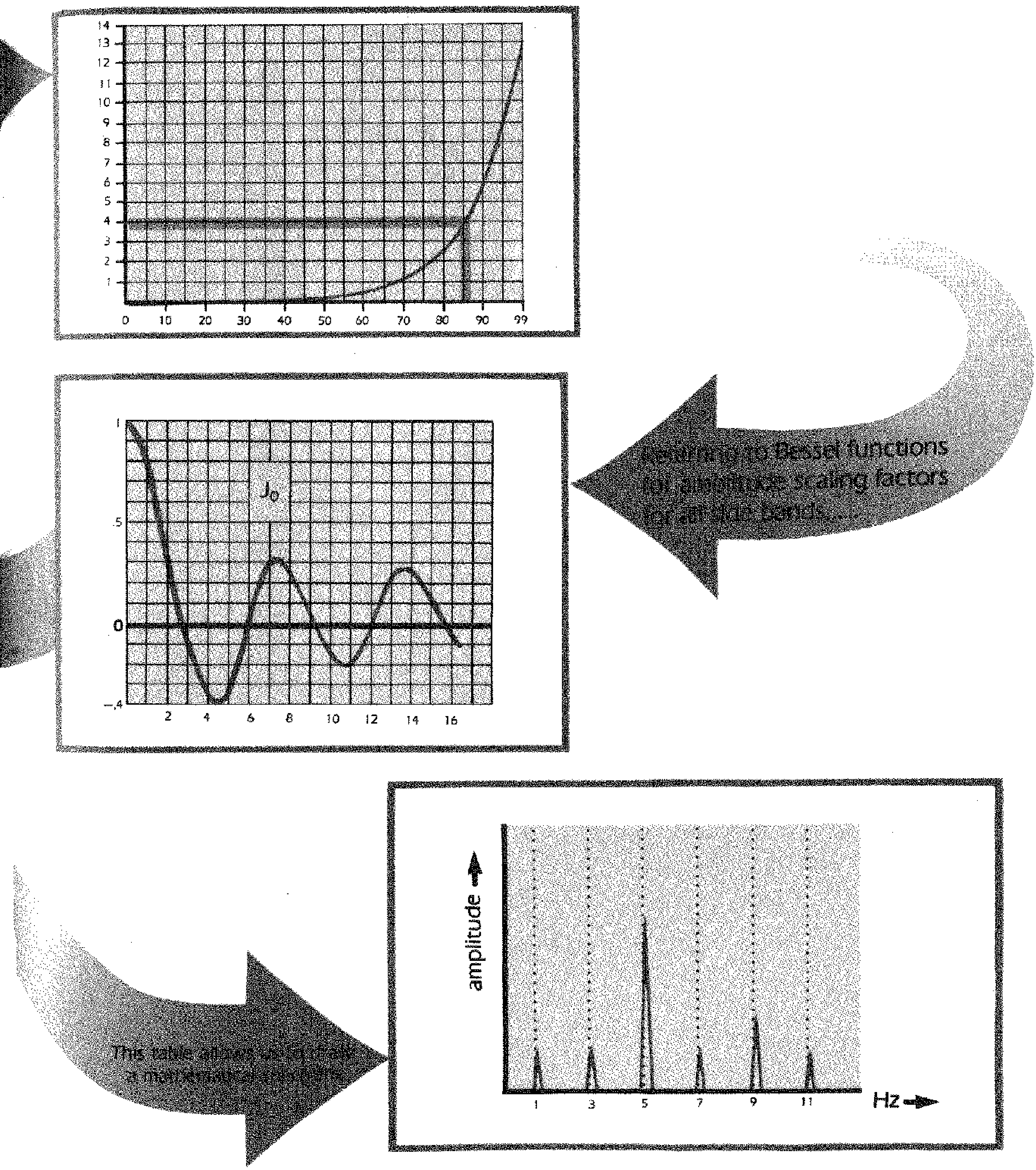
$$c \pm km$$

see page 65

...we can build a table of values.

AMPLITUDE	FREQUENCY		
Amplitude Coefficients	Side Frequencies		
(scaling)	odd order	Lower	Upper
$J_0(4) = -0.39$		$c = 1$	
$J_1(4) = 0.09$	(-1)	$1 - 2 = -1$	$1 + 2 = 3$
$J_2(4) = 0.36$		$1 - 4 = -3$	$1 + 4 = 5$
$J_3(4) = 0.43$	(-1)	$1 - 6 = -5$	$1 + 6 = 7$
$J_4(4) = 0.28$		$1 - 8 = -7$	$1 + 8 = 9$
$J_5(4) = 0.14$	(-1)	$1 - 10 = -9$	$1 + 10 = 11$
$J_6(4) = 0.05$		$1 - 12 = -11$	$1 + 12 = 13$

In creating timbres with the FM synthesis technique it would be rare to plot spectra as we are doing here. But in order to make helpful generalizations we are obliged to go through this process once in order to understand the basis of it all. The flow chart below recaps in graphic form the process of calculating a spectrum for simple FM.



So now, all the mathematics needed have been covered, and we are ready to go on with some practical exercises in calculating FM spectra, and to gain an understanding, in terms of simple FM, of some acoustic and psychoacoustic phenomena such as beating (using both fixed frequency carriers and detune), residual pitch, bandwidth and the use of envelope generators.

CHAPTER 5

SIMPLE FM
The Practice

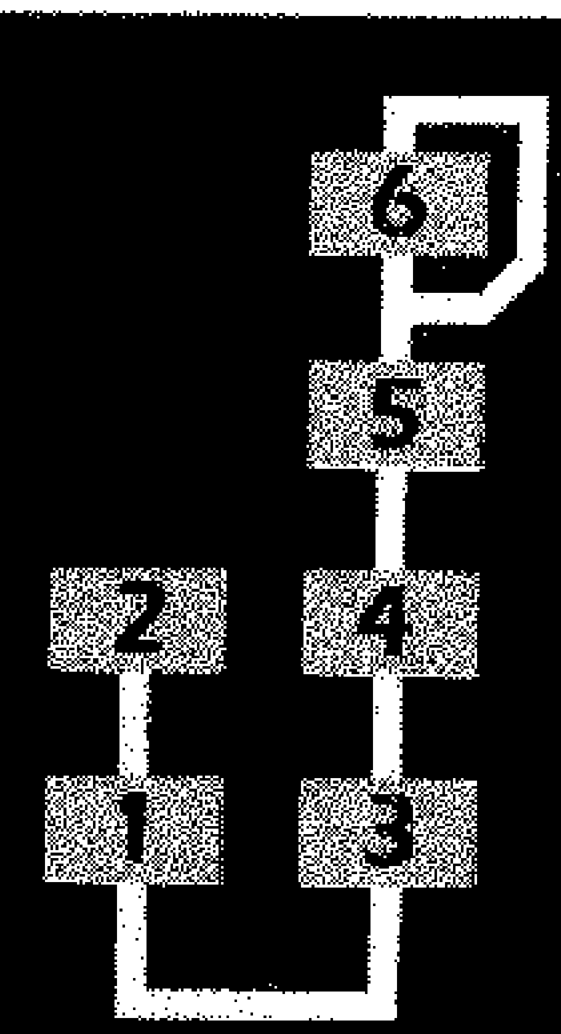
The means by which we learned to calculate the spectra of simple FM in the last chapter will now be used as applied to some selected examples. While these examples are chosen to show fundamental aspects of both the formation and control of sound synthesis by means of FM, much of what we will learn is applicable to any technique of synthesis. This is so because sounds that are interesting, dynamic, and vital are sounds having complexities that change according to basic musical dimensions: effort and spectral richness (velocity sensitivity), pitch height and spectral simplicity (level scaling), pitch height and sharpness of attacks (rate scaling), etc. FM provides great complexity that is surprisingly simple to control and to connect to such musical dimensions. To become comfortable with *simple* FM, as it is extended from the theoretical to the practical in this section, is our goal. Furthermore, understanding through experimentation with simple FM is essential to the understanding and effective use of complex FM.

(Some of these "X"-amples will appear to be tedious and perhaps redundant. Nonetheless we strongly recommend that they all be followed. Even after many years of experience with FM, we are still surprised by what simple "ear things" can be learned by working through simple examples. And of course, always, always, follow your own imagination and extend and distort our examples to suit your musical purposes).

At this point we will plot another spectrum having different values and, in so doing, learn why we can profit from some understanding of the FM theory. But first, set up the following "X"-ample in order to raise an important question about "beating" and the effects of detune.

"X"-ample 5.1

	FREQUENCY	OUTPUT
op 1	1.00	99
op 2	2.00	72
op 3		
op 4		
op 5		
op 6		



INSTRUCTIONS: Starting from the VOICE INIT ? position....

Select algorithm 1 and set up the values shown, and select parameter DETUNE for operator 2. Now press a key and increase the detune to +7. Notice the beating effect of the sound. Leaving the detune at +7, select Frequency Coarse for operator 2 and change it from 2 to 4. The beating stops!

Well, two important questions are raised by this simple experiment. What is this beating effect, and why does it disappear with the ratio of *1:4*? We have already plotted a spectrum based on a *1:2* ratio, so now we will plot a spectrum based on a ratio of $c:m = 1:4$ and $I = 1.5$, and have a look at what is going on inside the sound.

We will plot a spectrum based on a ratio of $c:m = 1:4$ and $I = 1.5$. Therefore:-

$$J_k\,(1.5),\ (1 \pm k4)$$

$$k = 0,1,2,3 \ldots n$$

where

$$n\ approx = I + 2 = 4 \text{ (round up)}$$

Now we form our table again . . . . . .

Frequency Components for Simple FM where

$$c:m = 1:4 \text{ and } I = 1.5$$

AMPLITUDE	FREQUENCY		
Amplitude Coefficients	Side Frequencies		
(scaling)	odd order	Lower	Upper
$J_0(1.5) = +.51$		$c = 1$	
$J_1(1.5) = .56$	(-1)	$1-4=-3$	$1+4=5$
$J_2(1.5) = .24$		$1-8=-7$	$1+8=9$
$J_3(1.5) = .05$	(-1)	$1-12=-11$	$1+12=13$
$J_4(1.5) = .01$		$1-16=-15$	$1+16=17$

In Figs. 5.1a-c we see that, unlike the ratio of $c:m = 1:2$, in this case the components reflected from the negative frequency domain do not fall on, and therefore sum with, components in the positive domain; rather, they interleave. Of what importance is it to realize this difference?

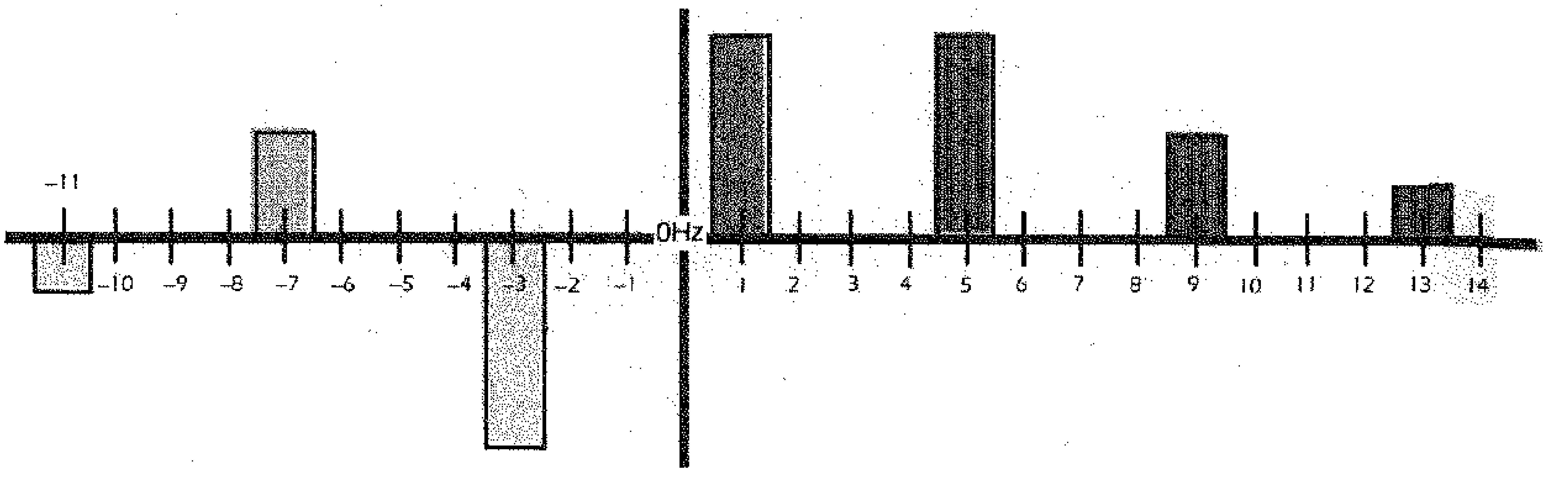


Fig. 5.1a

Plot of a ratio of $c{:}m = 1{:}4$ and $I = 1.5$. Note that this index produces Bessel coefficients for the 0th and 1st orders that are approximately equal as the two functions intersect at about this point.

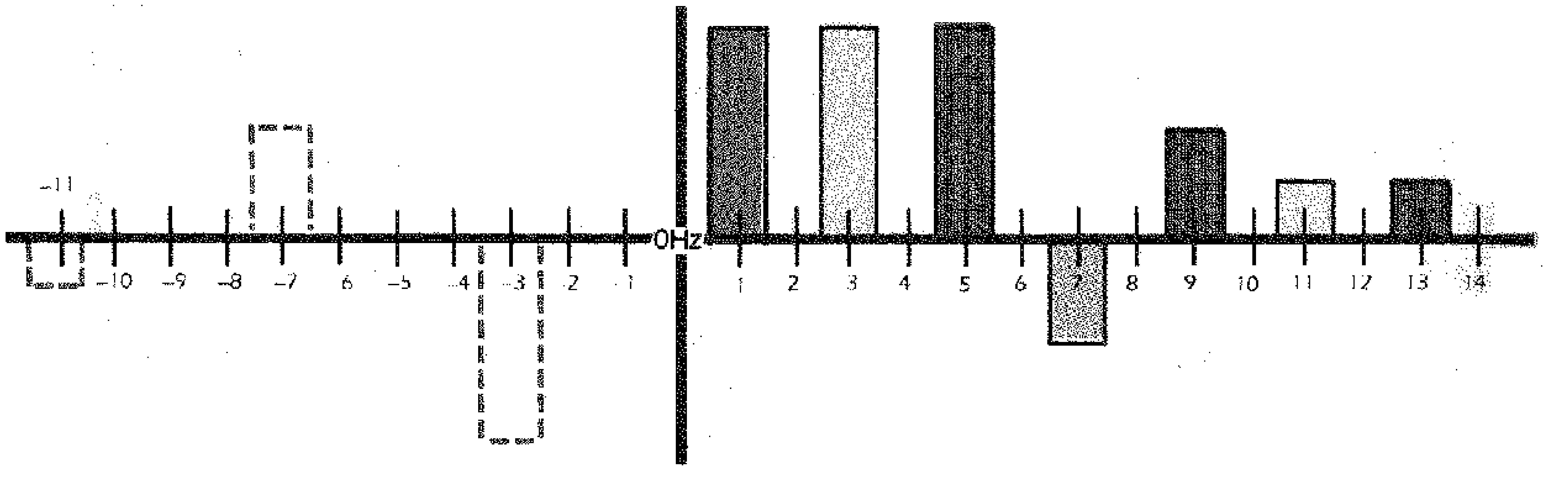


Fig. 5.1b

Now reflect around 0Hz.

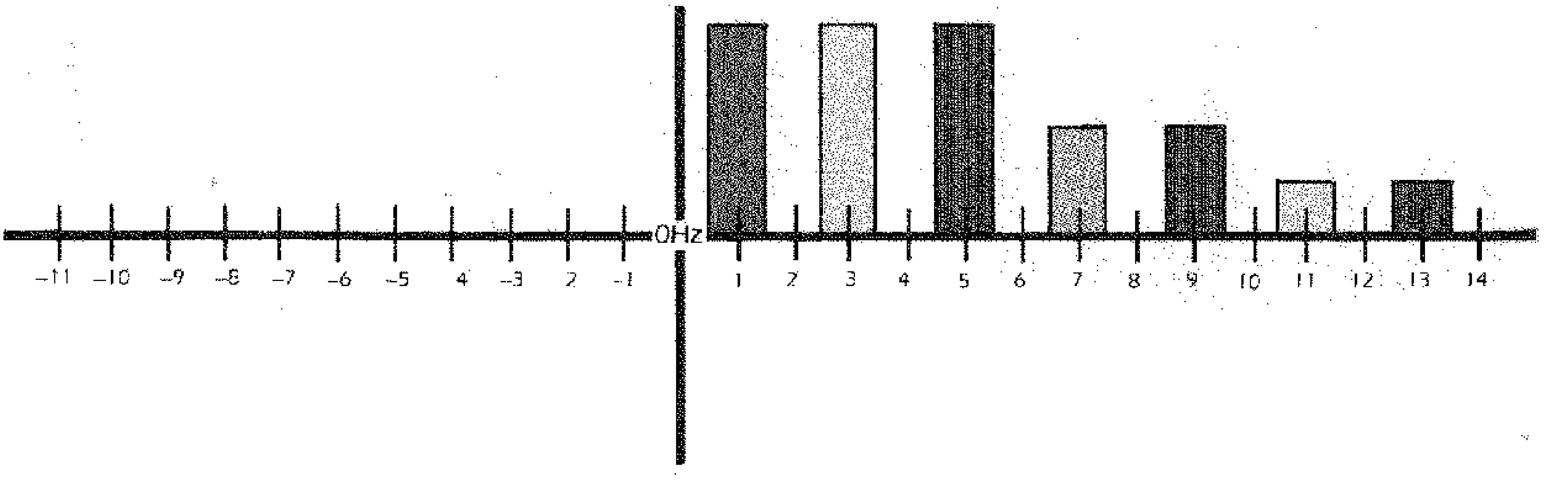
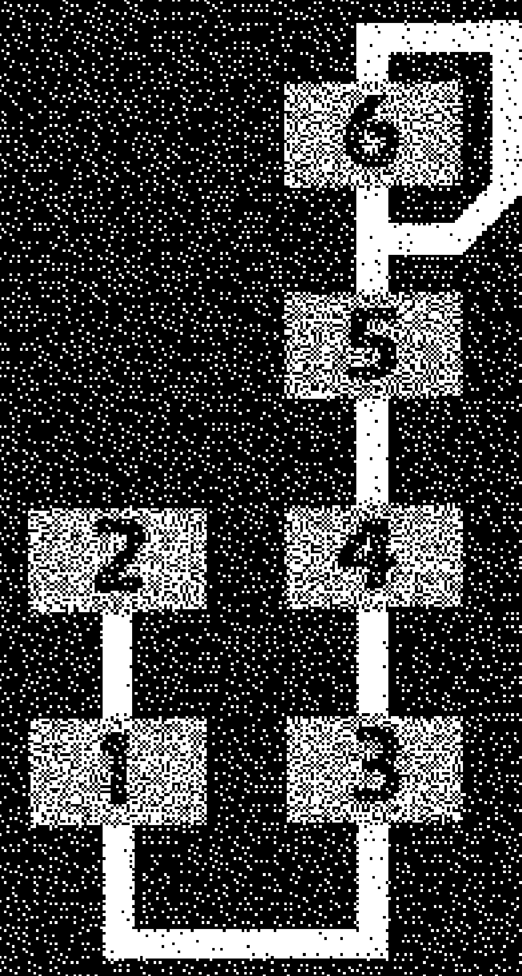


Fig. 5.1c

And finally, make them all positive (or take the magnitude of the spectrum).

Both of the spectra (Figs. 4.11 and 5.1) we have plotted consist of only odd harmonics. We will now discover why there is a very great difference in the effect when the two ratios are slightly mistuned. First, let's have a close look at "beating" ...

"X"-ample 5.2



	FREQUENCY	OUTPUT
op 1	1.00	99
op 2		
op 3	1.00	99
op 4		
op 5		
op 6		

INSTRUCTIONS: Starting from the VOICE INIT? position.....

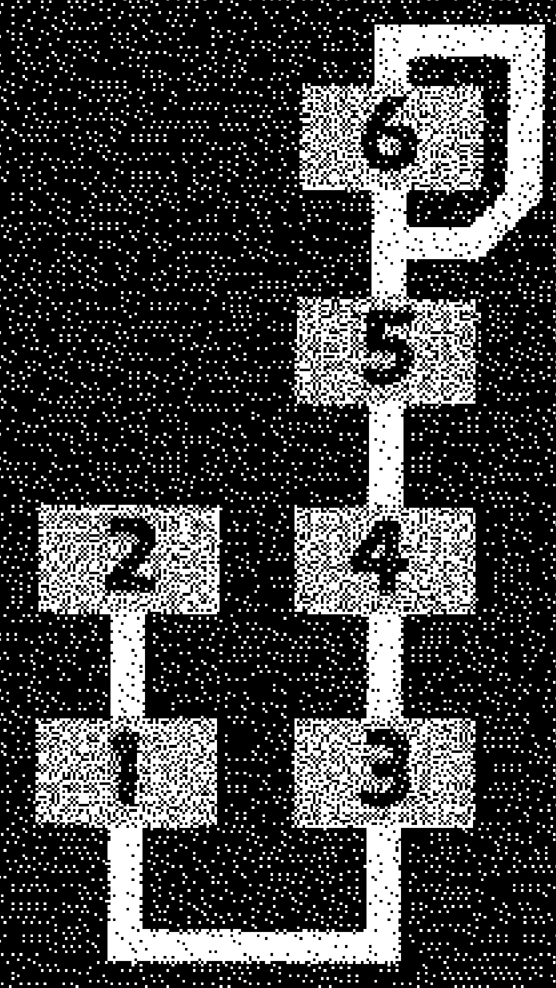
Select algorithm 1 and turn off all operators except 1 and 3, and set the values as shown. Select detune for op 3 and increase it to 7. Now change its frequency coarse value to 2.00. As with our previous "X"-ample, the beating disappears.

In this "X"-ample, we have taken two operators whose outputs sum. The beating effect is only really noticeable when the two frequencies are very close. When one of the frequencies is changed to twice that of the other, the effect is barely heard, even though the detune is still there. Beating, then, is not the result simply of a non-integral relationship – that is, between frequencies which do not fall *exactly* in the harmonic series – but rather an effect of the *close proximity* of two frequencies. In Fig. 5.2 overleaf we can see the result of adding two sinusoids whose frequencies differ by 1.5 cycles per second. There is a very gradual change of phase producing reinforcement and cancellation.

Now we will set up the synthesizer to produce the sounds corresponding to the two spectra that we have calculated.

With "X"-ample 5.3 we have in operators 1 and 2 a sound corresponding to the spectrum calculated in chapter 4 where ***c:m* = *1:2***, and in operators 3 and 4 the sound corresponding to the spectrum in Fig. 5.1 with a ratio of ***c:m* = *1:4***. Now set the detune in both operators 2 and 4 to +7 and listen alternately to the difference between the two spectra by turning operators 1 and 3 off and on.

"X"-ample 5.3



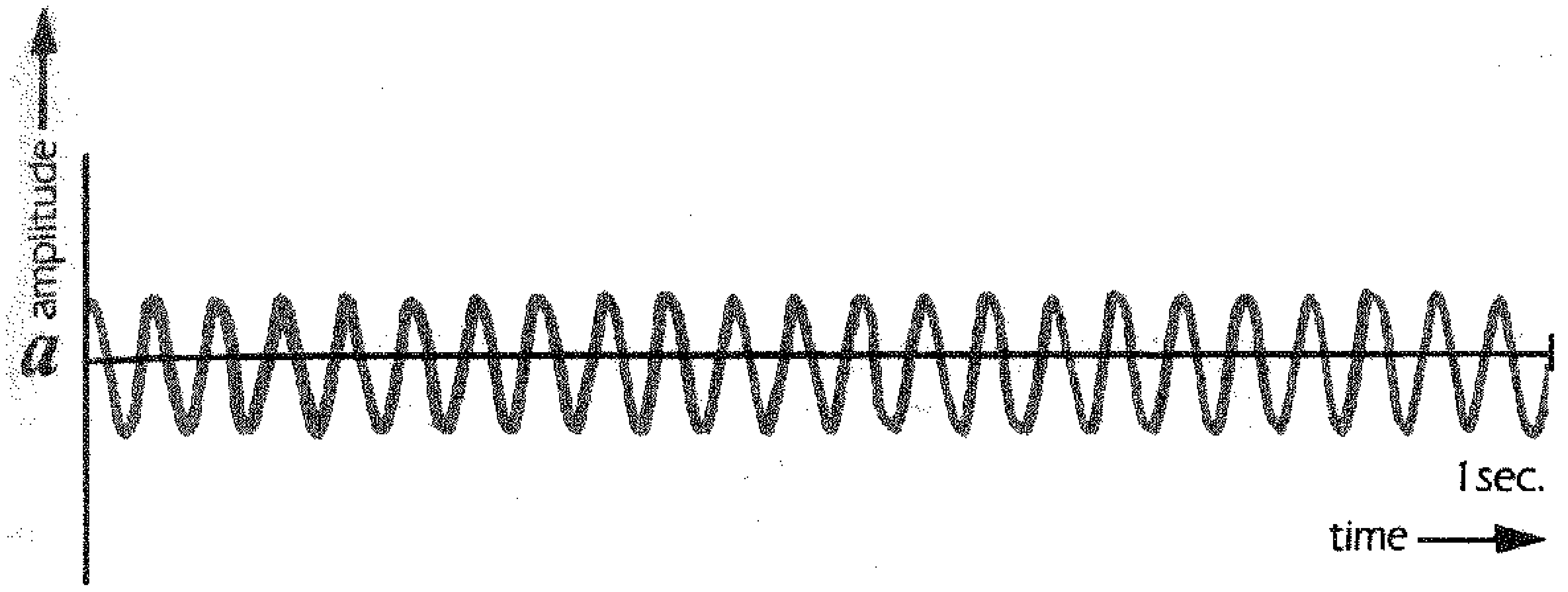
	FREQUENCY	OUTPUT
op 1	1.00	99
op 2	2.00	85
op 3	1.00	99
op 4	4.00	72
op 5		
op 6		

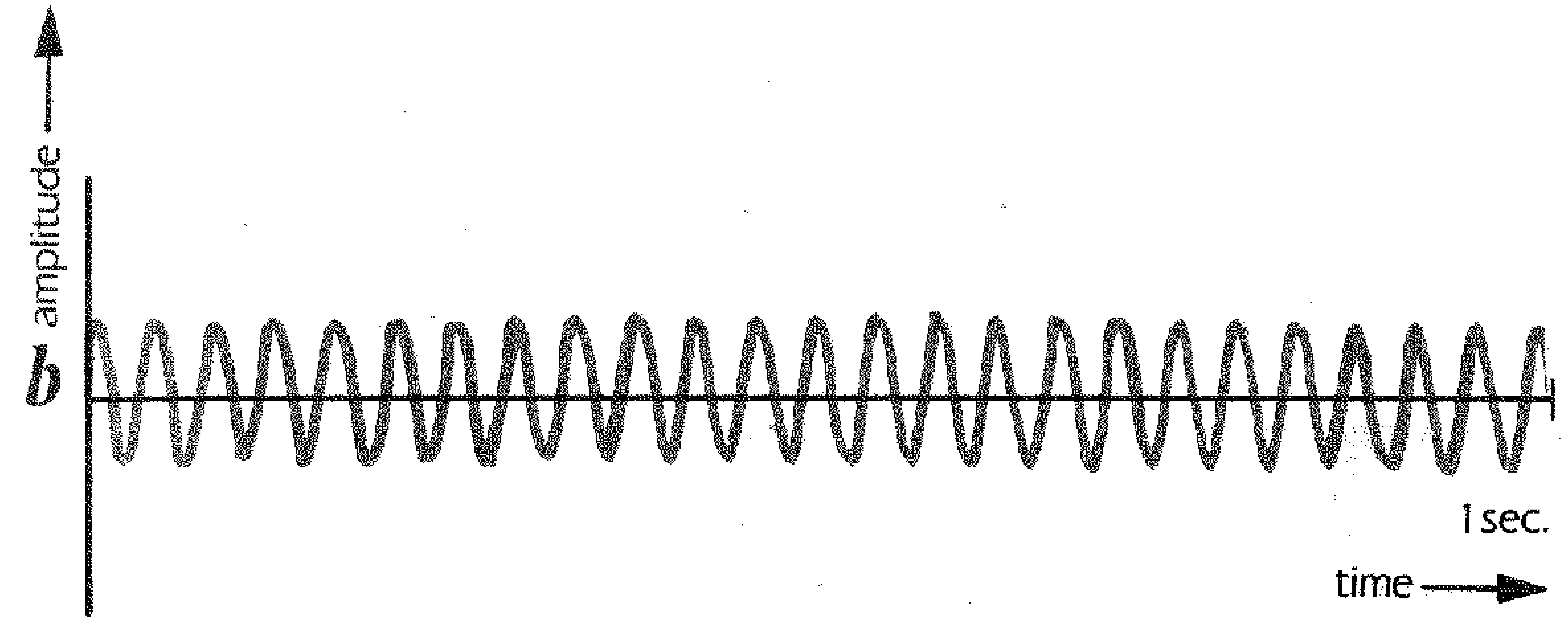
INSTRUCTIONS: Starting from the VOICE INIT? position....

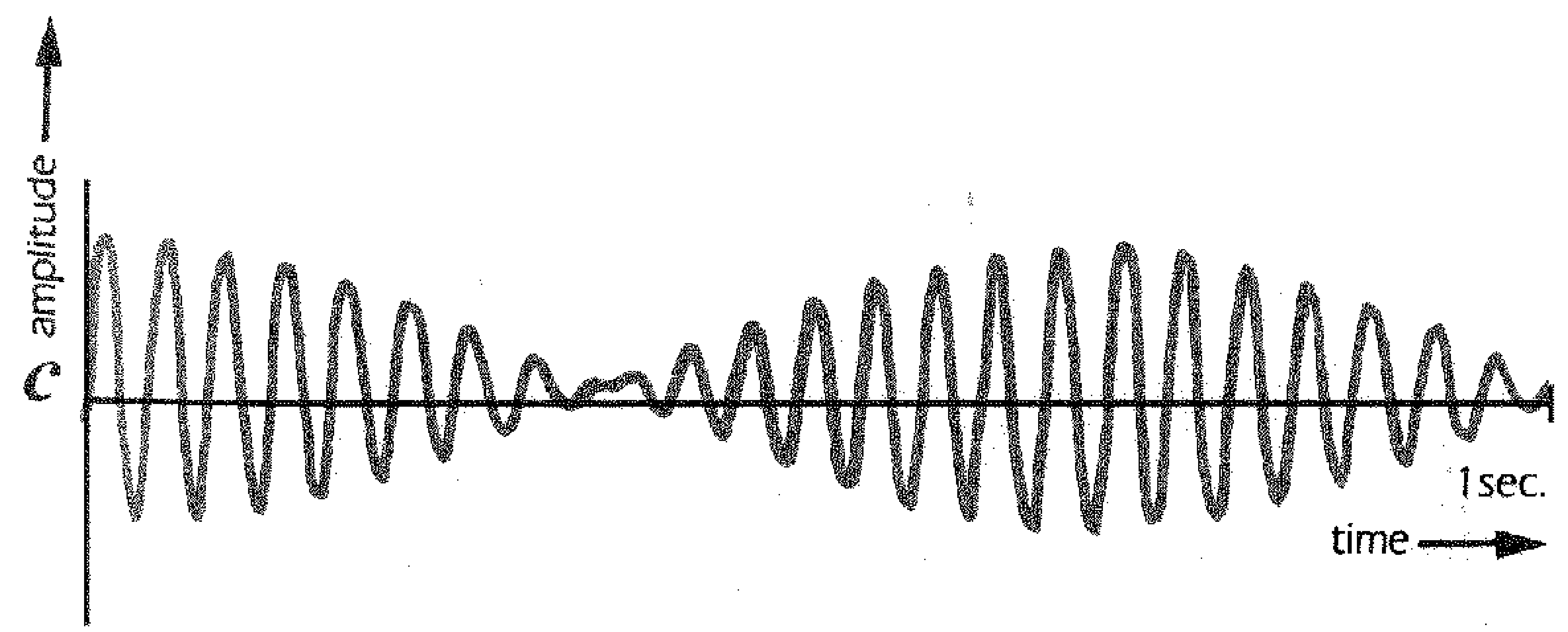
Select algorith 1 and turn off operators 5 and 6 and set up the values as shown.

Based upon our experiment with beating we can see why the ratio of $c : m = 1 : 2$ produces strong beating while the ratio of $c : m = 1 : 4$ does not. In the first case, the detune causes the components which are reflected from the negative frequency domain to fall just next to, but not directly on top of, components in the positive domain. The specral plots on page 90 (Fig. 5.3) are taken from a spectrum analyser into which was fed the sound in this "X" -ample. We can reveal the presence of the reflected components in the case of $c{:}m = 1{:}2$ by gradually increasing the detune over time as shown by the 3D plot.

The reason for distinguishing reflected components now becomes clear, for only a spectrum which has reflected components that fall on positive components will be strongly affected by detune. Table 4.3 on page 77 can be very useful in that it shows clearly those ratios which produce components that reflect and add to other components.







Wave a, having 23 cycles, adds to wave b, which has 24 cycles per second, to produce wave c, which has 23.5 cycles per second and a modulation of amplitude, or beat, of 1 cycle per second.

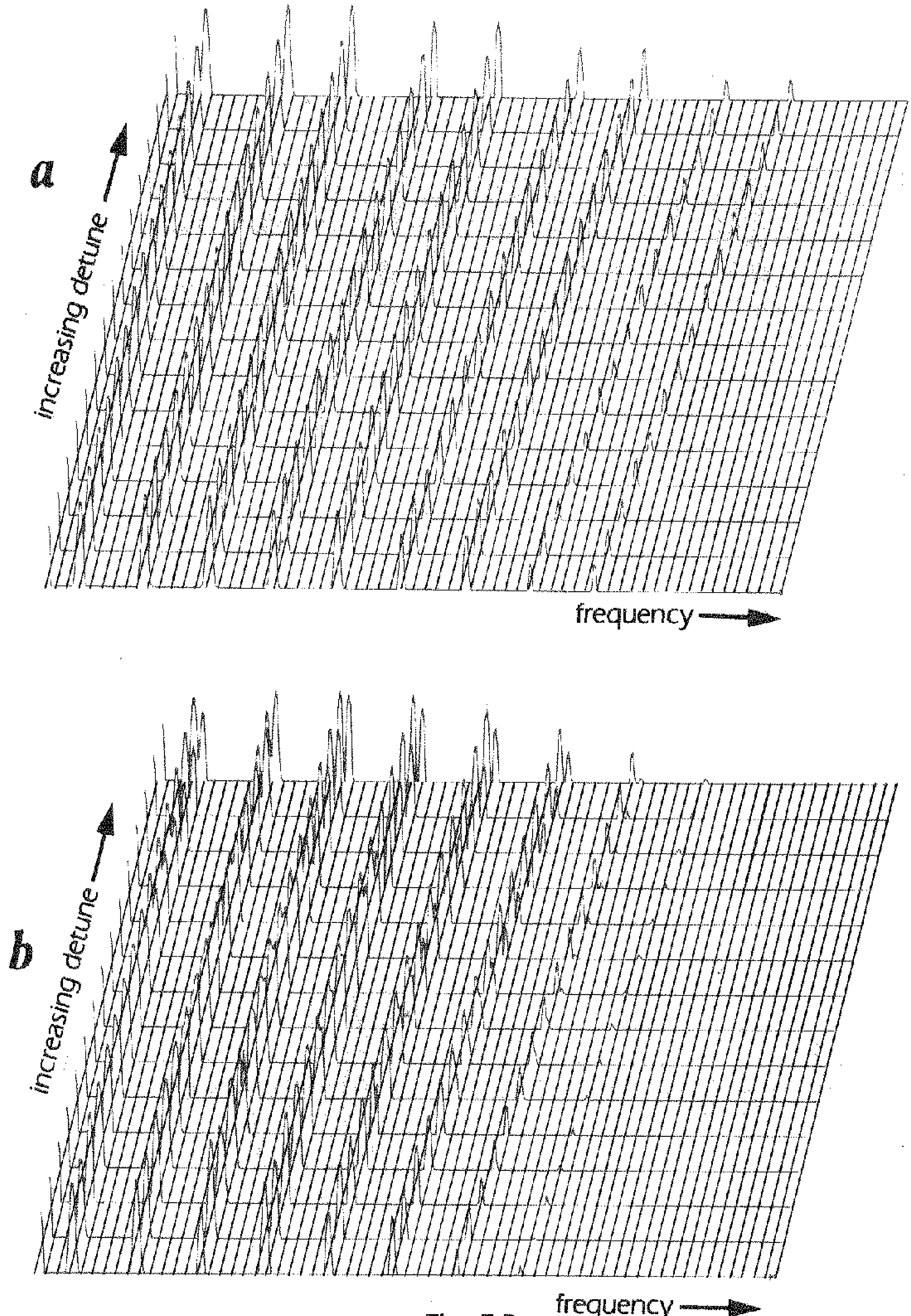


Fig. 5.3

The effect of increasing detune as a ratio of (a) 1: 4, where the reflected components interleave with the positive components, and as a ratio of (b) 1: 2, where the reflected components fall directly on top of, or rather just alongside, the positive components. Successive traces show increasing modulator detune.

We have seen that for any given ratio of ***c:m***, increasing the modulation index ***I*** increases the number of significant components in the spectrum. In comparing the two spectra of Figs. 4.11 and 5.1 we can learn something else in regard to the bandwidth of the spectra.

While the index of Fig. 4.11 is larger ($I = 4$ compared to $I = 1.5$ in Fig. 5.1), the bandwidth of the former is in fact smaller. The reason is that the side band components are at intervals of the modulating frequency, and in Fig. 5.1 the modulating frequency is two times that of Fig. 4.11. The general rule for bandwidth of spectrum in units of ratios is:

RULE 3

The bandwidth (*BW*) of the spectrum can be estimated approximately according to the following relations:

1. Where there are reflections about 0Hz:

   $$BW \text{ approx} = c + m \times (I + 2)$$

2. Where the carrier is greater than the modulator, and there are no reflections:

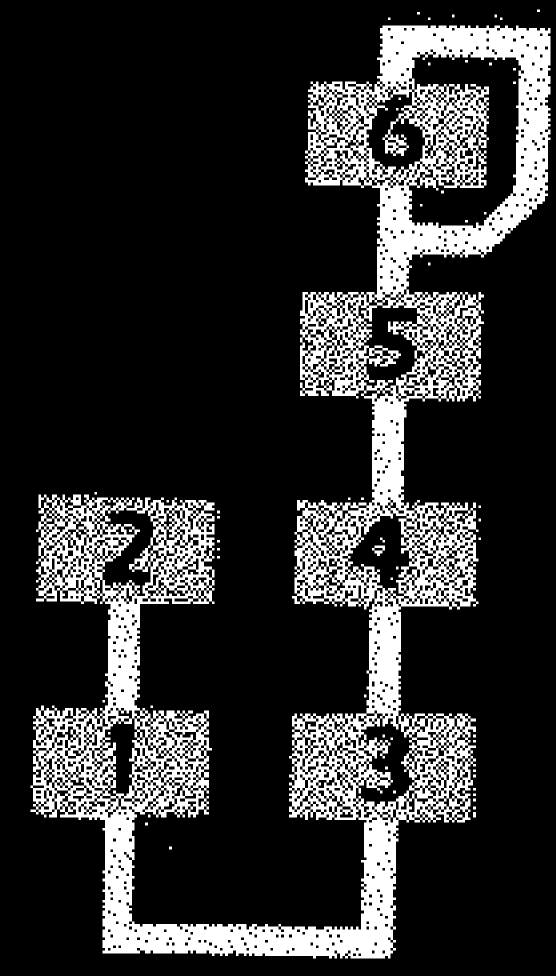
   $$BW \text{ approx} = 2m \times (I + 2)$$

This concept of bandwidth is easier to visualize with the help of some spectral plots and exercises. Each of the following "X"-amples is accompanied by a real time plot from a spectrum analyser, and we can see how certain simple parameter changes (increasing the carrier frequency; increasing the modulating frequency; increasing the modulation index)affect the bandwidth and nature of the spectrum:

A general observation which we can make from this exercise is that changing the frequency ratios, while affecting the bandwidth, does not increase the number of elements in the spectrum (apart from the sense of revealing those components which may have been concealed as reflections falling directly onto positive components).

Modulation index, however, affects the bandwidth by adding more components to the spectrum. It is interesting to speculate for a moment whether such semantic descriptions of tones such as "richness," "breadth" and so on *do* relate in any way to the visual images portrayed here. While we are thinking about how we relate to sounds, let's move on to another interesting acoustic phenomenom.

"X"-ample 5.4a



	FREQUENCY	OUTPUT
op 1	1.00	99
op 2	1.00	74
op 3		
op 4		
op 5		
op 6		

INSTRUCTIONS: Starting from the VOICE INIT ? position.....

Select algorithm 1 and set up the parameters as shown, then increase the Frequency Coarse value of operator 2 (modulator) by 1. 00 repeatedly with data entry + 1, which corresponds to successive traces of the 3D plot shown below.

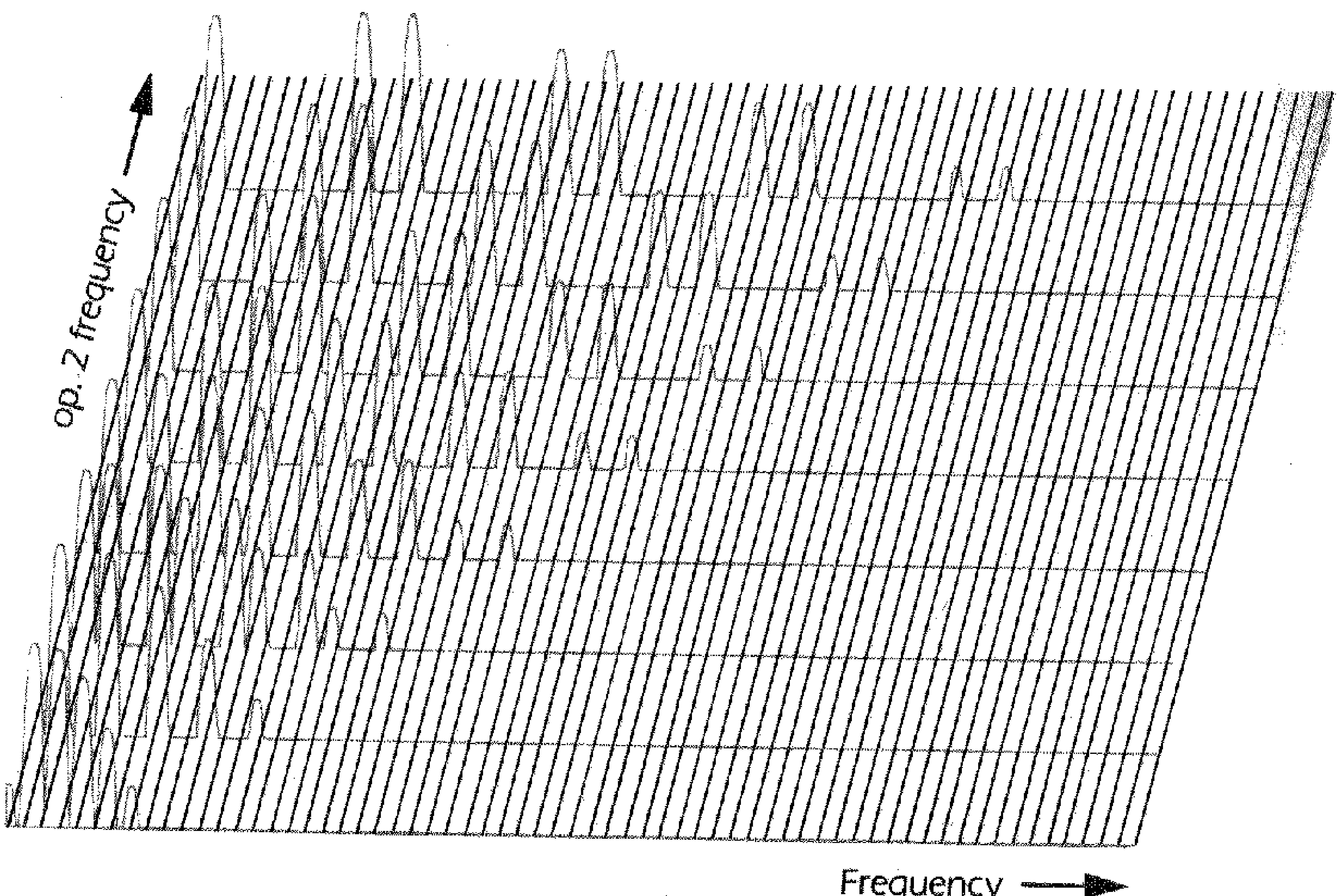
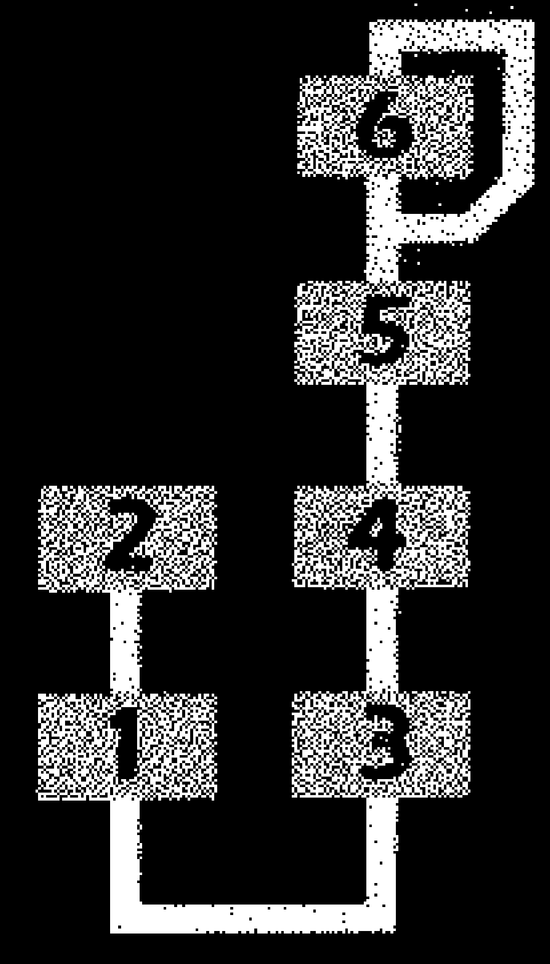


Fig 5.4a

"X"-ample 5.4b



	FREQUENCY	OUTPUT
op 1	1.00	99
op 2	1.00	74
op 3		
op 4		
op 5		
op 6		

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select algorithm 1 and set up the parameters as shown, then increase the Frequency Coarse value of operator 1 (carrier) by 1.00 repeatedly with data entry + 1 from 1 to 8, which corresponds to successive traces of the 3D plot shown below.

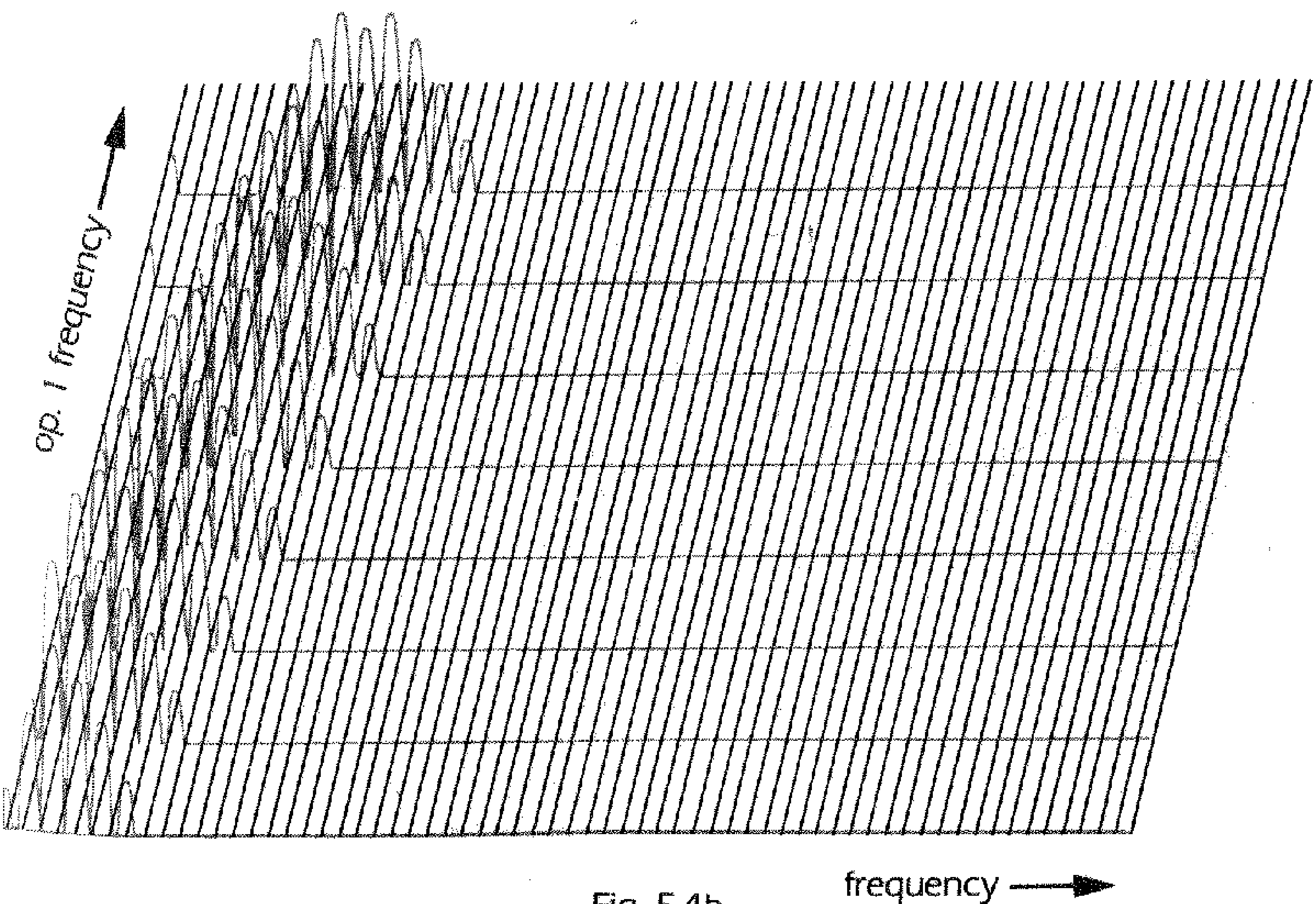
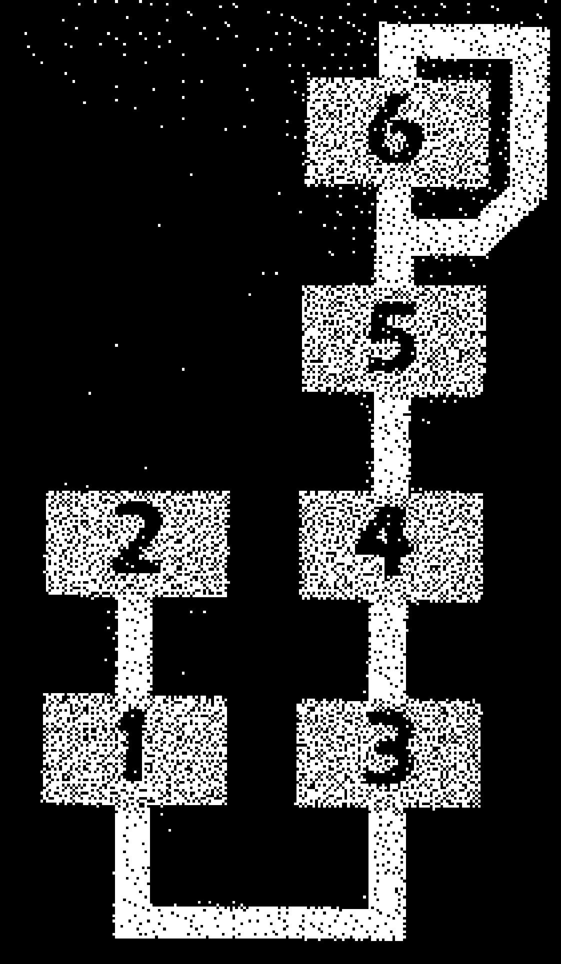


Fig. 5.4b

"X"-ample 5.4c



	FREQUENCY	OUTPUT
op 1	1.00	99
op 2	1.00	74
op 3		
op 4		
op 5		
op 6		

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select algorithm 1 and set up the parameters as shown, then increase the modulation index by increasing the output level of operator 2 (modulator) in steps of 3, which correspond to successive traces of the 3D plot shown below.

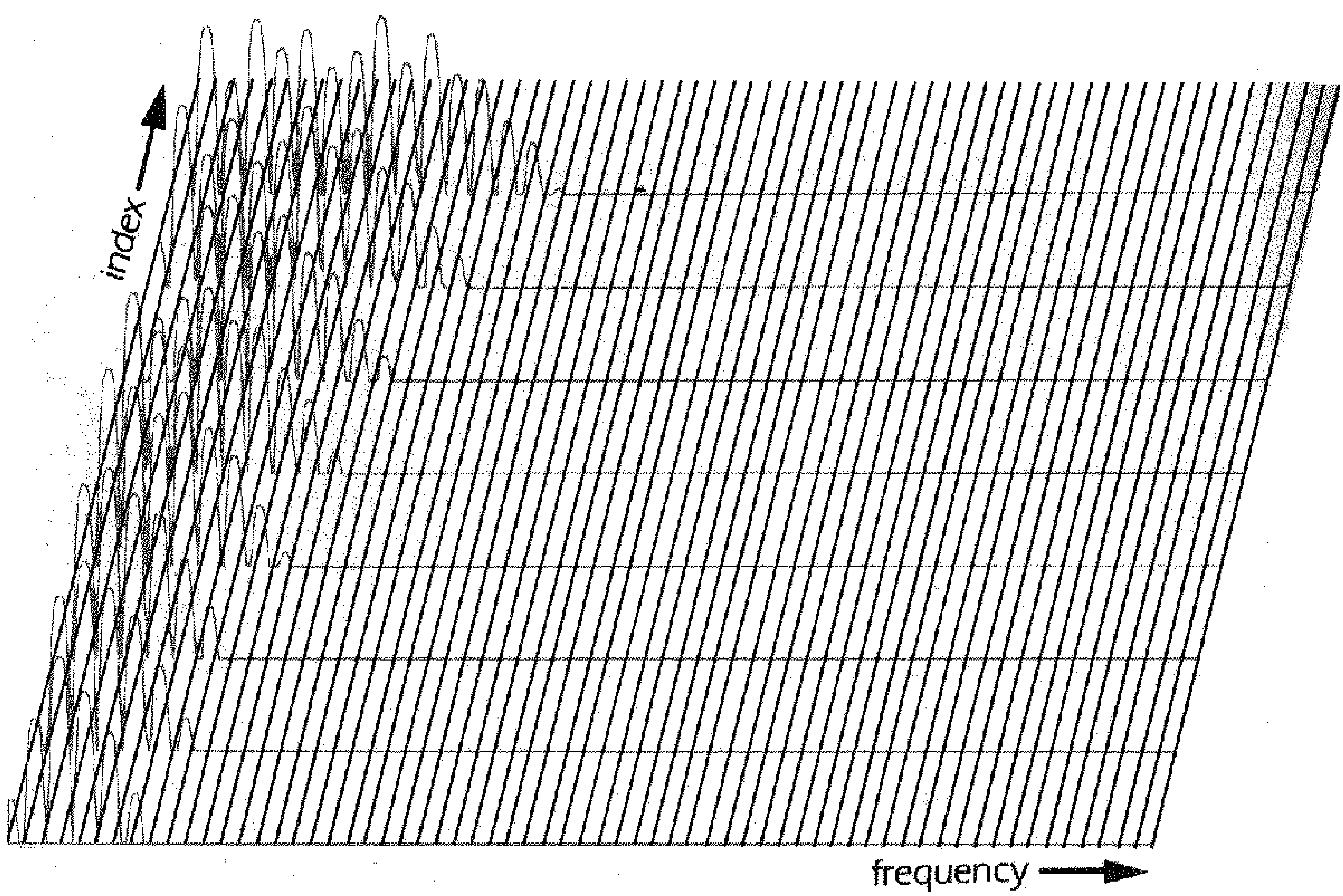


Fig 5.4c

Set up the following "x"-ample which reminds us that what we describe in the physical world of sound is not always what we seem to hear!

"X"-ample 5.5

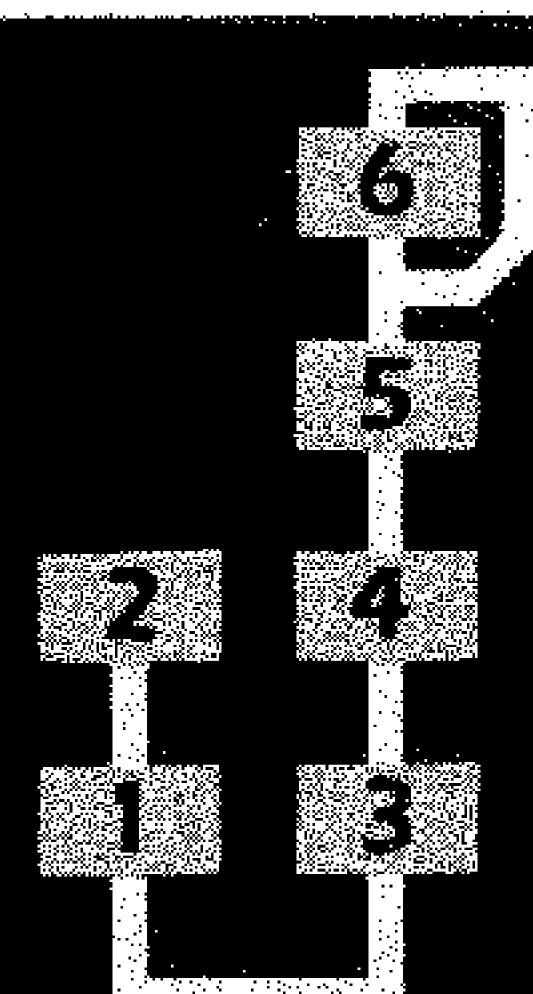
	FREQUENCY	OUTPUT
op 1	1.00	90
op 2	2.00	61
op 3	3.00	85
op 4	4.00	99
op 5	5.00	86
op 6	6.00	63

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select ALGORITHM 32. Now set the values as shown above. Listen to the result and then turn off operator 1. Notice that while there is no energy at 1 (the "pitch frequency"), we nonetheless perceive the same pitch. This is called residual pitch.

Now, by setting up the following Xample, we can see that it is possible to achieve the same spectrum that in "X"-ample No. 5.5 used five operators in an additive configuration, with just two in a simple FM configuration.

"X"-ample 5.6



	FREQUENCY	OUTPUT
op 1	4.00	92
op 2	1.00	62
op 3		
op 4		
op 5		
op 6		

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select algorithm 1 and turn off all operators except 1 and 2, then set up the values as shown. Store this sound, and compare it with that in the previous example.

A plot of the spectrum from the sound of the last "X"-ample (5.6) is shown in Fig. 5.5.

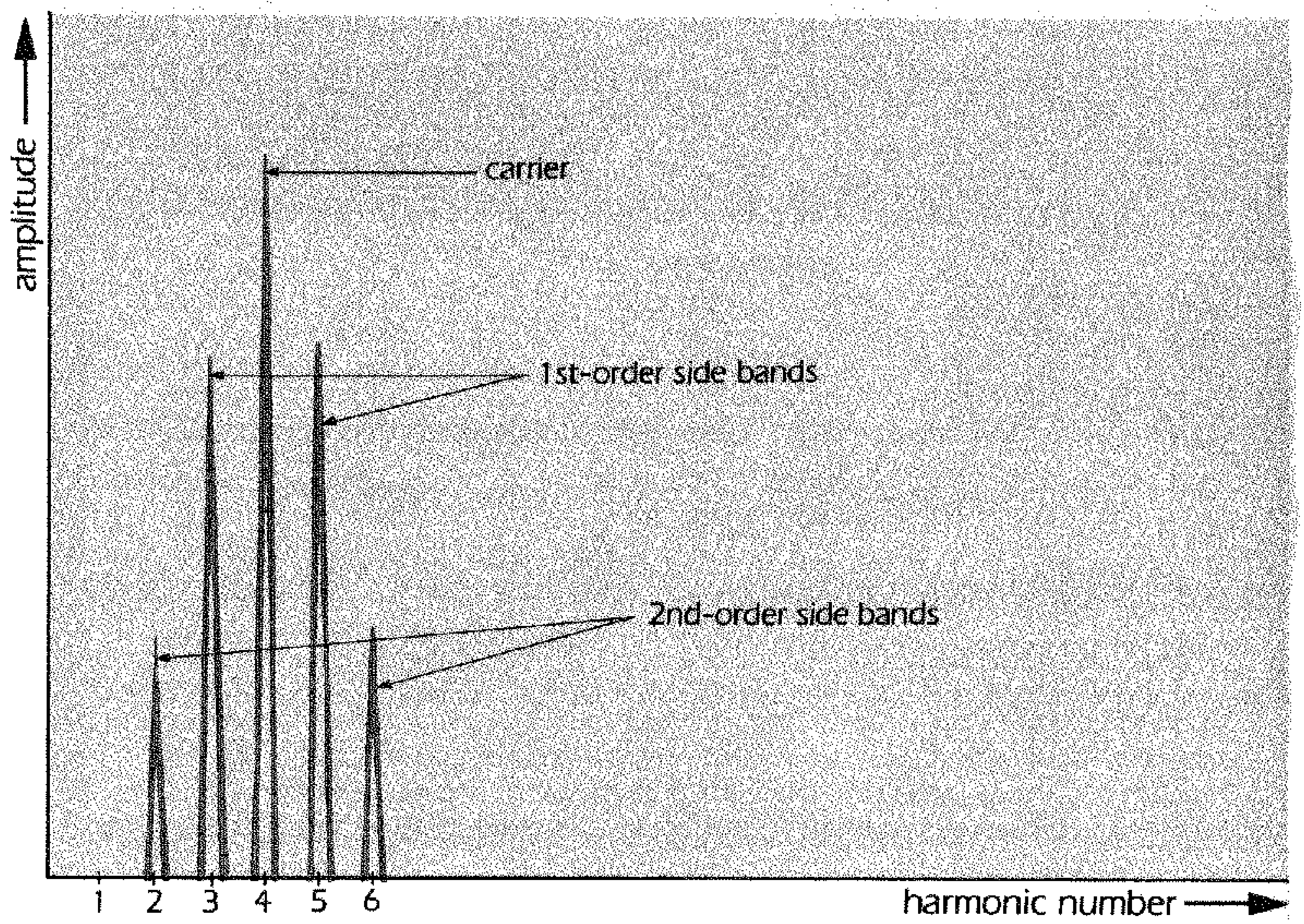


Fig. 5.5

Spectrum for $c : m = 4 : 1$ and $I = 1$. While there is no energy at the 1st harmonic, the pitch is nonetheless perceived to be at that frequency, called residual pitch.

If one of the two operators is detuned, will there be a strong effect of beating? Not unless we increase the index such that there are reflected side frequencies. Return to "X"-ample No. 5.6. Now select detune for operator 2, and increase it to 3 or 4. There is no discernible effect, as would be expected – a glance at the spectrum shows that there are no reflected components. But now increase the index by raising the output level of operator 2 a step at a time up to about 90. You can hear the beating effect increasing as more side bands are produced which reflect back from the negative domain to fall just alongside positive components.

The theory works! A simple application of the first rule, $c \pm km$ (for $I + 2$ side bands), will show you that an index of more than 4 is enough to increase the bandwidth of the spectrum from a pair of FM operators at a ratio of $4 : 1$ such that there will be reflections from the negative domain. Try this for yourself.

The final spectrum that we will plot in the context of simple FM is based upon a ratio of $c{:}m = 0{:}1$. This is a bit curious because of the fact that the carrier frequency is 0Hz. It is important to understand, however, because there are "X"-Series synthesizer applications which can only be understood in terms of this ratio.

So we do the obvious by simply "plugging" the values into our, by now familiar, system and seeing what results. Any index will do, but we will choose a small value to make it simple, $I = 1$:

$$J_k(1), (0 \pm k1)$$

for

$$k = 0,1,2,3 \ldots n$$

where

$$n \text{ approx} = I + 2 = 3$$

Frequency Components for Simple FM where

$$c{:}m = 0{:}1 \text{ and } I = 1$$

AMPLITUDE	FREQUENCY		
Amplitude Coefficients	Side Frequencies		
(scaling)	odd order	Lower	Upper
$J_0(1) = 0.78$		$c = 0$	
$J_1(1) = 0.42$	(-1)	$0-1=-1$	$0+1=1$
$J_2(1) = 0.1$		$0-2=-2$	$0+2=2$
$J_3(1) = 0.03$	(-1)	$0-3=-3$	$0+3=3$

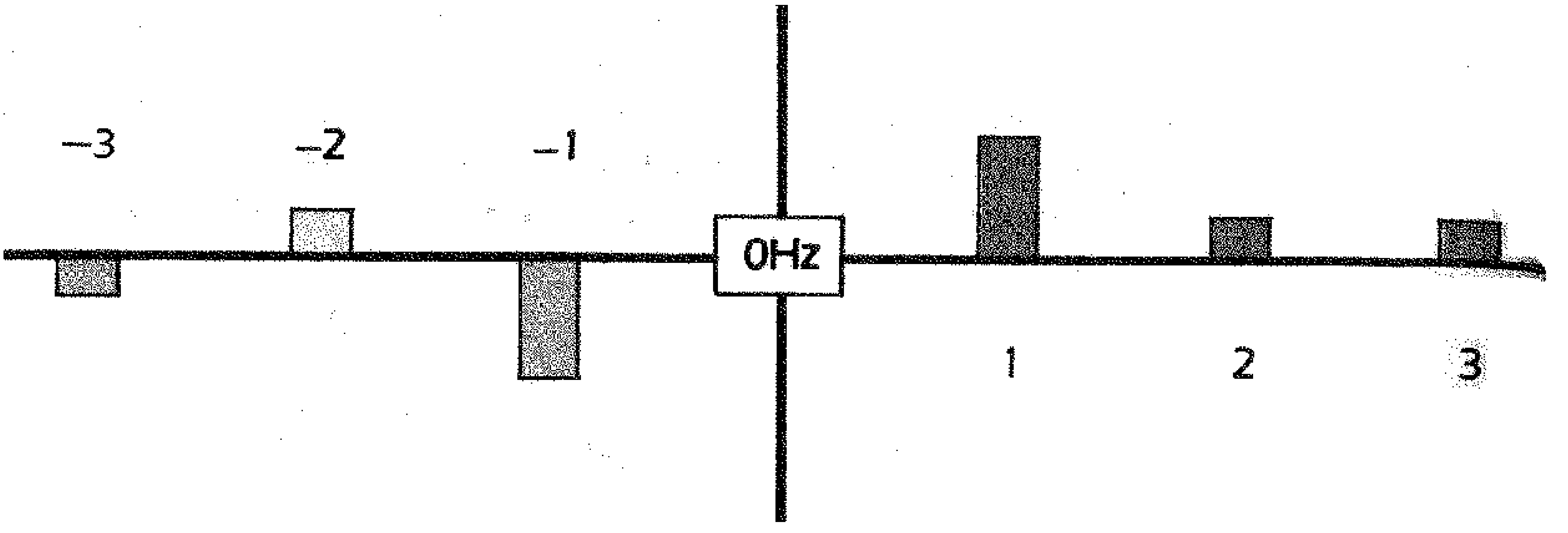


Fig. 5.6a
Plot of a ratio of ***c:m = 0:1*** and ***I = 1***. Because the carrier frequency is at 0Hz, frequencies are symmetrical around 0Hz.

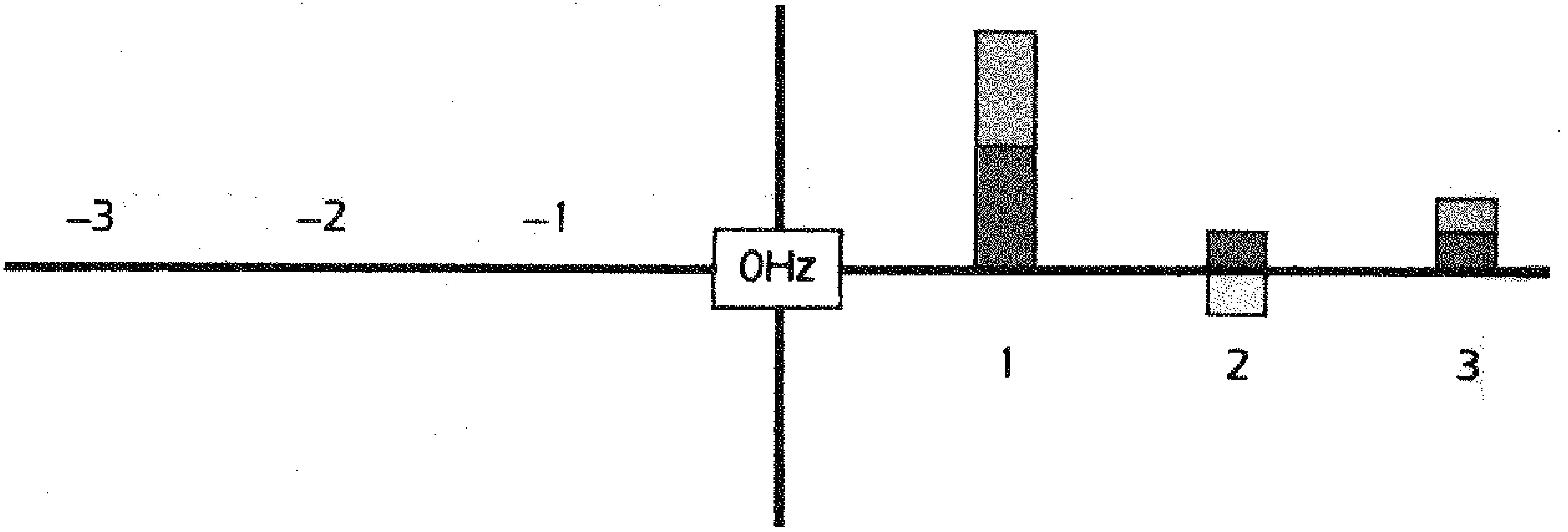


Fig. 5.6b
Now the components are shown reflected about 0Hz into the positive region by changing the sign, and summing with the positive components. All of the even numbered harmonics will cancel, leaving in Fig. 5.6c . .

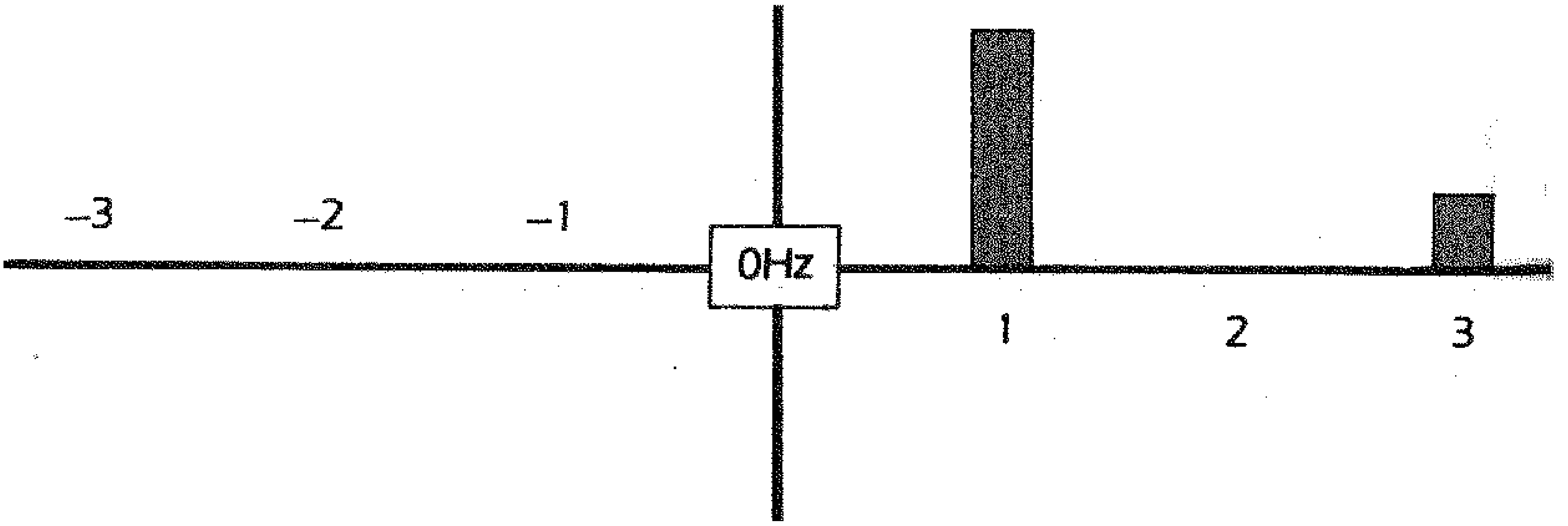


Fig. 5.6c
The normalised spectrum.

What would result if, instead of a carrier frequency of 0Hz, we set the carrier at 1Hz, which is possible on the DX7 (not a ratio of 1 but a fixed frequency of 1Hz)? In order to understand, we will consider a given key on the keyboard. Suppose that we press the key for A2 on the synthesizer (just below middle C). Then with a frequency ratio of 1, the modulating frequency would be 220Hz. Now, using *these* values in our table:

AMPLITUDE	FREQUENCY		
Amplitude Coefficients	Side Frequencies		
(scaling)	odd order	Lower	Upper
$J_0(1) = 0.78$		$c = 1$	
$J_1(1) = 0.42$	(-1)	$1 - 220 = -219$	$1 + 220 = 221$
$J_2(1) = 0.1$		$1 - 440 = -439$	$1 + 440 = 441$
$J_3(1) = 0.03$	(-1)	$1 - 660 = -659$	$1 + 660 = 661$

We can see that the exact cancellation of Fig. 5.6 will not occur, but rather there will be two components one cycle above and below each harmonic at 220, 440, 660Hz, etc. as shown in Fig. 5. 7.

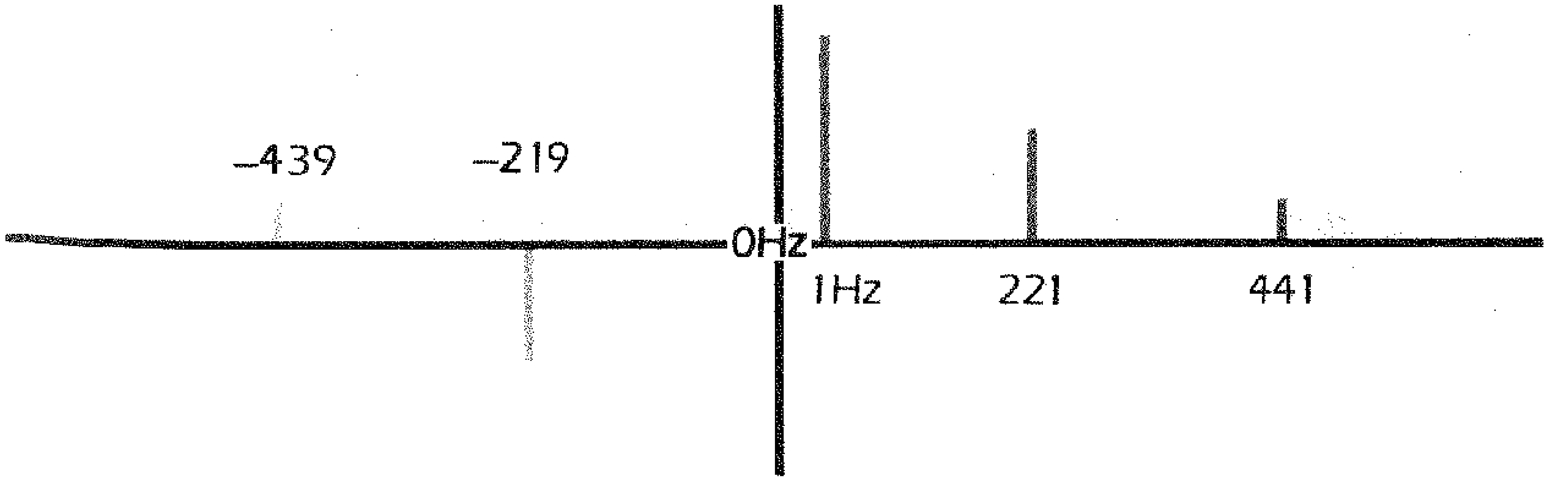


Fig. 5.7a

Plot of the spectrum where $f_c = 1$ Hz, $f_m = 220$ Hz and $I = 1$.

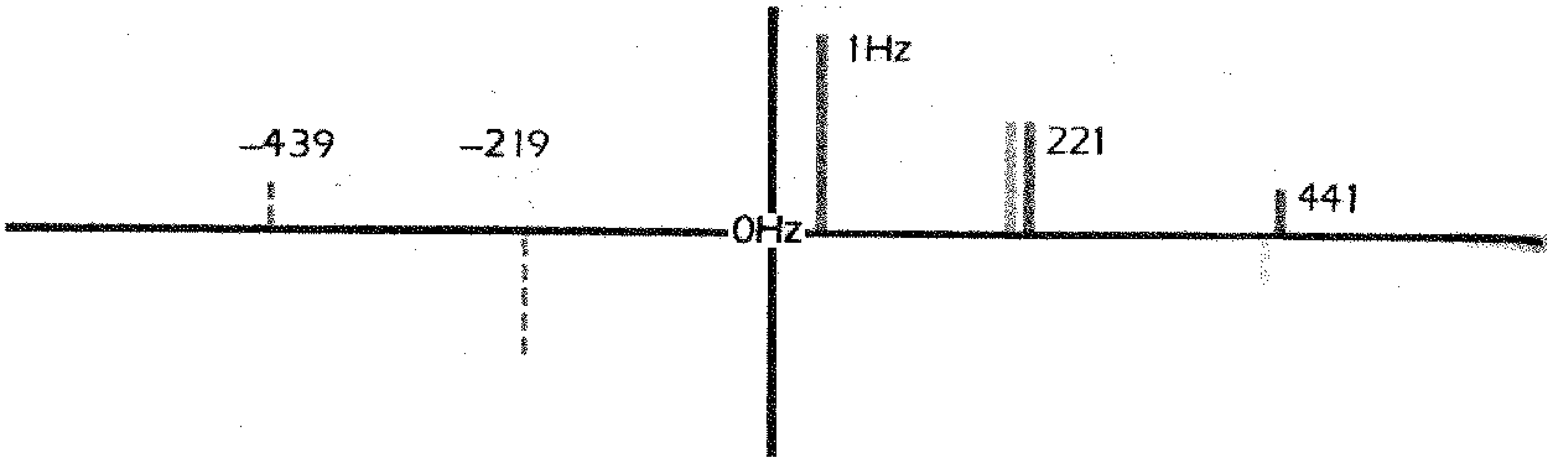


Fig. 5.7b

Because the carrier frequency is no longer at 0Hz, the reflected side band components no longer add to the positive components.

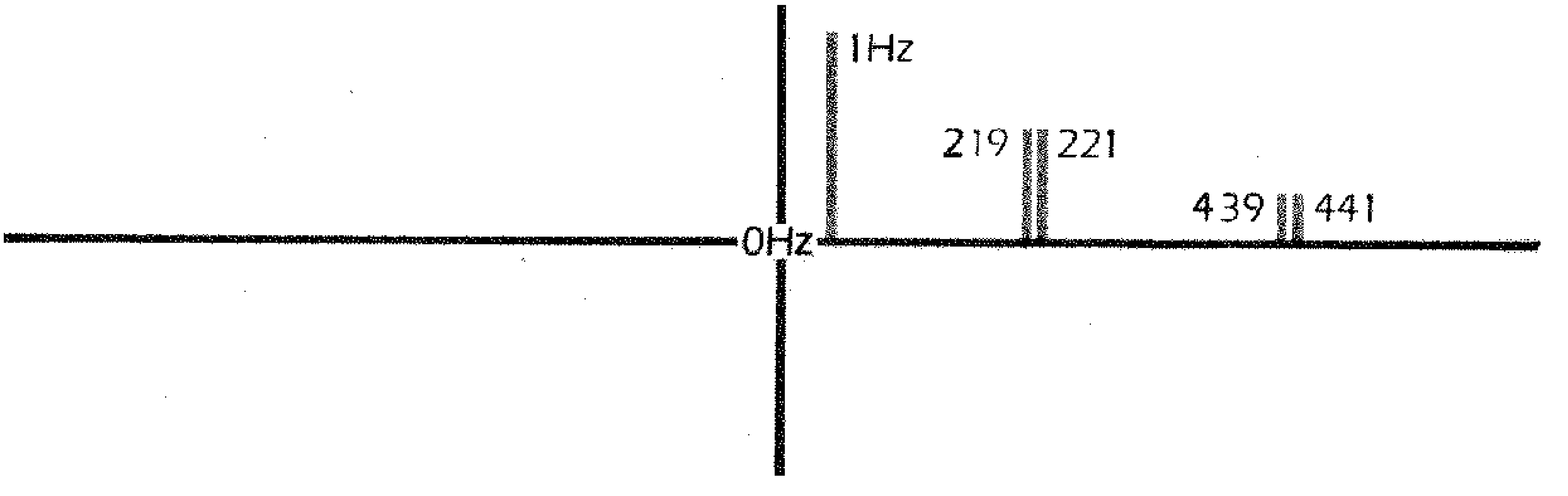


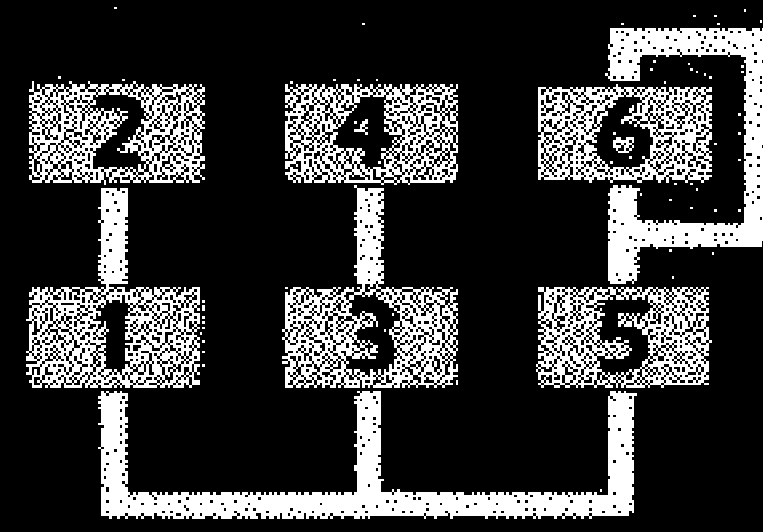
Fig. 5.7c

The magnitude (normalized version) of the spectrum. Beating will result. (Note that the component at 1Hz is too low to be heard.)

The rich beating which results from the use of a low fixed frequency carrier can be used to great advantage when building sounds on the DX7. You can investigate this with the following "X"-ample ("X"-ample 5.7).

Note that when the wheel is at 0 there is no sound. Why? Because the carrier is not being modulated ($I = 0$) and it is below the lower threshold of hearing, which for a sine tone is at about 20Hz. As you increase the output by turning the wheel you are increasing the modulation index from 0 upwards, adding more and more side band components, half of which are reflected and beating against the positive upper side band components. This is a pleasing effect, not unlike that given by a rotary speaker cabinet. OK! Let's complete the electric organ sound before going on to the next topic! First, set AMS = 0 on operator 2, then repeat the parameters for operators 1 and 2 on the remaining pairs. Just increase the Frequency Coarse values of operators 4 and 6 to increase the overall bandwidth, and choose different, low

"X"-ample 5.7



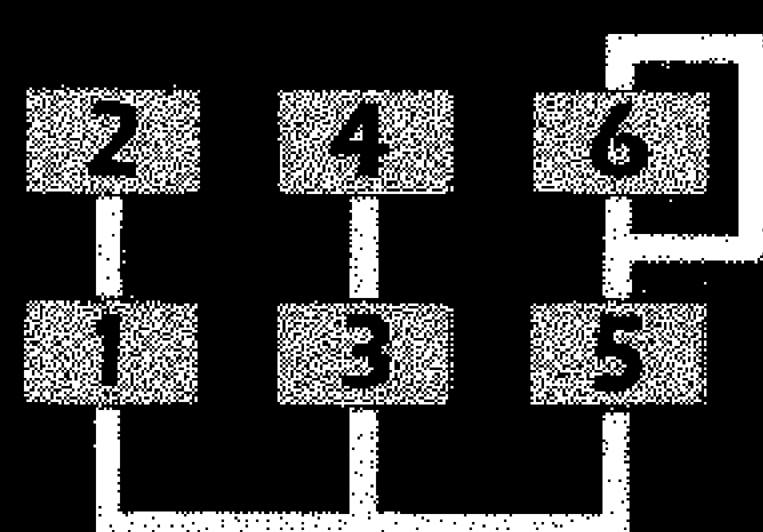
	FREQUENCY	OUTPUT	AMS
op 1	1.000Hz (fixed)	99	0
op 2	1.00 (ratio)	84	3
op 3			
op 4			
op 5			
op 6			

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select ALGORITHM 5 then set up the parameters shown. Using the modulation wheel, EG BIAS = ON, hold the key A3 and gradually turn the wheel from min. to max.

fixed frequencies for the carriers, operators 3 and 5, to mimic the different rotational speeds of the speakers in our imaginary cabinet. Your final settings look something like"X"-ample 5.8, but you should be beginning to understand a little about simple FM now in order to make your own adjustments.

"X"-ample 5.8



	FREQUENCY	OUTPUT
op 1	1.000Hz	99
op 2	1.00 (ratio)	84
op 3	1.288Hz	99
op 4	2.00 (ratio)	84
op 5	2.042Hz	99
op 6	3.00 (ratio)	84

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select Algorithm 5, then set up the above values (you should already be halfway there if you have done "X"-ample 5.7!).
Try putting some velocity sensitivity on operators 2, 4 and 6. And perhaps this "organ" would be more musically-useful if it were transposed down by one octave. (The plots in Fig. 5.8 show visually the beating effects obtained in a sound like this.)
Well, that was easy! Let's move on . . . .

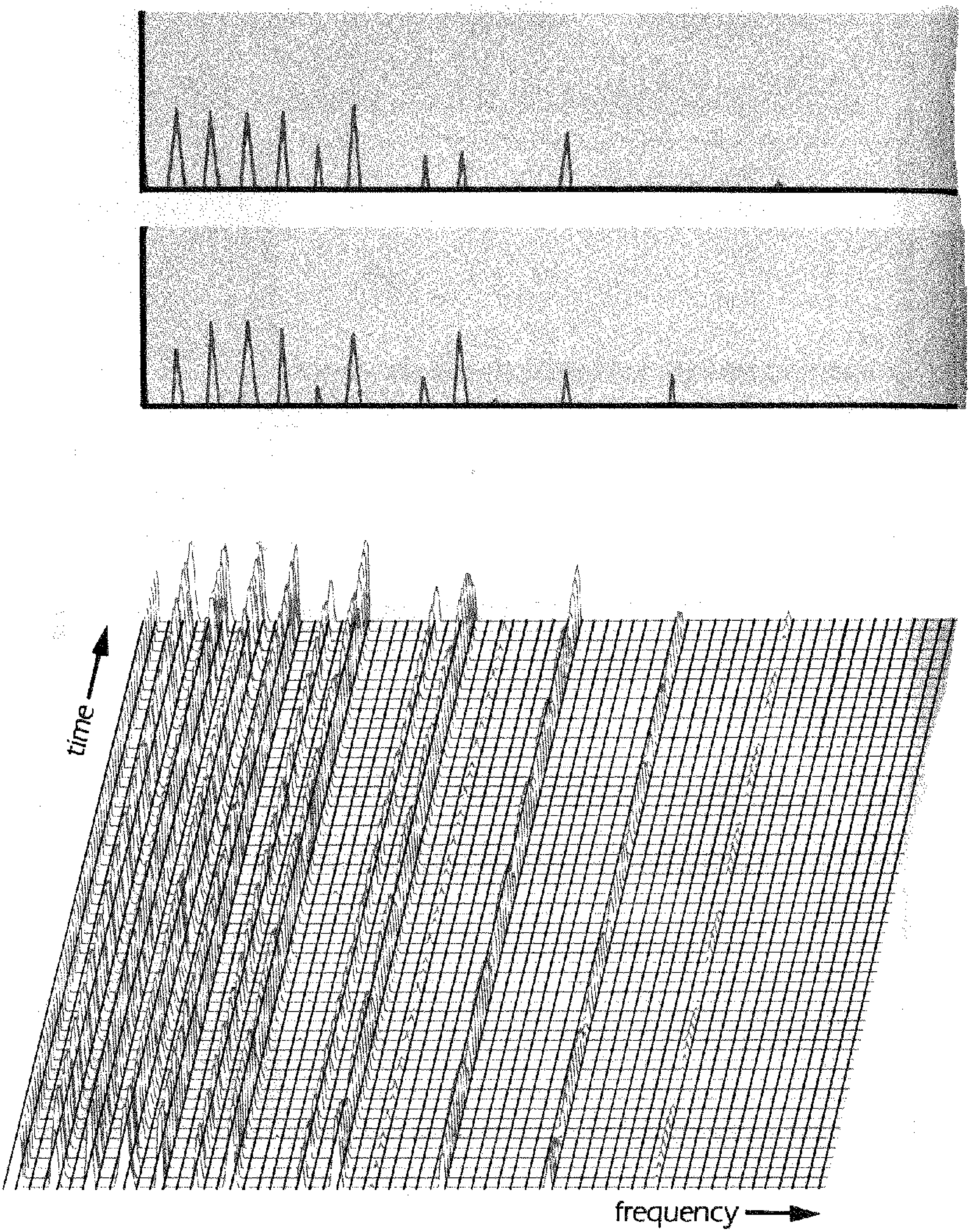


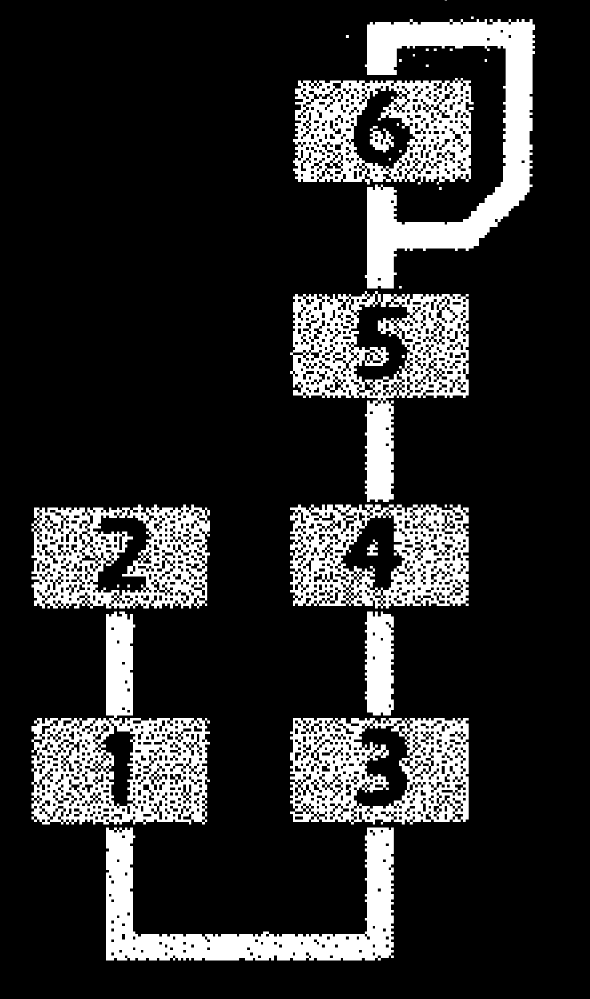
Fig. 5.8

This 3D plot of the sound made in "X"-ample 5.8, clearly shows the amplitude of the components due to beating. The smaller inserts are simply two different traces extracted from this plot.

Set up the following "X"-ample:

"X"-ample 5.9

	FREQUENCY	OUTPUT	AMS
op 1	0.50	99	0
op 2	0.68	99	3
op 3			
op 4			
op 5			
op 6			



INSTRUCTIONS: Starting from the VOICE INIT ? position.....

Select algorithm 1 and set up the parameters shown, and ensure that the MOD WHEEL Range is 99 and the EG BIAS = ON. Press a key around the centre of the keyboard and move the modulation wheel back and forth from min. to max.

The index is now changing from 0 to 12 and back to 0 as you move the wheel. Look at Fig. 5.9 as you do this, noting that you are moving *through* the traces (into the page, as it were) as you do so. It is important to remember that when we change the output of an operator, when it is acting as a modulator, it does not cause a change in the *amplitude* of the carrier wave which it modulates, but rather in the richness of the spectrum.

When the wheel is at minimum the spectrum consists of a single sinus component at the frequency of the carrier, and when the wheel is at maximum the spectrum consists of many components which, in this case, are inharmonically related because the ratio of $c : m = 0.50 : 0.68$.

One of the very great attractions of FM synthesis is that we have such a simple control over the bandwidth (*BW*) of the spectrum. Now suppose we want to make such a change in *BW* of the spectrum in a more controlled way, and possibly in a very short time. That is exactly what the envelope generator allows us to do! (Perhaps this would be a good moment to re-read your owners' manual regarding the *practical* control of the envelopes in your particular synthesizer, as here we shall be more concerned with the *concept*.)

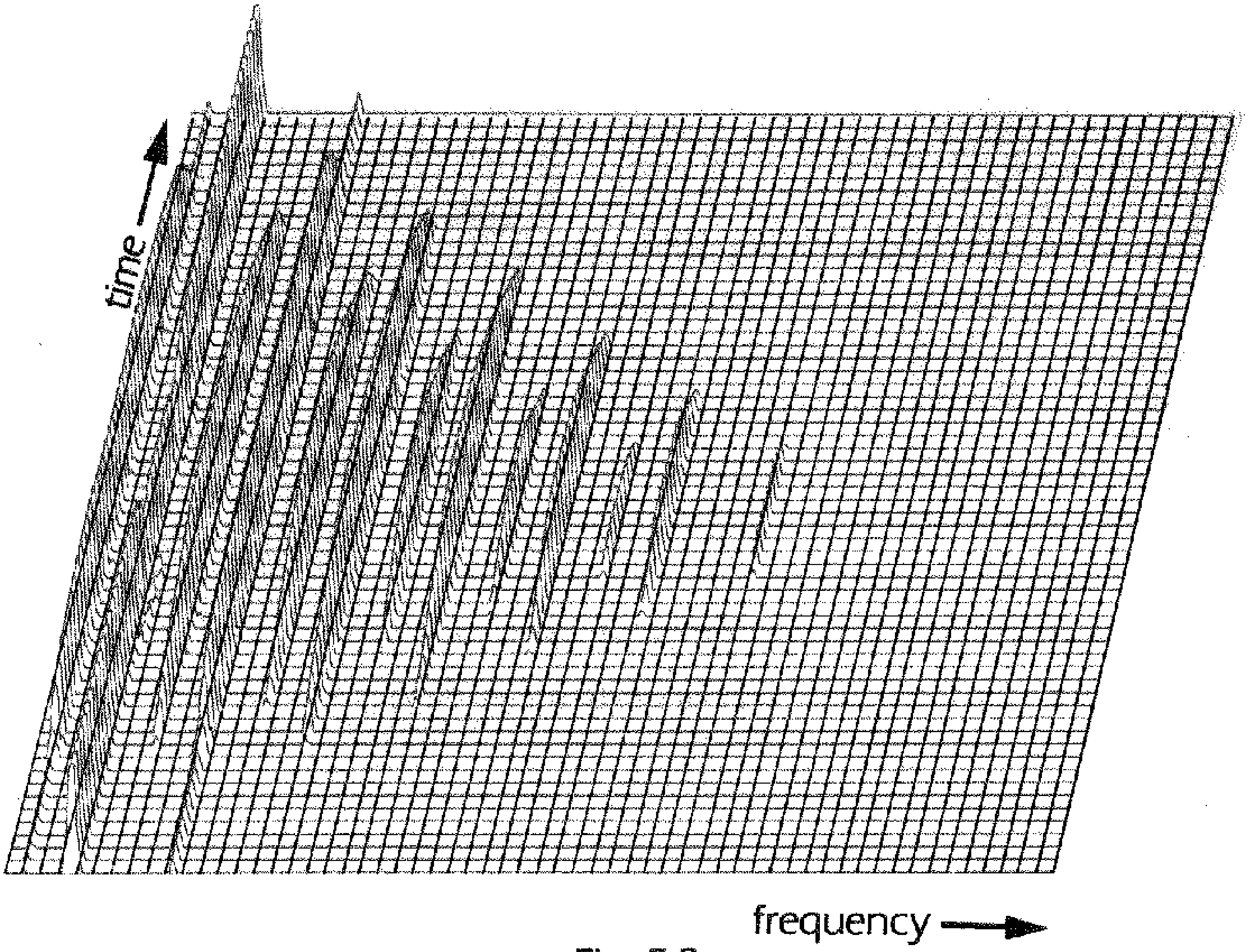


Fig. 5.9

An envelope controlling the amplitude of the output of a generator has one effect when the operator is a carrier, and another effect when the operator is a modulator. When a carrier, the change is one of intensity or loudness, and when a modulator, the change is one of spectral *BW* or timbre. It is very important to grasp this concept and perhaps a visual representation would be useful here, so have a look at Fig. 5.10.

Hint

The envelopes of the carrier and modulator work in different dimensions, the first giving the loudness contour, the second controlling the change of bandwidth – tone colour – over time. The only common points they must share are the "key on" and "key off" events.

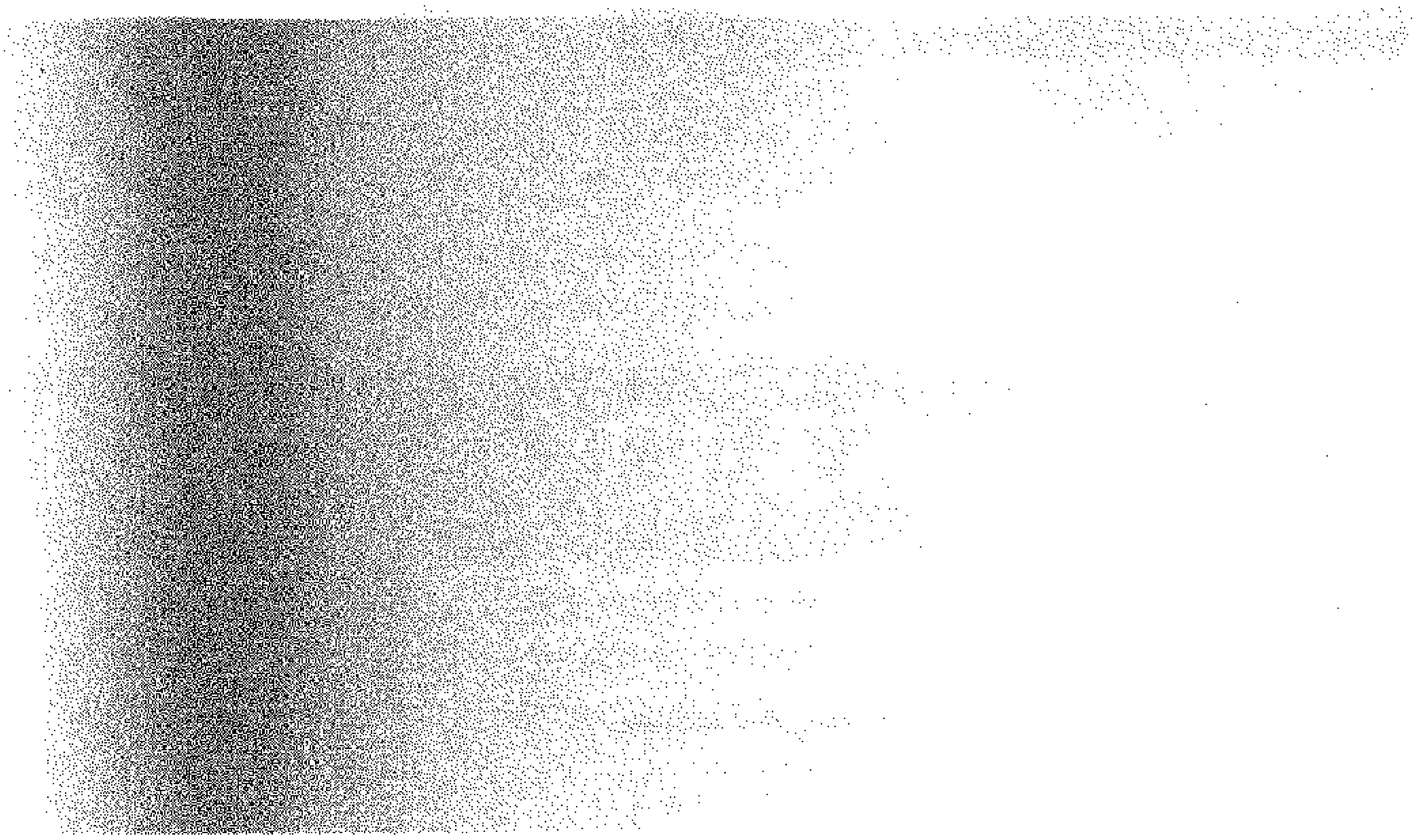


Fig. 5.10a

We can imagine our sound starting out as a blank canvas upon which we paint the tone colour — the depth of the colour as we proceed from left to right representing the richness or depth of the tone as it develops (the changing modulation index or output level of the modulator, as governed by the envelope). The envelope, or shape of the colour change, is the shape of the envelope of our *modulating* operator. (The depth of colour represents the index and consequently bandwidth.)

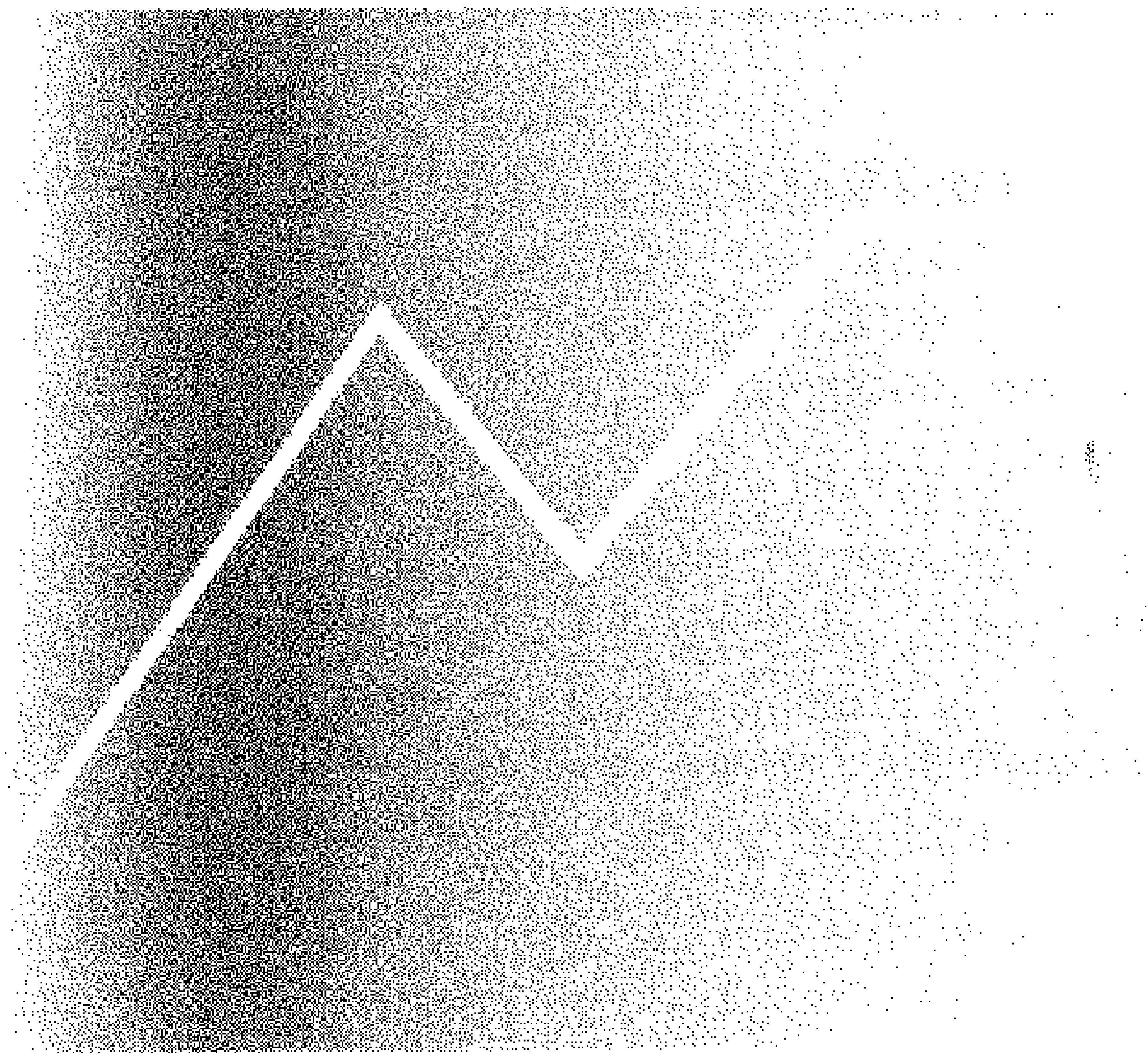


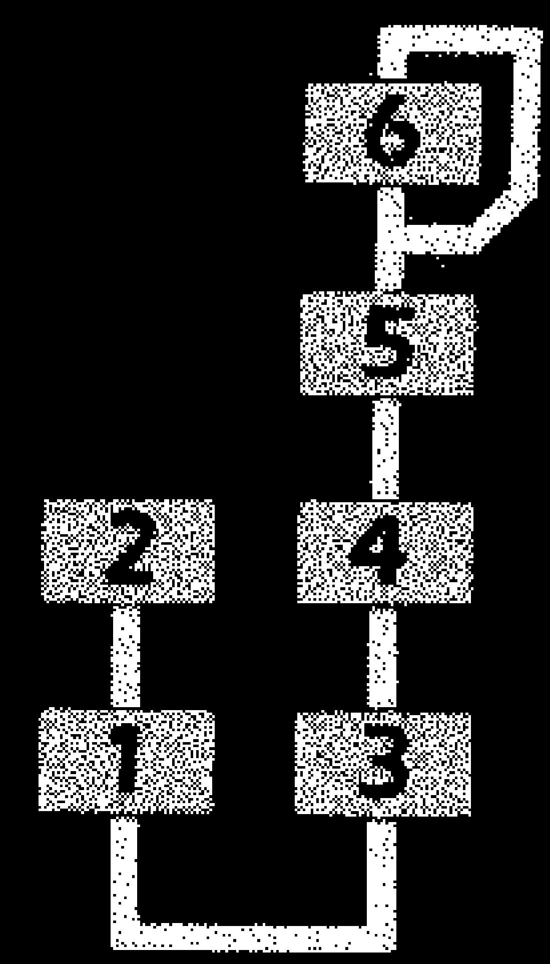
Fig. 5.10b

We now take a pair of scissors and cut out the loudness contour of our sound from the "tone-painted" canvas. This, of course, is the envelope of our *carrier* operator. Notice that the two operators, or envelopes, are working in entirely different dimensions and are independent of each other. The movement or change in bandwidth, governed by the modulation envelope, is completely independent of the carrier envelope.

Now back to our "X"-amples . . .

By placing an envelope on the output of operator 2 in "X"-ample 5.10, we can do exactly what we were doing with the wheel in "X"-ample 5.9, except now in a programmed way. Let's do it!

"X"-ample 5.10



	OUTPUT	AMS
op 2	99	0

L1	L2	L3	L4
99	99	0	0
R1	R2	R3	R4
35	99	35	99

INSTRUCTIONS:

Start from "X"-ample 5.9. Change the parameters of operator 2 to those shown above, not forgetting to make the Amplitude Modulation Sensitivity = 0.

Now press and hold a key. The synthesizer is making the same change of *BW* that we made with the wheel, only now under program control. Increase the rate of increase of the *BW* by raising R1, operator 2, to 99 (nearly instantaneous). Listen and notice that the spectrum changes in a manner similar to that of a bell. What do we need to do to make a more realistic bell, then?

First, we will program an envelope on operator 1 that causes a change in loudness or intensity that is appropriate. Then we shall copy it to operator 2. An important concept is implied here, that in natural sounds **there is often a correlation between the change of intensity in time and the change of BW in time.** (Continue working with the basic setting of "X"-ample 5.10.)

So, let's construct our envelope on operator 1 using the following values: L1=99, L2=99, L3=0, L4=0, R1=99, R2=99, R3=30, R4=30. Notice that we have made R3=R4. This is simply to allow you to play the sound in different ways depending on your style (holding a key or tapping it like a bell will result in the same envelope). So, having made the envelope on operator 1, copy it to operator 2. Now we will reduce the index a bit by reducing the output of operator 2 from 99 to perhaps 75. Make the change in fairly small, successive steps as you repeatedly press the key so that you hear the gradual reduction of the *BW* of the sound. Store this simple FM bell, as we shall be developing it in the final chapter.

We will now shorten the whole event in the following way, leaving the levels for both operators the same (select R3 for operator 2). Gradually, as you repeatedly press a key, eg. A3, change the value of R3 of operator 2 from 30 to about 60 or 70. The physical effect is to change the index at a very rapid rate from about 4 to 0, thereby decreasing the *BW* to a single sinusoid just as we were doing at a very much slower rate with the bell-like tone. Perceptually, we are no longer able to follow the decrease in *BW* in time, but rather hear the rapid change as a totality.

Set the values of R3 and R4 on operator 1 to about 50, and to about 60 on operator 2, so that the change of bandwidth occurs regardless of how the key is played (this is what we did with the bell). The effect of causing a rapid change in the *BW* of the sound is to produce a burst of noise at the onset of the tone which is then followed by a sinusoid whose intensity decreases exponentially. Notice that with very small changes in the envelope we have converted our bell sound into a hand drum type sound.

Store these simple FM sounds, as they will be developed in the final chapter. A spectral plot of this simple hand drum sound can be seen in Fig. 5.11, with the partials falling away rapidly. A plot of the completed bell sound can be found in chapter 7, Fig. 7.4.

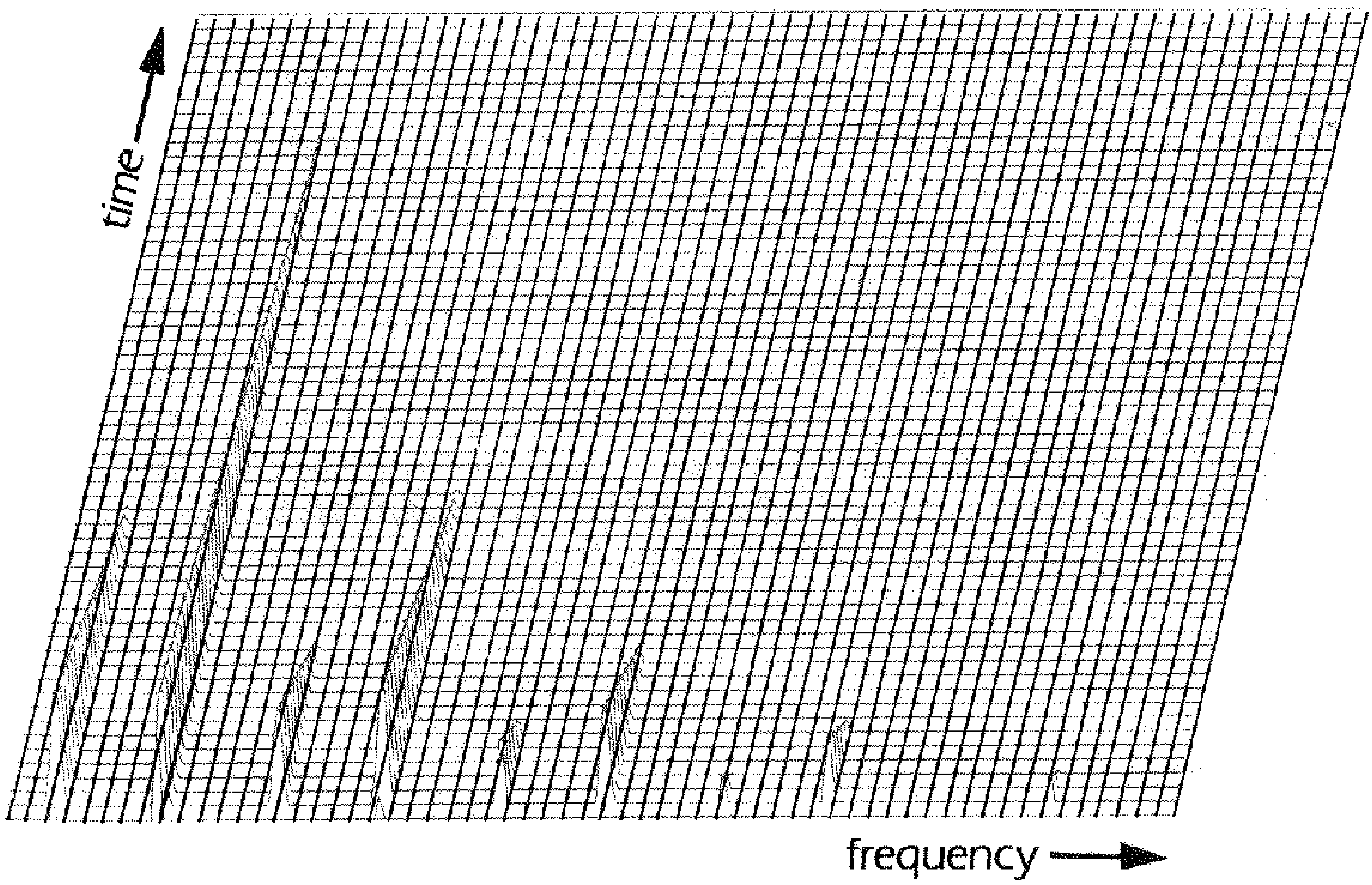


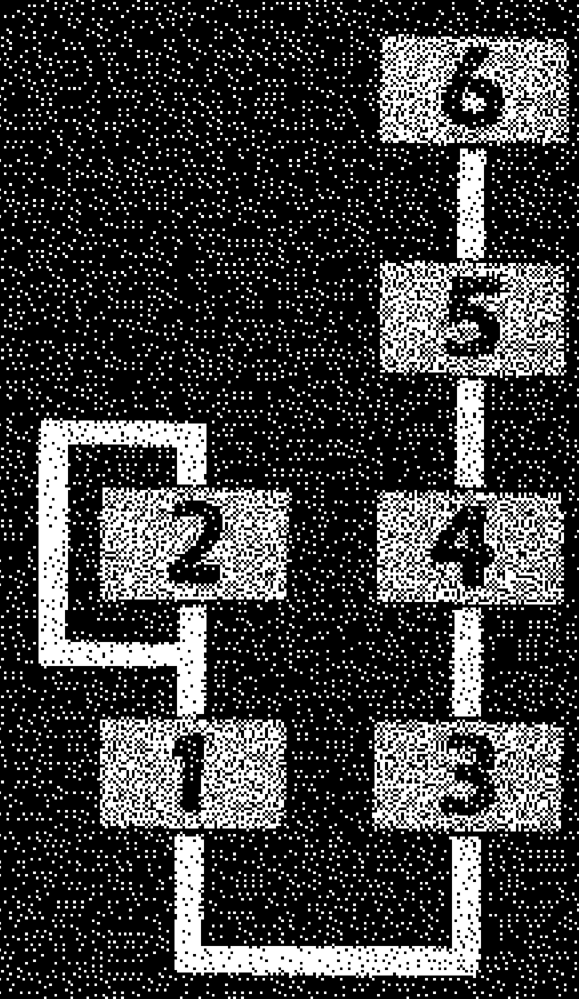
Fig. 5.11

A 3D plot of the simple hand drum sound developed from "X"-ample 5.10.

Before we move on to complex FM, set up the following example and see how the control of bandwidth can convert an "organ sound" into a "brass sound". Don't worry about the use of feedback in this "X"-ample, just observe the envelope control.

"X"-ample 5.11

	FREQUENCY	OUTPUT	R1
op 1	1.00	99	99
op 2	1.00	74	99
op 3			
op 4			
op 5			
op 6			



INSTRUCTIONS: Starting from the VOICE INIT? position....

Select Algorithm 2 and set the feedback values to 7. Now, set the parameters as shown. The sound is very organ-like, but by reducing R1, op.2 to 50 as you repeatedly tap a key, you will notice that the development of the bandwidth in those first few milliseconds of the tone imparts a brass-like quality to the sound.

This, of course, is a very simple version of a trumpet, and we haven't even looked any further into the sound than its opening attack characteristics. But you will see that by the simple control of *bandwidth* over time- the development of the spectrum through the modulation index as governed by the envelope of the modulating operator- is one of the keys to good programming.

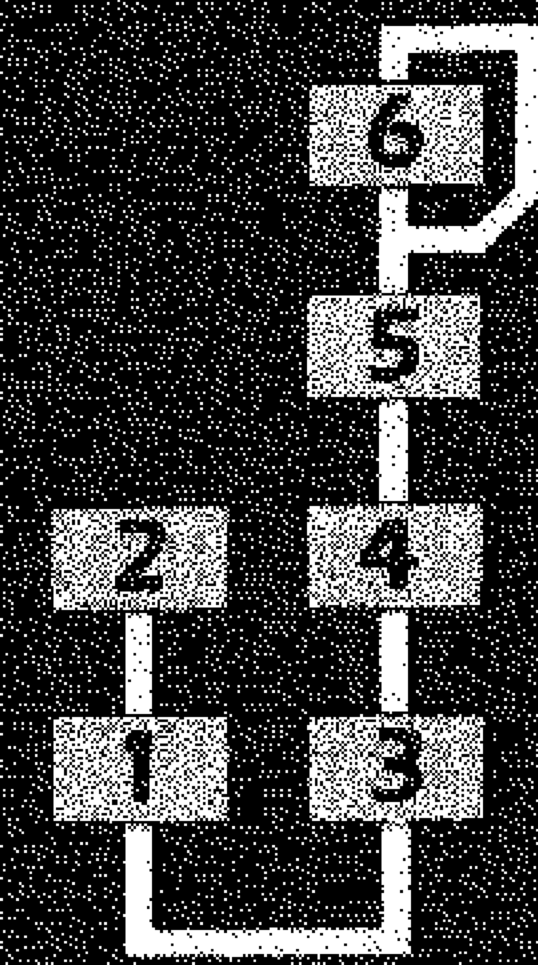
It should also be becoming clear that the tone possibilities with a simple FM pair are enormous, and indeed, in practical programming, it is useful to proceed as far as possible in the search for a sound with simple FM.

We have noted from Table 4.3 that several ratios will produce only odd-numbered harmonics. We have noted in comparing ratios of 1:2 and 1:4 that beating will be apparent in the former and not in the latter. Are there other useful differences that can be made use of?

We have shown above how the envelope controlling the output level of an operator which is in the position of a modulator is in fact a control of the modulation index over time. If we program an index envelope that begins at 0 and reaches its maximum rather slowly, then we hear first the carrier alone (0 modulation produces no side frequencies) and then the gradual appearance of the side frequencies as the modulation takes effect. Now we will do it.

"X"-ample 5.12

	FREQUENCY	OUTPUT	L1	R1
op 1	1	99	90	70
op 2	2	70	90	10
op 3				
op 4				
op 5		(L4	is	at 0)
op 6				



INSTRUCTIONS: Starting from the VOICE INIT ? position...

Select Algorithm 1 and turn off ops. 3–6. Play a pitch and notice at the attack there is only the component at the carrier which is present. Then as the very slow attack of the modulating operator grows from a minimum of 0 to a maximum of 70 the odd harmonics become apparent. (Remember that even though the levels, L1, L2, and L3 are all maximum at 99, the final output of an operator is determined by the interaction of envelope level, output level, and keyboard level scaling. In this case the envelope level reaches 99 but the output level is set at 70. If we were to change the envelope level to90.then there would be a corresponding decrease in the final output.)

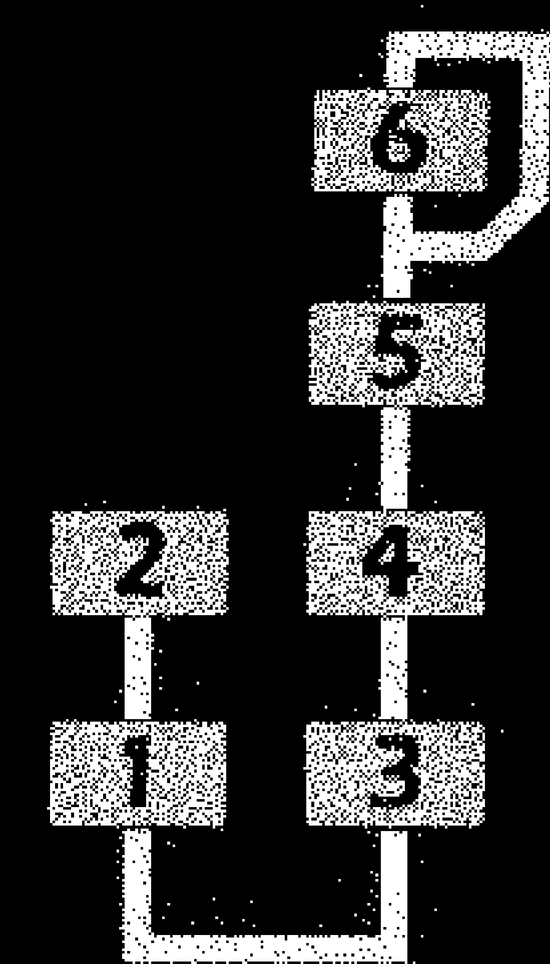
Now how can we use the various ratios that produce similar frequency distributions? Look at Table 4.3 and notice that a ratio of c:m = 3:2 also produces odd-numbered harmonics. Also notice that the carrier is at 3 rather than at 1 as in the case of c:m = 1:2. In fact the fundamental frequency will only appear when there is a bit of modulation. If we change the freq ratio of operator 1 from 1 to 3 and then do the above "X"-ample again the difference is very apparent. Now at the attack we hear a pitch an octave and a fifth above the fundamental (the third harmonic), and a little time later, the fundamental and remaining odd harmonics. As some instruments such as woodwinds produce a higher harmonic prominently during the attack before the fundamental is activated, we can simulate such a rapid transition of the spectral envelope by changing the rate R1 of operator 2 to a value of 50-rapid, but not as rapid as that of the carrier.

Make the change and by repeatedly striking a key with an immediate release you can easily hear the subtle effect. The 3rd harmonic is just audible at the attack. Now, with the data entry slide, experiment with other values for R1 as you continue to strike a key.

Before continuing on to complex FM, there is another important application which is most easily understood here with simple FM but which is extensible to all forms of the algorithms.

At the very end of chapter 2 we introduced briefly the idea of aliasing, using an example which we left in great need of repair. Now we have the knowledge to repair it, so let's do so First, the example again:

"X"-ample 5.13a



	FREQUENCY	OUTPUT
op 1	1.00	99
op 2	5.00	99
op 3		
op 4		
op 5		
op 6		

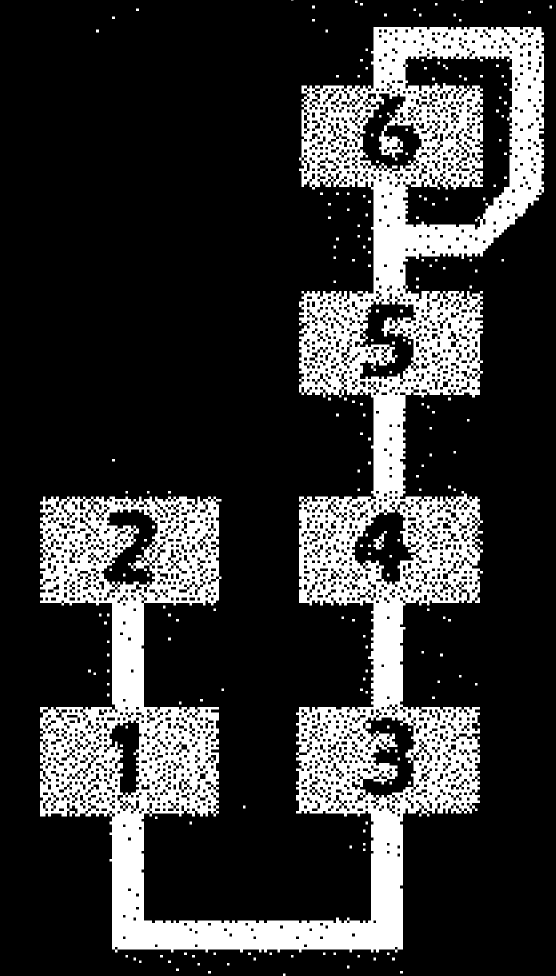
INSTRUCTIONS: Starting from the VOICE INIT? position....

Select Algorithm 1 and turn off op. 3, set ops. 1 & 2 to the above settings. This is a repetition of the example used to demonstrate aliasing in Chapter 2.

Now note by note, beginning at the lowest key (C1), slowly play an ascending chromatic scale. What do you hear? The tone itself is rather rich in harmonics, and as you would expect, the tone rises in pitch as you slowly ascend. Then, at about B3, there seems to be a slight change to the sound, a roughness that was not previously present. Continuing to ascend chromatically, at Eflat4 there is a very noticeable change where there seems to be an additional pitch. All of the remaining notes have similarly noticeable perturbations of the initial sound in a seemingly random manner. Why should that be?

The answer, you will remember, had to do with the fact that we were trying to produce components at frequencies higher than one half the sampling rate. The result was that these frequencies reflected back where we could hear them but not where they would not be noticed. The solution would be to decrease the bandwidth as pitch ascends, so that no side frequencies are produced which are above the half sampling rate. How would we accomplish that? With keyboard level scaling.

"X"-ample 5.13b



	break point	R curve keyscale	R depth
op 1			
op 2	C3	–LIN	40
op 3			
op 4			
op 5			
op 6			

Starting from "X"-ample 5.13a, add the above parameters to Op. 2.

The effect here is to begin to decrease the index of operator 2 (output level) beginning at C3 with a linear curve to a depth of 40. The choice of depth value was made by ear to keep as much timbral richness as possible without allowing a bandwidth great enough to produce the contextually undesirable aliasing ("contextually" because aliasing can in some cases be useful). Key scaling of this sort can be useful in many ways beyond correcting for aliasing. Timbral change as a function of pitch or small differences in tone color from note to note are easily achieved in this way and do much to make the instrument lively and interesting to listen to.

In many of the voices of X-Series, you will find a simple FM pair which constitute the basis of the tone quality of the voice. Turn off the operators one by one until you isolate the pair that makes the greatest difference. As we have seen, even simple FM can be complicated, given the multitude of choices within the FM parameters e.g. ratios and indices) and choices in the domain of control of the parameters e.g. envelopes and level scaling). One very good way to profit from the experience of others is to analyse the available voices (factory voices and ROMs), where you look for the basic FM pair and discover how it works in all of the dimensions which we have considered.

CHAPTER 6

COMPLEX FM

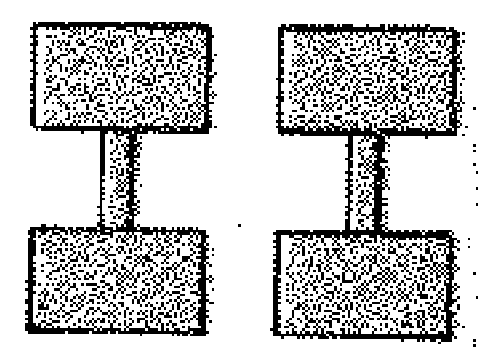
We now have an understanding of simple FM (one carrier and one modulator), which is essential to our understanding of complex FM, where we may have two or more carriers and/or two or more modulators. In particular, we will see how a simple rearrangement of operators can make an enormous difference in the *potential complexity of the resulting spectrum.* The effect of some of these arrangements is fairly obvious, whereas with others it is not. Our purpose is to gain sufficient understanding so that we may make informed choices in regard to synthesis algorithms – it is probably useful in this chapter to see complex FM as an *extension* of simple FM, and thus see algorithms as extensions of one or more simple FM pairs. Also we can effectively analyse the work of others, remembering that one of the most striking attributes of this new technology is the easy communicability of voice data.

We will continue in this section to be rather precise in constructing the spectra, with the understanding that, in actual use, we think in a much more general way about the contribution of the operators to the resulting sound. This *exercise in precision*, while a bit repetitive, should provide us then with the means to make useful generalisations and to explain certain phenomena which must otherwise remain mysterious.

We will begin with a general presentation where we extend the basic concepts of simple FM to include complex FM. We shall stick to the theory in this chapter, leaving the developments and applications to the final section of this book, where we shall finally absorb our understanding into the world of music. The concepts we have learned can find myriad perceptual meanings as they are applied to the making of voice data, both in forming the overall spectral envelope and in adding what has appropriately been called "**stuff**" – meaning those details of a sound such as a bit of noise or frequency skew during the attack which can make a sound lively and palatable.

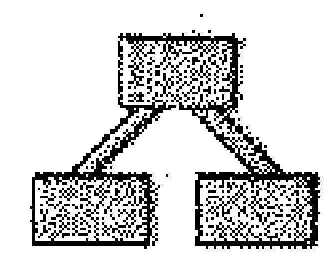
Complex Carrier Wave

Parallel carriers, independent modulators



The simplest of the forms of complex modulation consists of parallel carriers, which is the summing of simple FM spectra. That is, we consider the output of each carrier as simple FM and simply add them together. First we will consider the form where every carrier has its own modulator, as in algorithm No. 5 of the X-Series synthesizer. In Fig. 6.1 we see that the spectral representation of the outputs of the three carriers simply add to form the final spectrum. That is, based on the $c : m$ ratio and the index I for each pair, we compute the spectrum as with simple FM and add them together.

Parallel carriers, one modulator



Next, we will consider the form where there are, again, multiple carriers, but now each is modulated by the same modulator, as is the case in algorithm No. 22 of the synthesizer, where operators 3, 4, and 5 are carriers modulated by operator 6 (Fig. 6.2). In this case we consider each carrier separately and compute the spectrum as we did above. With this algorithm we can exactly duplicate the spectrum of Fig. 6.1 and have two operators left over.

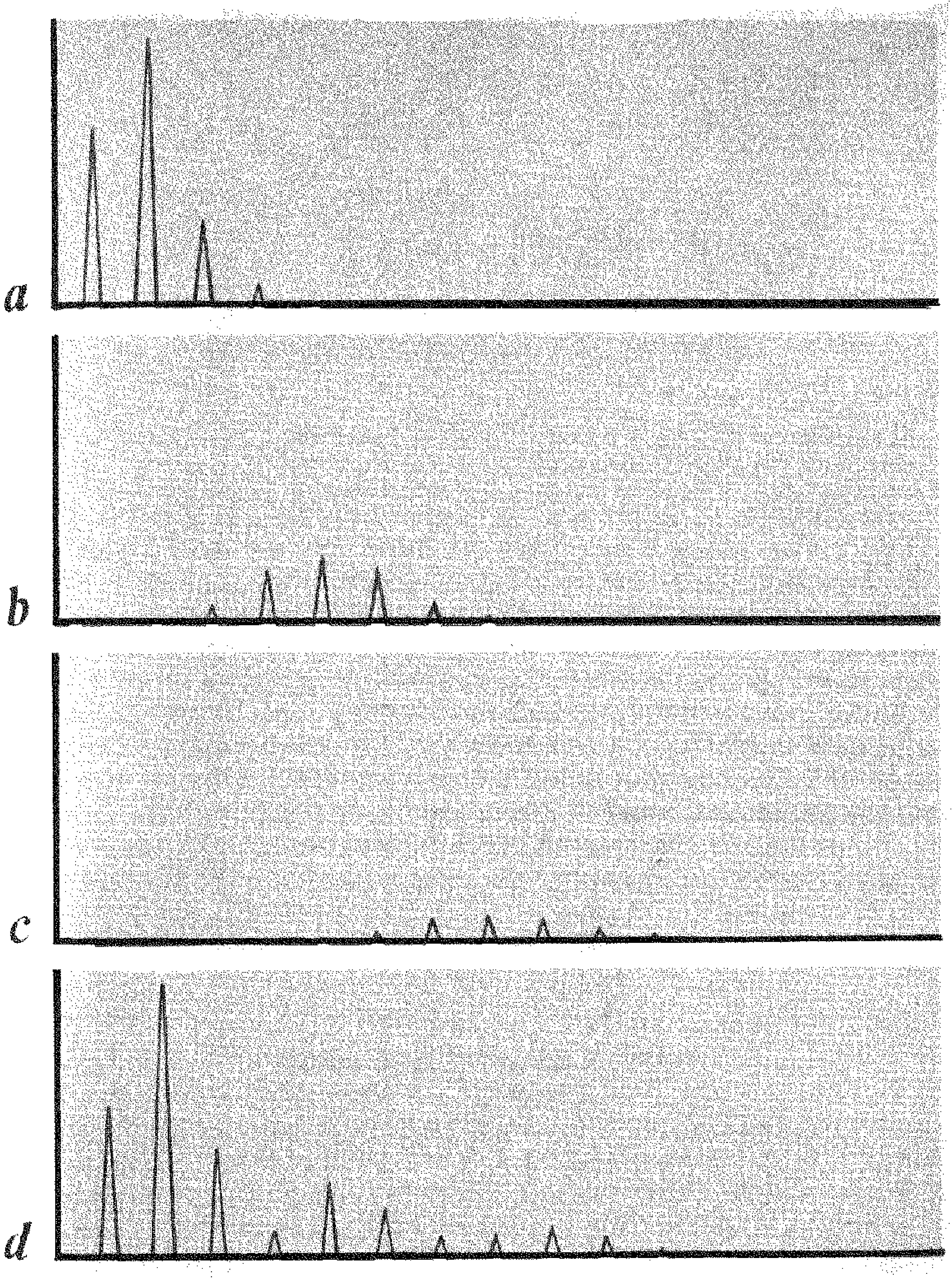


Fig. 6.1

Three simple FM pairs each produce a spectrum as in 6.1a-c, which sum to produce the final spectrum as shown in 6.1d. The ***c* : *m*** ratios for the three pairs are $c_1 : m_1 = 1 : 1$, $c_2 : m_2 = 5 : 1$, $c_3 : m_3 = 9 : 1$. Notice that while the outputs (indices) of modulators in each pair are the same, OL = 70 or $I \neq 1$, and therefore have the same spectral shape, their contribution to the final spectrum is different because of the different output levels of their respective carriers. Try setting this up on your DX7; the carrier values are op 1 = 99, op 3 = 75, op 5 = 55.

Alg. 22

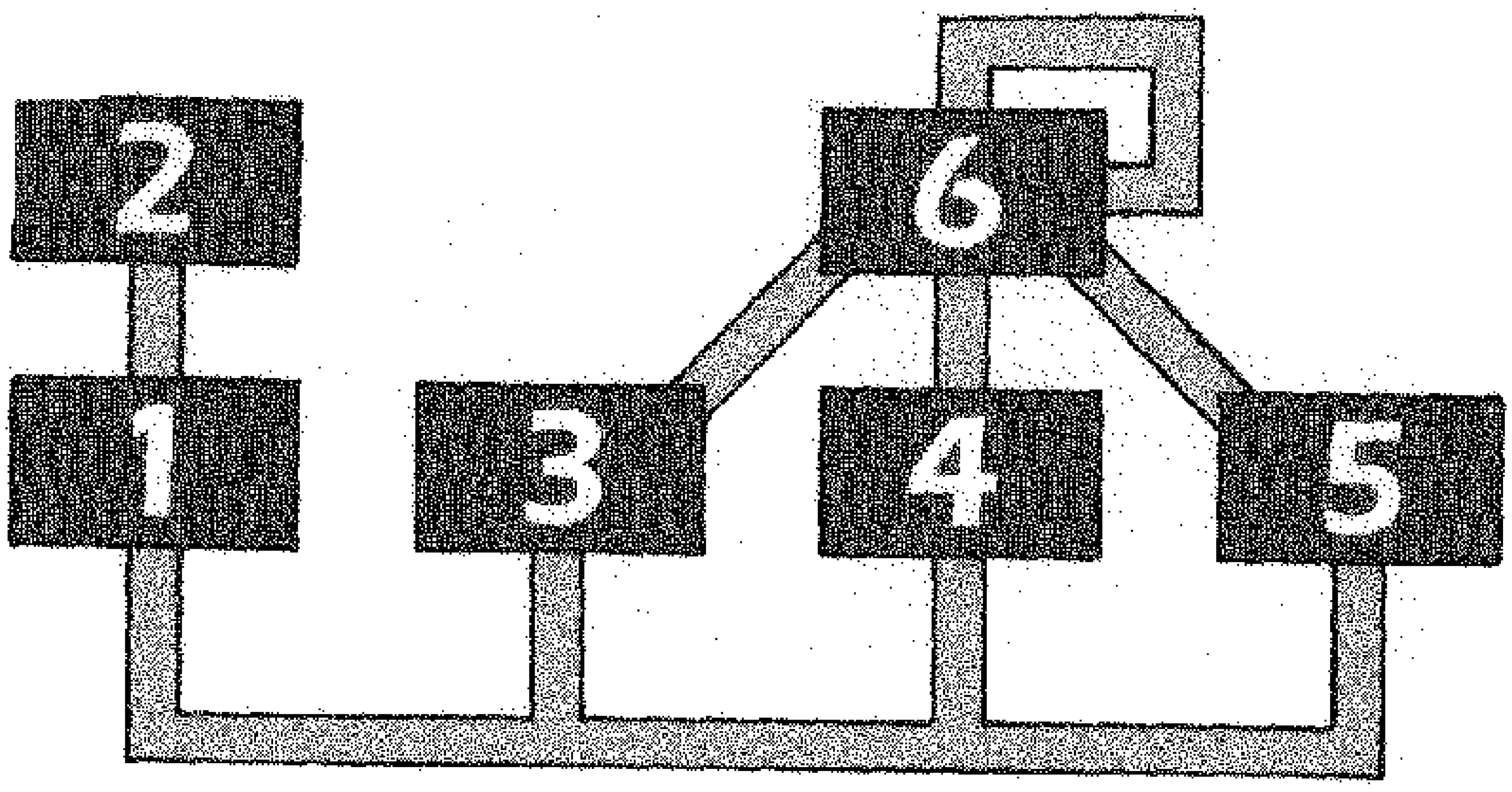


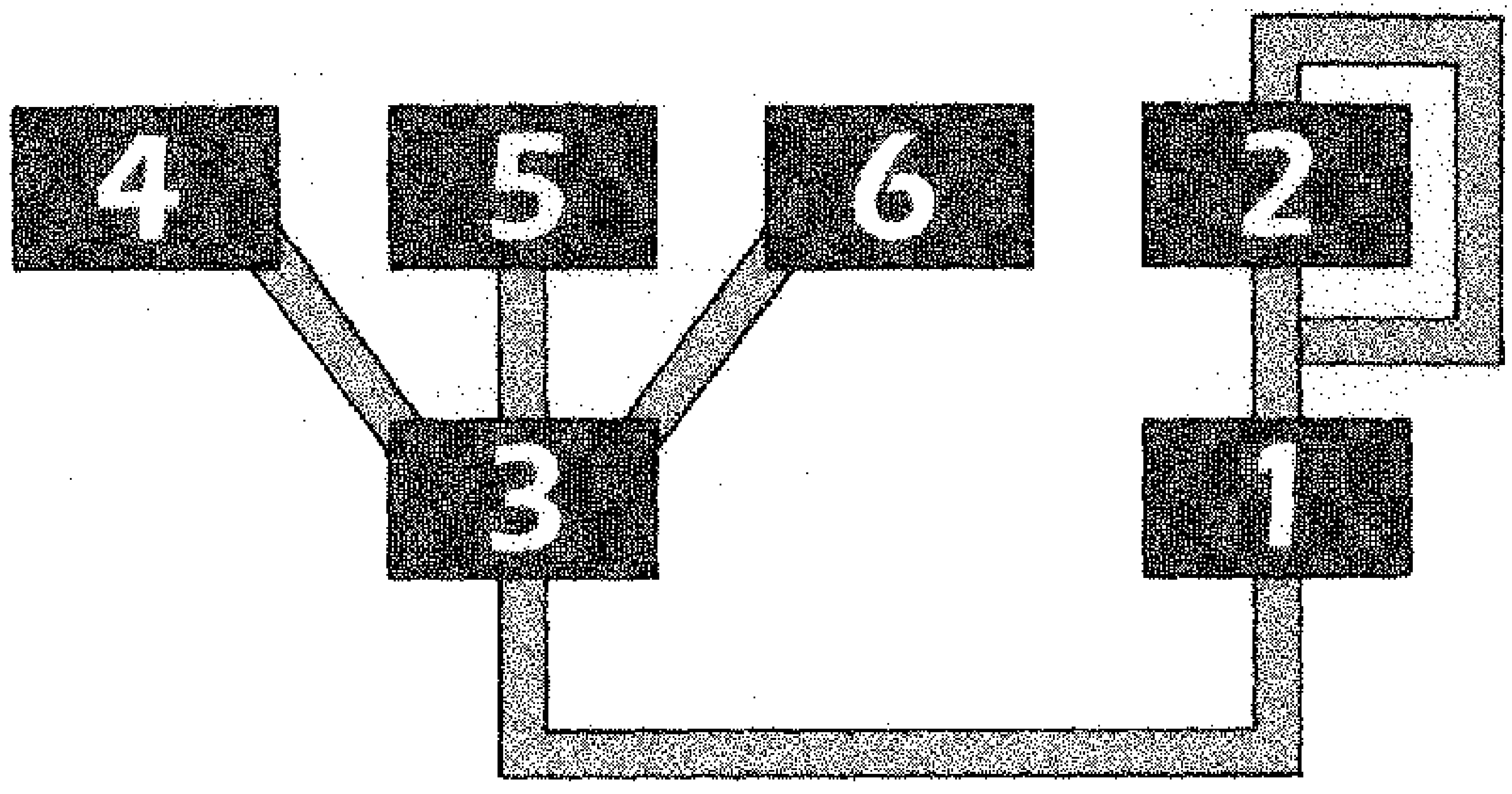
Fig. 6.2

The carrier operators are used exactly as they are in Fig. 6.1, however only one modulating operator is used here instead of three since the same frequency and output levels are applied to all carriers. There are therefore two operators which are not used. It is important to understand, when comparing the results of the above two configurations, that the two different algorithms offer different possibilities. We see that while we gain an FM pair with algorithm No. 22, we do lose the option of having the different indexes (and indeed frequency ratios) given by the three modulators in algorithm No. 5 to each carrier. Given the capability of key scaling with the X-Series synthesizer, this might be an important difference should it be desirable, for example, that the BW of one region of the spectrum decreases while another increases as notes are played in different regions of the keyboard. On the other hand, where this sort of control is not required, the extra "pair" will be useful in extending the complexity of the sound, thus even at this early stage of the chapter we can begin to see the value of theory for efficient or economical programming.

PARALLEL MODULATORS

With parallel modulation we come to perhaps the most difficult aspect of FM synthesis, and perhaps the most useful. It is not difficult because of conceptual or mathematical complexity but rather because a complex modulating wave results in a kind of "explosion" in the number of frequency components which are produced. (Remember that any wave which is not a sinusoid is a complex wave. Therefore if two operators, each of which is a sinusoid, sum to modulate a single carrier operator then the modulating wave is complex).

All that we have learned in regard to simple FM is extended to explain parallel modulators or complex modulation. What we learn in this section will in turn be extended to explain both cascade modulation and feedback. In fact, an understanding of parallel modulators requires an additional but final piece of theory — everything else in this book is extension and application.



Alg. 12

Fig. 6.3

In algorithm No. 12 operators 4, 5, and 6 are modulating operator 3. Here we will only consider operators 3, 4 and 5. As there are two independent outputs there must be two indices, but they are not independent when considering the amplitudes of frequency components in the spectrum.

When there are two modulators in parallel and one carrier (as in the case of algorithm No. 12 & 13, operators 3, 4, 5), we must make expand the relation which we have learned for simple FM to include two modulating frequencies and Bessel coefficients for each. In looking at algorithm No. 13 (we use this algorithm for practical convenience when switching from parallel modulators to a stack or cascade of the same operators in algorithm 14) in Fig. 6.3 we note that as there are two modulating operators (4, 5), each having an independent output level as well as frequency, and there must therefore be two indices. The indices in this case, however, are *not* independent as they were in the case of parallel carriers.

J_{k1} (I_1) times J_{k2} (I_2) is the amplitude scaling factor for each of the frequency components given by:

$$c \pm k_1 m_1 \pm k_2 m_2$$

for

$k_1 = 0,1,2,3 \ldots n_1$ where n_1 approx $= I_1 + 2$

$k_2 = 0,1,2,3 \ldots n_2$ where n_2 approx $= I_2 + 2$

The frequencies which result can be many more than one might think based upon our examination of parallel carriers. Indeed it seems that the simple spectrum from each of the modulators is produced when $k_2 = 0$ and k_1 ranges to n_1 and when $k_1 = 0$ and k_2 ranges to n_2, except that the amplitudes result from the *multiplication* of Bessel coefficients. But then in addition, there must be component frequencies resulting from all combinations of the frequencies in the simple spectrum as well. These we call **combination side frequencies** which occur when neither k_1 nor k_2 is 0. That is, we let $k_2 = 1$ and k_1 range from 1 *to* n_1 then let $k_2 = 2$ and again let k_1 range from 1 *to* n_1. This process continues until $k_2 = n_2$. Obviously, the larger the values for I_1 and I_2, the greater the number of components which will result. Now, we will form this relation as a table where the explicit representation extends through to $n_1 = 3$ and $n_2 = 3$.

Table 6.1

Frequency Components for Complex FM
parallel modulators
$c : m_1 : m_2$

Amplitude Coefficients (scaling)	Simple Side Frequencies			
	odd order	Lower		Upper
$J_0(I_1) \times J_0(I_2)$		c (carrier)		
$J_1(I_1) \times J_0(I_2)$	(−1)	$c - m_1$		$c + m_1$
$J_2(I_1) \times J_0(I_2)$		$c - 2m_1$		$c + 2m_1$
$J_3(I_1) \times J_0(I_2)$	(−1)	$c - 3m_1$		$c + 3m_1$
$J_0(I_1) \times J_1(I_2)$	(−1)	$c - m_2$		$c + m_2$
$J_0(I_1) \times J_2(I_2)$		$c - 2m_2$		$c + 2m_2$
$J_0(I_1) \times J_3(I_2)$	(−1)	$c - 3m_2$		$c + 3m_2$
	Combination Side Frequencies			
	odd order		odd order	
$J_1(I_1) \times J_1(I_2)$	(−1)	$c - m_1 + m_2$		$c + m_1 + m_2$
$J_1(I_1) \times J_1(I_2)$	$(-1)^2$	$c - m_1 - m_2$	(−1)	$c + m_1 + m_2$
$J_2(I_1) \times J_1(I_2)$		$c - 2m_1 + m_2$		$c + 2m_1 + m_2$
$J_2(I_1) \times J_1(I_2)$	(−1)	$c - 2m_1 - m_2$	(−1)	$c + 2m_1 - m_2$
$J_3(I_1) \times J_1(I_2)$	(−1)	$c - 3m_1 + m_2$		$c + 3m_1 + m_2$
$J_3(I_1) \times J_1(I_2)$	$(-1)^2$	$c - 3m_1 - m_2$	(−1)	$c + 3m_1 - m_2$
$J_1(I_1) \times J_2(I_2)$	(−1)	$c - m_1 + 2m_2$		$c + m_1 + 2m_2$
$J_1(I_1) \times J_2(I_2)$		$c - m_1 - 2m_2$		$c + m_1 - 2m_2$
$J_2(I_1) \times J_2(I_2)$	(−1)	$c - 2m_1 + 2m_2$		$c + 2m_1 + 2m_2$
$J_2(I_1) \times J_2(I_2)$		$c - 2m_1 - 2m_2$		$c + 2m_1 - 2m_2$
$J_3(I_1) \times J_2(I_2)$	(−1)	$c - 3m_1 + 2m_2$		$c + 3m_1 + 2m_2$
$J_3(I_1) \times J_2(I_2)$	(−1)	$c - 3m_1 - 2m_2$		$c + 3m_1 - 2m_2$
$J_1(I_1) \times J_3(I_2)$	(−1)	$c - m_1 + 3m_2$		$c + m_1 + 3m_2$
$J_1(I_1) \times J_3(I_2)$	$(-1)^2$	$c - m_1 - 3m_2$	(−1)	$c + m_1 - 3m_2$
$J_2(I_1) \times J_3(I_2)$		$c - 2m_1 + 3m_2$		$c + 2m_1 + 3m_2$
$J_2(I_1) \times J_3(I_2)$	(−1)	$c - 2m_1 - 3m_2$	(−1)	$c + 2m_1 - 3m_2$
$J_2(I_1) \times J_3(I_2)$	(−1)	$c - 2m_1 + 3m_2$	(−1)	$c + 2m_1 - 3m_2$
$J_3(I_1) \times J_3(I_2)$	(−1)	$c - 3m_1 - 3m_2$		$c + 3m_1 + 3m_2$
$J_3(I_1) \times J_3(I_2)$	$(-1)^2$	$c - 3m_1 - 3m_2$	(−1)	$c + 3m_1 - 3m_2$

The table shows the pattern of the frequency components which result from one carrier and two modulators in parallel. First, the simple side frequencies when $k_2 = 0$ and then when $k_1 = 0$. However, unlike simple FM, there are two Bessel coefficients which are multiplicative. That is, the amplitude of $c - m_1$ is determined by $J_1(I_1)$ ***times.*** $J_0(I_2)$ and $c + m_1$ is determined by $J_1(I_1)$ ***times*** $J_0(I_2)$. The first is negative because it is a lower side frequency and one of the two Bessel coeffcients is odd order.

There are cases in the combination side frequencies where both modulating frequencies are lower and odd order, therefore $(-1) \times (-1)$, ***or*** $(-1)^2$, which equals $+1$ (multiplication of like signs).

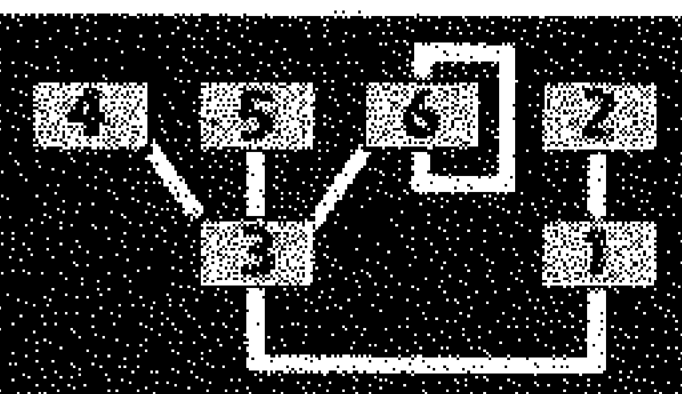
It is clear from Table 6.1 that the re-positioning of operators from two carriers and one modulator (as in Fig. 6.2) to one carrier and two modulators (as in Fig. 6.3), has a very great effect regarding the number of frequency components which result. Because the operators do *exactly* the same amount of computation in both cases, we can begin to see more clearly why FM synthesis is so very powerful. (Compare this table to Table 4.2 which describes simple FM). When we add yet another modulator or form the operators in cascade or series (as in algorithm No. 1, operators 3,4,5,6) the number of frequency components that result can become literally astronomical! But, one step at a time!

We will now plot a spectrum based upon this relationship of one carrier and two modulators in parallel. We will use *frequency* values for $c : m_1 : m_2$ of **500Hz : 100Hz : 10Hz** and indices of $I_1 = 1$ and $I_2 = 0.5$ This time we choose *frequencies* for the carrier and modulators, rather than ratios, in order most clearly to reveal the combinatorial pattern in parallel modulation. Therefore:-

$J_{k1}(1)$ *times* $J_{k2}(0.5)$ for $(500 \pm k_1\ 100 \pm k_2\ 10)$
for
$k_1 = 0,1,2,3 \ldots n_1$ where n_1 approx $= I_1 + 2$
$k_2 = 0,1,2,3 \ldots n_2$ where n_2 approx $= I_2 + 2$

We can follow this exercise on the synthesizer with this "X"-ample . . . .

"X"-ample 6.1



	FREQUENCY	OUTPUT
op 1		
op 2		
op 3	501.2Hz	99
op 4	100Hz	70
op 5	10Hz	63
op 6		

INSTRUCTIONS: Starting from the VOICE INIT? position...

Select algorithm 13 and turn off ops 1,2,6, then set the values as shown above.

Again, we form the frequencies and amplitudes in a table from which we can realize the spectrum. You surely notice that we are not insisting upon much precision when determining the Bessel coefficients. The reason is that the ear is only sensitive to relatively large differences in amplitude except in very simple contexts. Visual approximation of the coefficients from Fig. 4.3 (the Bessel function graphs on page 64), then, is sufficient.

Frequency Components for Complex FM *parallel modulators* $c:m_1:m_2 = 500Hz:100Hz:10Hz \quad I_1 = 1 \quad I_2 = 0.5$				
Amplitude Coefficients (scaling)	Simple Side Frequencies			
	odd order	Lower	Upper	
$J_0(1)\times J_0(0.5)=0.76$		500 (carrier)		
$J_1(1)\times J_0(0.5)=0.42$	(-1)	500−100=400	500 + 100=600	
$J_2(1)\times J_0(0.5)=0.10$		500−2×100=300	500+2×100=700	
$J_3(1)\times J_0(0.5)=0.03$	(-1)	500−3×100=200	500+3×100=800	
$J_0(1)\times J_1(0.5)=0.20$	(-1)	500−10=490	500+10=510	
$J_0(1)\times J_2(0.5)=0.02$		500−2×10=480	500+2×10=520	
	Combination Side Frequencies			
	odd order		odd order	
$J_1(1)\times J_1(0.5)=0.11$	(-1)	500−100+10=410		500+100+10=610
$J_1(1)\times J_1(0.5)=0.11$	$(-1)^2$	500−100−10=390	(-1)	500+100−10=590
$J_2(1)\times J_1(0.5)=0.03$		500−2×100+10=310		500+2×100+10=710
$J_2(1)\times J_1(0.5)=0.03$	(-1)	500−2×100−10=290	(-1)	500+2×100−10=690
$J_3(1)\times J_1(0.5)=.008$	(-1)	500−3×100+10=210		500+3×100+10=810
$J_3(1)\times J_1(0.5)=.008$	$(-1)^2$	500−3×100−10=190	(-1)	500+3×100−10=790
$J_1(1)\times J_2(0.5)=0.01$	(-1)	500−100+2×10=420		500+100+2×10=620
$J_1(1)\times J_2(0.5)=0.01$	(-1)	500−100−2×10=380		500+100−2×10=580
$J_2(1)\times J_2(0.5)=.003$		500−2×100+2×10=320		500+2×100+2×10=720
$J_2(1)\times J_2(0.5)=.003$		500−2×100−2×10=280		500+2×100−2×10=680
$J_3(1)\times J_2(0.5)=.001$	(-1)	500−3×100+2×10=220		500+3×100+2×10=820
$J_3(1)\times J_2(0.5)=.001$	(-1)	500−3×100−2×10=180		500+3×100−2×10=780

Table 6.2

The Bessel coefficients are based upon a visual approximation of the values from Fig. 4.3, where:-

$$\mathbf{J_{k1}(I_1)} \quad J_0(1)=0.8 \quad J_1(1)=0.44 \quad J_2(1)=0.1 \quad J_3(1)=0.03$$
$$\mathbf{J_{k2}(I_2)} \quad J_0(0.5)=0.95 \quad J_1(0.5)=0.25 \quad J_2(0.5)=0.03 \quad J_3(0.5)=\textit{insignificant}$$

Therefore the Bessel coefficient for the carrier comes from 0.8 × 0.95, and for the first simple side frequencies it comes from 0.44 × 0.95, etc. We have shown three significant decimal places in some cases simply to make clear the pattern. Normally we would round to two places, which is sufficient accuracy for purposes of hearing.

In Fig. 6.4 we see the spectral representation of the frequencies from Table 6.2. Note that neither index is large enough to produce frequencies in the negative frequency domain. Reflected frequencies would occur if the carrier frequency were lowered.

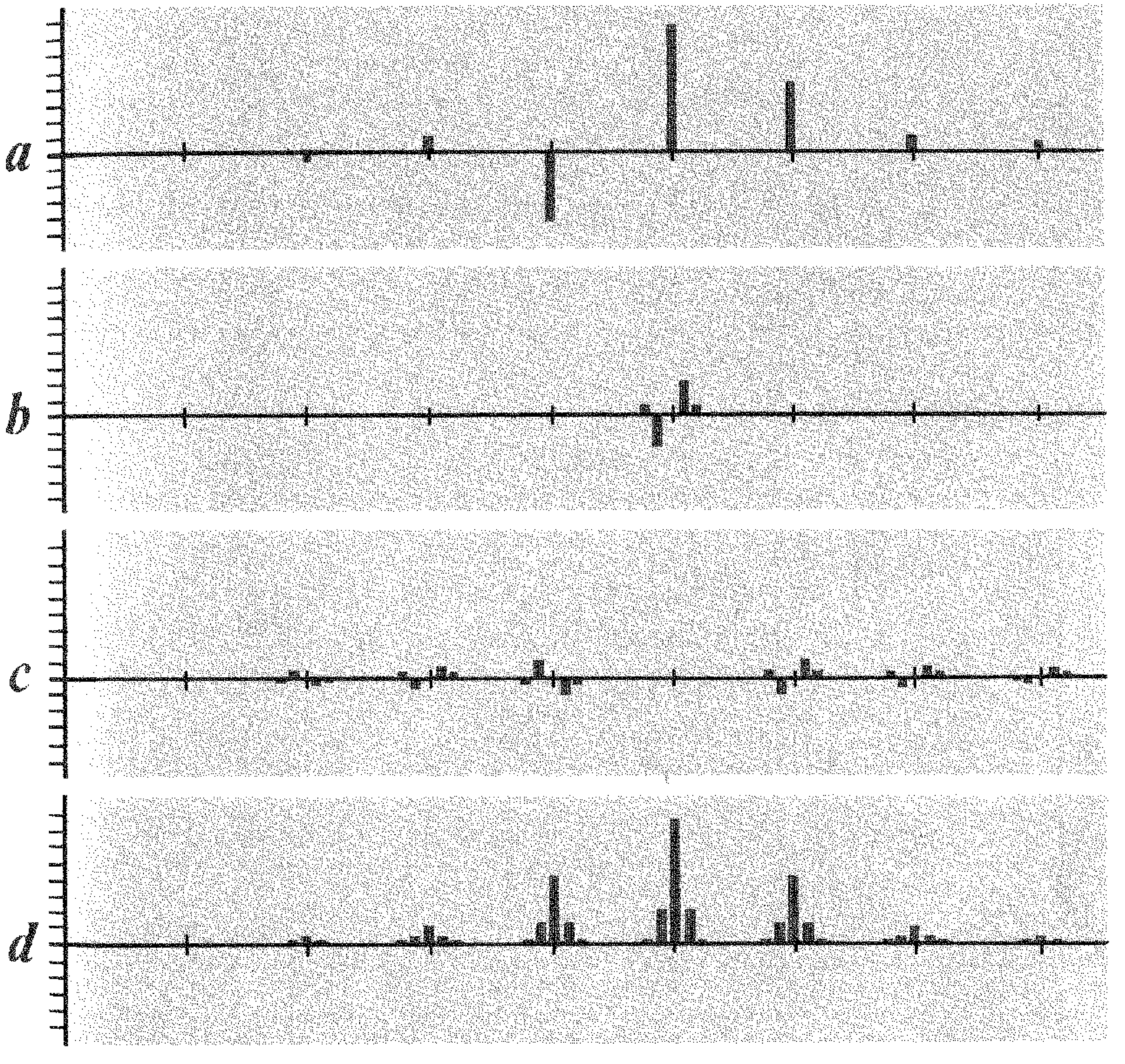


Fig. 6.4

Here we see the spectrum resulting from the values in Table 6.2. The simple side frequencies from m_1 are shown in ***a*** at 200, 300, 400, 600, 700, and 800Hz with the carrier at 500Hz. The simple frequencies from m_2 are at 480, 490, 510, and 520Hz are shown in ***b***. The combination side frequencies are shown in ***c***. In ***d*** we see the magnitude of the sum of the previous three. It is apparent from this representation that each component frequency of one simple spectrum acts as a local carrier to each frequency component of the other spectrum.

Now we will change one value, m_2, in the relationship such that $c:m_1:m_2$ are $500Hz:100Hz:500Hz$ and the indices remain $I_1 = 1$ and $I_2 = 0.5$. In fact, we can now think in terms of ratios of frequencies, $5:1:5$.

Frequency Components for Complex FM *parallel modulators* $c:m_1:m_2 = 5:1:5$ $I_2 = 1$ $I_2 = .5$				
Amplitude Coefficients (scaling)	Simple Side Frequencies			
	odd order	Lower	Upper	
$J_0(1) \times J_0(0.5) = 0.76$		5 (carrier)		
$J_1(1) \times J_0(0.5) = 0.42$	(–1)	5–1=4		5 + 1=6
$J_2(1) \times J_0(0.5) = 0.10$		5–2×1=3		5+2×1=7
$J_3(1) \times J_0(0.5) = 0.03$	(–1)	5–3×1=2		5+3×1=8
$J_0(1) \times J_1(0.5) = 0.20$	(–1)	5–5=0		5+5=10
$J_0(1) \times J_2(0.5) = 0.02$		5–2×5=–5		5+2×5=15
	Combination Side Frequencies			
	odd order		odd order	
$J_1(1) \times J_1(0.5) = 0.11$	(–1)	5–1+5=9		5+1+5=11
$J_1(1) \times J_1(0.5) = 0.11$	$(-1)^2$	5–1–5=–1	(–1)	5+1–5=1
$J_2(1) \times J_1(0.5) = 0.03$		5–2×1+5=8		5+2×1+5=12
$J_2(1) \times J_1(0.5) = 0.03$	(–1)	5–2×1–5=–2	(–1)	5+2×1–5=2
$J_3(1) \times J_1(0.5) = 0.008$	(–1)	5–3×1+5=7		5+3×1+5=13
$J_3(1) \times J_1(0.5) = 0.008$	$(-1)^2$	5–3×1–5=–3	(–1)	5+3×1–5=3
$J_1(1) \times J_2(0.5) = 0.01$	(–1)	5–1+2×5=14		5+1+2×5=16
$J_1(1) \times J_2(0.5) = 0.01$	(–1)	5–1–2×5=–6		5+1–2×5=–4
$J_2(1) \times J_2(0.5) = 0.003$		5–2×1+2×5=13		5+2×1+2×5=17
$J_2(1) \times J_2(0.5) = 0.003$		5–2×1–2×5=–7		5+2×1–2×5=–3
$J_3(1) \times J_2(0.5) = 0.001$	(–1)	5–3×1+2×5=12		5+3×1+2×5=18
$J_3(1) \times J_2(0.5) = 0.001$	(–1)	5–3×1–2×5=–8		5+3×1–2×5=–2

Table 6.3

The table is formed from the same values as in Table 6.2 except for m_2 which has been changed to equal the carrier, and the relations are expressed as ratios rather than frequencies.

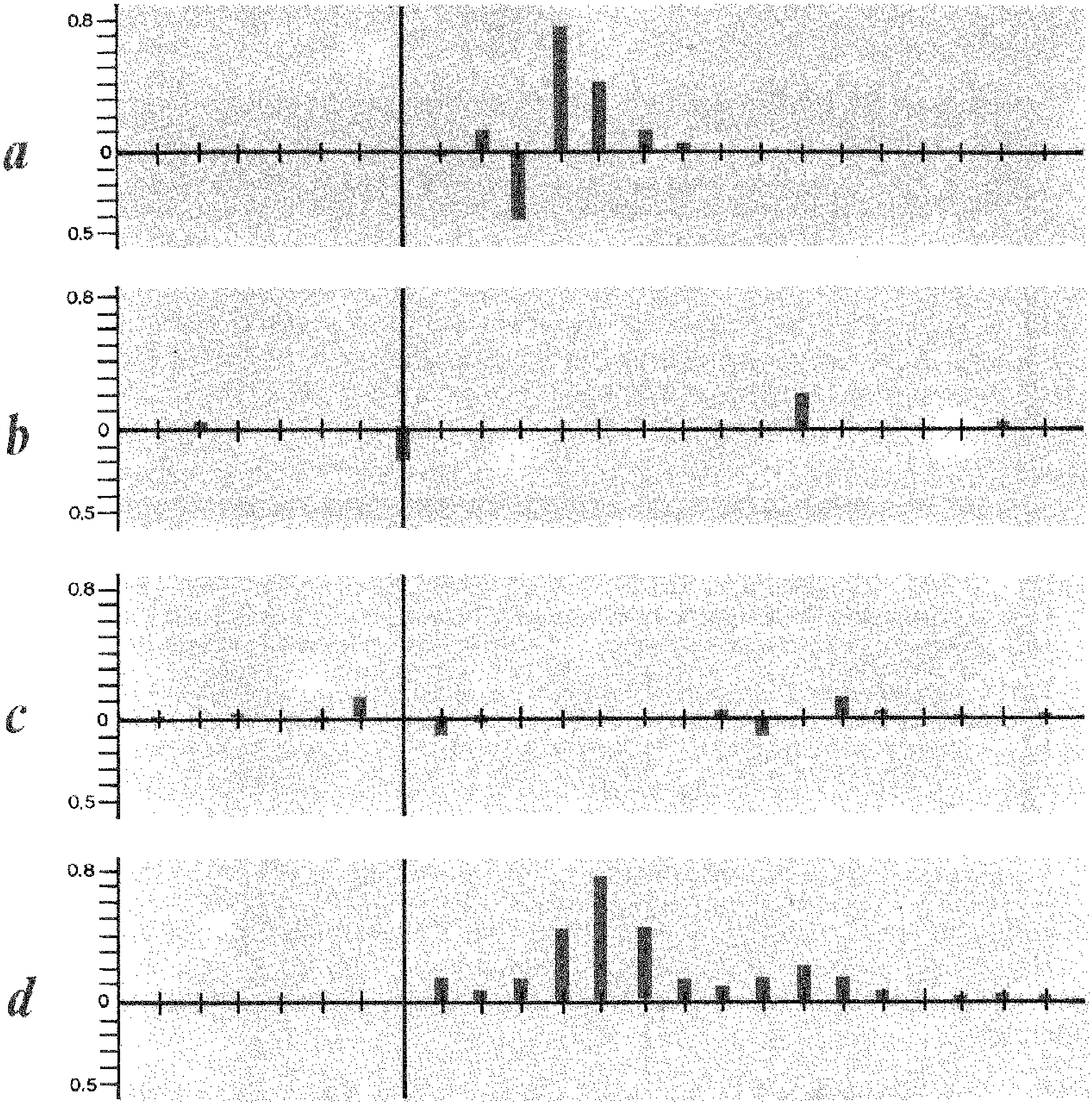
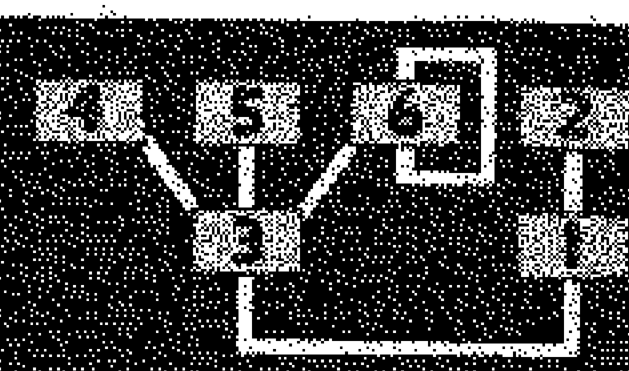


Fig. 6.5

In the spectrum resulting from the values in Table 6.3, the relationship of the simple side frequencies and the combination side frequencies is no longer clear, as was the case in Fig. 6.4, since there are components which fall at the same frequencies and therefore are "hidden". As in Fig. 6.4, a. and b. are the simple side frequencies, c. the combination side frequencies and d. the normalised spectrum.

In Fig. 6.5 we see the spectral representation of the frequencies from Table 6.3. Note that the ratio of c : m_1 alone is like Fig. 5.5 (from the previous chapter where we were looking at the phenomenon of residual pitch). which allows us to compare the effect of the second modulator in parallel with the first. As its ratio to the carrier is 1:1 (=5:5) and the index is small, there is an increase in the bandwidth of the spectrum compared to that of Fig. 5.15. In particular there is a "bump" in the spectrum around the 10th harmonic which results from the combinatorial relation of the sideband components. Let's hear this effect with an "X"-ample . . . .

"X"-ample 6.2



	FREQUENCY	OUTPUT	AMS
op 1			
op 2			
op 3	5.00 (ratio)	99	
op 4	1.00 (ratio)	70	
op 5	5.00 (ratio)	64	3
op 6			

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select Algorithm 13, turn off ops. 1,2 and 6, and set the values shown. Given the fact that the indices are exactly the same as the previous "X" - ample, there are exactly the same number of side band components. However, since there are now many components which fall at the same frequencies, the complexity of the sound is reduced. By moving the wheel from min. and max., you will hear that the bandwidth of the spectrum has been increased in a general way without drastically changing the spectral form, as would have been the case of an increase of bandwidth brought about by an increase of index in the simple FM situation (ops. 3 and 4 alone).

Let us continue to explore this example further by treating the envelopes of the three operators in slightly different ways. Starting from **"X"-ample 6.2**, turn off the AMS on operator 5 and govern the changing indexes automatically by setting the following envelopes (leaving L values unchanged from their Voice Init settings except for operator 5).

	R1	R2	R3	R4
op 3	80	99	99	99
op 4	10	99	99	99
op 5	99	99	10	99
op 5 only, L3 = 0				

If we turn off op. 4 the sound produced will be at a pitch that is two octaves and a major 3rd above the key that is pressed, since the ratio is 5:5. (Remember that the freq. ratios produce frequencies that are multiples of the pitch frequency of the key pressed). Now we will turn on op. 4 but with an envelope having EG values such that the rise time of the output (index!) is slower than the others.

Before listening to this example let's imagine what we are going to hear based on what we have learned. First, a rate of 10 is very slow, which means that the output level of op. 4 will slowly increase from a value of 0 to 70 or index from 0 to 1. Because the envelopes of ops. 3 & 5 quickly reach their maximum while op. 4 is slowly rising from 0, there is no noticeable modulation from op. 4 at the onset of the tone, and we have simple FM only at a ratio of 5:5. However, as op.4's significance increases, the simple side frequencies and the combination side frequencies become apparent and we hear the fundamental frequency change from that based upon the 5th harmonic to that at the fundamental, as shown in Fig. 6.5d. (Here, we compare Fig. 6.4a rather than Fig. 6.5b, since the latter represents one of the two sets of simple side frequencies in the context of complex modulation, where the amplitudes are the result of the multiplication of Bessel coefficients. However at the outset of the tone, the output of op. 4 is zero, therefore at least for an instant we have simple FM). As the tone progresses, we move into an area of complex FM with the resulting increase in bandwidth, and then finally, the output of op. 5 slowly decreases, leaving again simple FM at a ratio of 5:1, as seen in Fig. 6.4a.

It is an important feature of FM synthesis that we can simply interleave spectra in this way or emphasize different spectral regions. It is important because even small changes in spectral emphasis do much to impart character to a sound, even if these changes take place over only fractions of a second. For example try changing op. 4 R1 and op. 5 R3 to 55, thus speeding up the whole process of spectral shift. We may want to control the effect on the spectrum by the second modulator, not as a function of time as we do by using the envelopes, but as a function of pitch – and this is done by key scaling. As key scaling adjusts the output level of any operator as a function of its distance from a "break-point" (a selected note on the keyboard), this can be used to reduce the effect of chosen operators and thus we can implement a change of spectrum as a function of pitch.

Let's go back to the theory and extend it to include three modulating components, conceptually not more difficult – just more complicated. Remembering what was necessary to extend the theory from one to two modulating operators, it should not be surprising that for three (as indeed there are shown in fig. 6.6) we have the following expression:

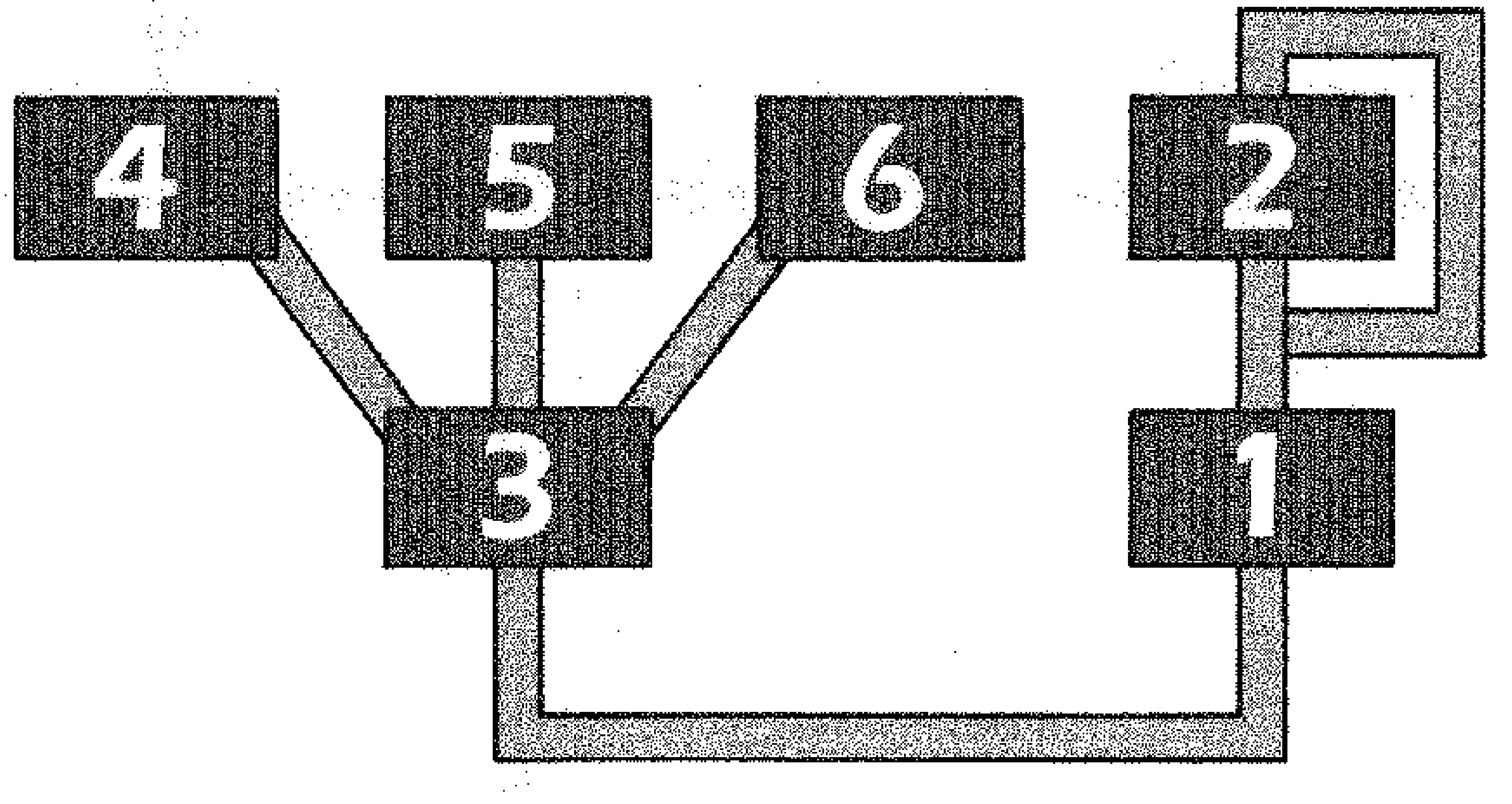


Fig. 6.6

In algorithm No. 12 operators 4, 5, and 6 are used to modulate operator 3.

$J_{k1}(I_1)$ times $J_{k2}(I_2)$ times $J_{k3}(I_3)$ = amplitude scaling factor for the frequency components given by:-

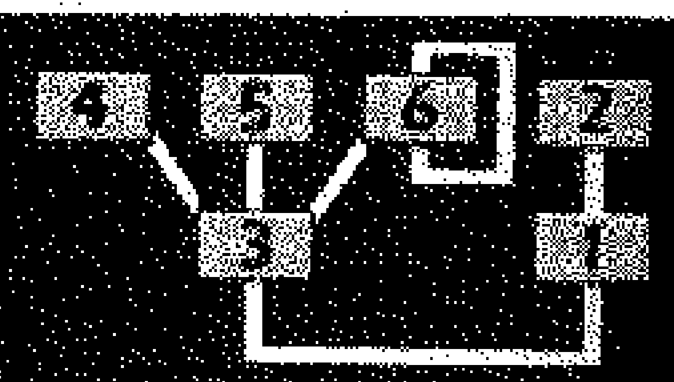
$$c \pm k_1 m_1 \pm k_2 m_2 \pm k_3 m_3$$

for

$k_1 = 0,1,2,3...n_1$ where $n_1 = I_1 + 2$
$k_2 = 0,1,2,3...n_2$ where $n_2 = I_2 + 2$
$k_3 = 0,1,2,3...n_3$ where $n_3 = I_3 + 2$

Extending this relation to our now familiar table we would observe a rather sizeable increase in the number of side band components! The point is that the simple addition of one operator greatly increases the complexity of the spectrum. However, if the ratios of frequencies are all of small integers then the complexity might not be apparent since there must necessarily be many common components. On the other hand if ops. 3, 4, and 5 are integer ratios and op. 6 were in some non-integral relation to the rest, for example, 1.41, then the additional operator will be apparent indeed, as it will produce an inharmonicity. Try the following "X"-ample:-

"X"-ample 6.3



	FREQUENCY	OUTPUT
op 1		
op 2		
op 3	5.00	99
op 4	1.00	70
op 5	5.00	64
op 6	1.41	45

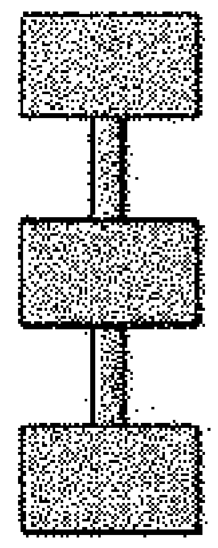
INSTRUCTIONS: Starting from the VOICE INIT? position....

Select Algorithm 13 and set the above parameters (notice that ops. 3, 4, 5 are as X-ample 6.2, except that now AMS = 0 on op.5 as we are not using the modulation wheel).

Notice that even with such a small output on op. 6 the inharmonicity is evident. Now turn off ops 4 and 5. The inharmonicity can now barely be heard. The reason is that it is now simple modulation and with such a small output the index is very small. The two side frequencies are there, one on either side of the carrier, but at very low amplitude. With ops 4 and 5 on, however, the inharmonic components resulting from op. 6, while still of low amplitude, are now easily audible because they are replicated at every side frequency produced by the other modulators. Remember that with two modulators every component produced by one is as a carrier to the other, The same is true for three modulators, only *there* it is three-way combinatorial.

In fact this concept (multiple parallel modulators) is how we come to visualise a stack or cascade of operators. But before we proceed to that, recall "X"-ample 6.2 and try one further experiment. Leaving the modulation wheel at maximum, simply call up Algorithm Select, and then try holding a key and switching between algorithms 13 and 14, which, as you will observe, alternates between parallel modulators and a stack or cascade. You will notice that the effect of operator 5 in the cascade configuration has much less effect on the carrier frequency than in the parallel modulator configuration where the energy seems to be more widely spread across the whole spectrum. We will continue our theory and find out what is happening inside the sound.

Cascade Modulators



The expressions describing the amplitudes and frequencies resulting from operators in cascade are nearly the same as those for the parallel. You will notice one important difference however. The *order* of the first modulator k_1, is used to scale the *index* of the second modulator, I_2.

Amplitude	Frequency
$J_{k1}(I_1) \times J_{k2}(k_1 \times I_2)$	$c \pm k_1 m_1 \pm k_2 m_2$

for

$k_1 = 0,1,2,3...n_1$ where n_1 $I_1 + 2$ and

$k_2 = 0,1,2,3...n_2$ where n_2 $I_2 + 2.$

With a little thought we can see in Table 6.4 the effect of this index scaling of the operator at the top of the stack. The index of the modulator m_2 is multiplied by the integer that indicates side band *order* of the modulator m_1. As a result there are some differences when compared to parallel modulation, as shown in Table 6.1, and we will list these differences in order of importance as far as the sound is concerned.

1. The amplitude of the carrier is determined by the index I_1 only. This is because in the expression $J_0(I_1) \times J_0(O \times I_2)$ the index I_2 is multiplied by 0 (the *order* of I_1) yielding an index of 0 which for J_0 is 1 (refer to Fig. 4.3). Therefore the carrier amplitude is always determined by $J_0(I_1) \times 1$ and unlike the case for parallel modulation, no matter how I_2 changes the amplitude of the carrier is unaffected (unless, of course, there are reflected side frequencies which fall at the carrier frequency).

2. Because orders of I_1 greater than 1 ($J_2(I_1) \cdots J_{k1}(I_1)$) cause I_2 to be ever larger ($J_{k2}(2 \times I_2) \cdots J_{k2}(k_1 \times I_2)$), there is greater energy in the higher order combination side frequency components, when compared to parallel modulation.

3. There will be no simple side frequencies around the carrier resulting from m_2 ($c \pm 1m_2 \cdots c \pm k_2 m_2$) The order for I_1 in this case is 0, while the order for I_2 ranges from 1 until k_2. Since the index I_2 is multiplied by 0 the effective index is 0, and for all orders of J greater than 0 the resulting coefficient is 0.

Frequency Components for Complex FM *cascade modulators* $c : m_1 : m_2$				
Amplitude Coefficients (scaling)	Simple Side Frequencies			
	odd order	Lower		Upper
$J_0(I_1) \times J_0(0 \times I_2)$		*c (carrier)*		
$J_1(I_1) \times J_0(1 \times I_2)$	(-1)	$c - m_1$		$c + m_1$
$J_2(I_1) \times J_0(2 \times I_2)$		$c - 2_{m1}$		$c + 2m_1$
$J_3(I_1) \times J_0(3 \times I_2)$	(-1)	$c - 3m_1$		$c + 3m_1$
$J_0(I_1) \times J_1(0 \times I_2)$	(-1)	$c - m_2$		$c + m_2$
$J_0(I_1) \times J_2(0 \times I_2)$		$c - m_2$		$c + 2m_2$
$J_0(I_1) \times J_3(0 \times I_2)$	(-1)	$c - 3m_2$		$c + 3m_2$
	Combination Side Frequencies			
	odd order		odd order	
$J_1(I_1) \times J_1(1 \times I_2)$	(-1)	$c - m_1 + m_2$		$c + m_1 + m_2$
$J_1(I_1) \times J_1(1 \times I_2)$	$(-1)(-1)$	$c - m_1 - m_2$	(-1)	$c + m_1 - m_2$
$J_2(I_1) \times J_1(2 \times I_2)$		$c - 2m_1 + m_2$		$c + 2m + m_2$
$J_2(I_1) \times J_1(2 \times I_2)$	(-1)	$c - 2m_1 - m_2$	(-1)	$c + 2m_1 - m_2$
$J_3(I_1) \times J_1(3 \times I_2)$	(-1)	$c - 3m_1 + m_2$		$c + 3m_1 + m_2$
$J_3(I_1) \times J_1(3 \times I_2)$	$(-1)(-1)$	$c - 3m_1 - m_2$	(-1)	$c + 3m_1 - m_2$
$J_1(I_1) \times J_2(1 \times I_2)$	(-1)	$c - m_1 + 2m_2$		$c + m_1 + 2m_2$
$J_1(I_1) \times J_2(1 \times I_2)$	(-1)	$c - m_1 - 2m_2$		$c + m_1 - 2m_2$
$J_2(I_1) \times J_2(2 \times I_2)$		$c - 2m_1 + 2m_2$		$c + 2m_1 + 2m_2$
$J_2(I_1) \times J_2(2 \times I_2)$		$c - 2m_1 - 2m_2$		$c + 2m_1 - 2m_2$
$J_3(I_1) \times J_2(3 \times I_2)$	(-1)	$c - 3m_1 + 2m_2$		$c + 3m_1 + 2m_2$
$J_3(I_1) \times J_2(3 \times I_2)$	(-1)	$c - 3m_1 - 2m_2$		$c + 3m_1 - 2m_2$
$J_1(I_1) \times J_3(1 \times I_2)$	(-1)	$c - m_1 + 3m_2$		$c + m_1 + 3m_2$
$J_1(I_1) \times J_3(1 \times I_2)$	$(-1)^2$	$c - m_1 - 3m_2$	(-1)	$c + m_1 - 3m_2$
$J_2(I_1) \times J_3(2 \times I_2)$		$c - 2m_1 + 3m_2$		$c + 2m_1 + 3m_2$
$J_2(I_1) \times J_3(2 \times I_2)$	(-1)	$c - 2m_1 - 3m_2$	(-1)	$c + 2m_1 - 3m_2$
$J_3(I_1) \times J_3(3 \times I_2)$	(-1)	$c - 3m_1 + 3m_2$		$c + 3m_1 + 3m_2$
$J_3(I_1) \times J_3(3 \times I_2)$	$(-1)^2$	$c - 3m_1 - 3m_2$	(-1)	$c + 3m_1 - 3m_2$

Table 6.4

The table shows the pattern of the frequency components which result from one carrier and two modulators in cascade (ops 3, 4 and 5 in algorithm 14). First, the simple side frequencies when $k_2 = 0$ and then when $k_1 = 0$. In the second case there is no energy, since the 0th order for I_1 is multiplied by I_2, resulting in an index of 0. Since the Bessel coefficient for an index of 0 for any order greater than 0 is equal to 0, there will be no energy of the second set of simple side band components. The combination side frequencies also differ from parallel modulation in that their amplitude increases slightly with higher orders of J_{k1}.

We will now use the same values as we used for parallel modulators to develop our understanding of the differences between them. Comparing Table 6.5 and Table 6.2 we see the difference in amplitudes of the components.

Frequency Components for Complex FM cascade modulators $c{:}m_1{:}m_2$ = 500Hz:100Hz:10Hz $I_1 = 1$ $I_2 = .5$				
Amplitude Coefficients (scaling)	Simple Side Frequencies			
	order	Lower		Upper
$J_0(1)\times J_0(0\times0.5=0)=0.8$		500 (carrier)		
$J_1(1)\times J_0(1\times0.5=0.5)=0.42$	(-1)	500−100=400		500+100=600
$J_2(1)\times J_0(2\times0.5=1.0)=0.10$		500−2×100=300		500+2×100=700
$J_3(1)\times J_0(3\times0.5=1.5)=0.165$	(-1)	500−3×100=200		500+3×100=800
$J_0(1)\times J_1(0\times0.5=0)=0$	(-1)	500−10=490		500+10=510
$J_0(1)\times J_2(0\times0.5=0)=0$		500−2×10=480		500+2×10=520
	Combination Side Frequencies			
	odd order		odd order	
$J_1(1)\times J_1(1=0.5=0.5)=0.11$	(-1)	500−100+10=410		500+100+10=610
$J_1(1)\times J_1(1\times0.5=0.5)=0.11$	$(-1)^2$	500−100−10=390	(-1)	500+100−10=590
$J_2(1)\times J_1(2\times0.5=1)=0.04$		500−2×100+10=310		500+2×100+10=710
$J_2(1)\times J_1(2\times0.5=1)=0.04$	(-1)	500−2×100−10=290	(-1)	500+2×100−10=690
$J_3(1)\times J_1(3\times0.5=1.5)=0.017$	(-1)	500−3×100+10=210		500+3×100+10=810
$J_3(1)\times J_1(3\times0.5=1.5)=0.017$	$(-1)^2$	500−3×100−10=190	(-1)	500+3×100−10=790
$J_1(1)\times J_2(1\times0.5=0.5)=0.01$	(-1)	500−100+2×10=420		500+100+2×10=620
$J_1(1)\times J_2(1\times0.5=0.5)=0.01$	(-1)	500−100−2×10=380		500+100−2×10=580
$J_2(1)\times J_2(2\times0.5=1)=0.01$		500−2×100+2×10=320		500+2×100+2×10=720
$J_2(1)\times J_2(2\times0.5=1)=0.01$		500−2×100−2×10=280		500+2×100−2×10=680
$J_3(1)\times J_2(3\times0.5=1.5)=0.008$	(-1)	500−3×100+2×10=220		500+3×100+2×10=820
$J_3(1)\times J_2(3\times0.5=1.5)=0.008$	(-1)	500−3×100−2×10=180		500+3×100−2×10=780

Table 6.5

As in Table 6.2 the Bessel coefficients are based upon a visual approximation of the values from Fig. 4.3, but we need an additional index of 1.5 because of the scaling of J_{k2} by the order of J_{k1}.

$J_{k1}(I_1)$	$J_0(1)=0.8$	$J_1(1)=0.44$	$J_2(1)=0.1$	$J_3(1)=0.03$
$J_{k2}(I_2)$	$J_0(0.5)=0.95$	$J_1(0.5)=0.25$	$J_2(0.5)=0.03$	$J_3(0.5)$
	$J_0(1.5)=0.55$	$J_1(1.5)=0.56$	$J_2(1.5)=0.25$	= ***insignificant***

For example, $J_3(1)=0.03$ and $J_1(3\times0.5=1.5)=0.56$, the product of which is $0.03\times0.56=0.017$. Compared to the same frequency components in parallel modulation (Table 6.2), these are significantly greater in amplitude.

The plot in Fig. 6.7 is made from the values of Table 6.5 and should, then, be compared to the plot of Fig. 6.4. Notice the three differences which were mentioned above.

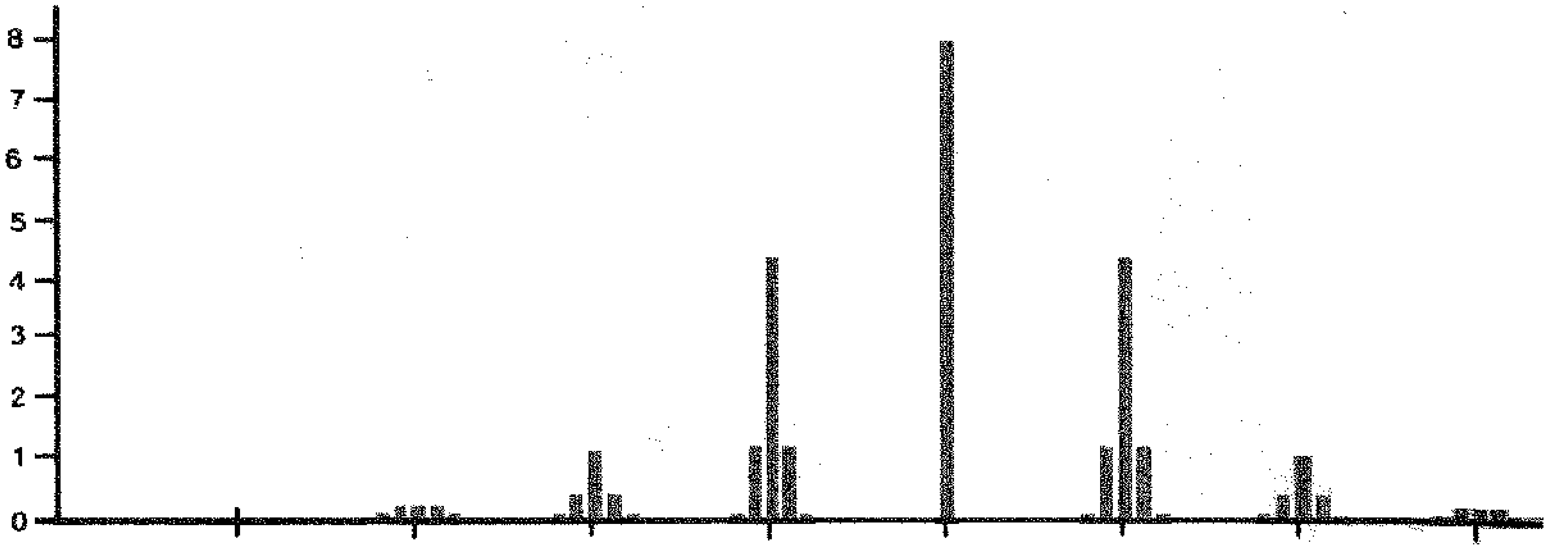


Fig. 6.7

The spectrum that results from cascade modulation where the ratios of $c{:}m_1{:}m_2$ are ***500Hz:100Hz:10Hz***. Notice that there are no simple sideband components from J_{k2} around the carrier.

In order to help explain parallel and cascade modulation we chose these ratios of frequencies because, for small indices, the carrier is high enough that there are no components in the negative frequency domain and they are sufficiently different in size that there are no common components. Since we understand how that works from simple FM there was no need to complicate our task. But common use of complex modulation does involve both negative frequency components and common components.

The next two figures together with an "X"-ample will help very much in our understanding of cascade modulation (and parallel since they are so similar).

In Fig. 6.8 we see a plot through time of the spectrum in Fig. 6.7 but as the carrier frequency changes from 500Hz to 100Hz. The distance of the side band components remains constant (plotted in linear frequency) but as the carrier approaches 100Hz the lowest side band components begin to rise as they are reflected around 0Hz.

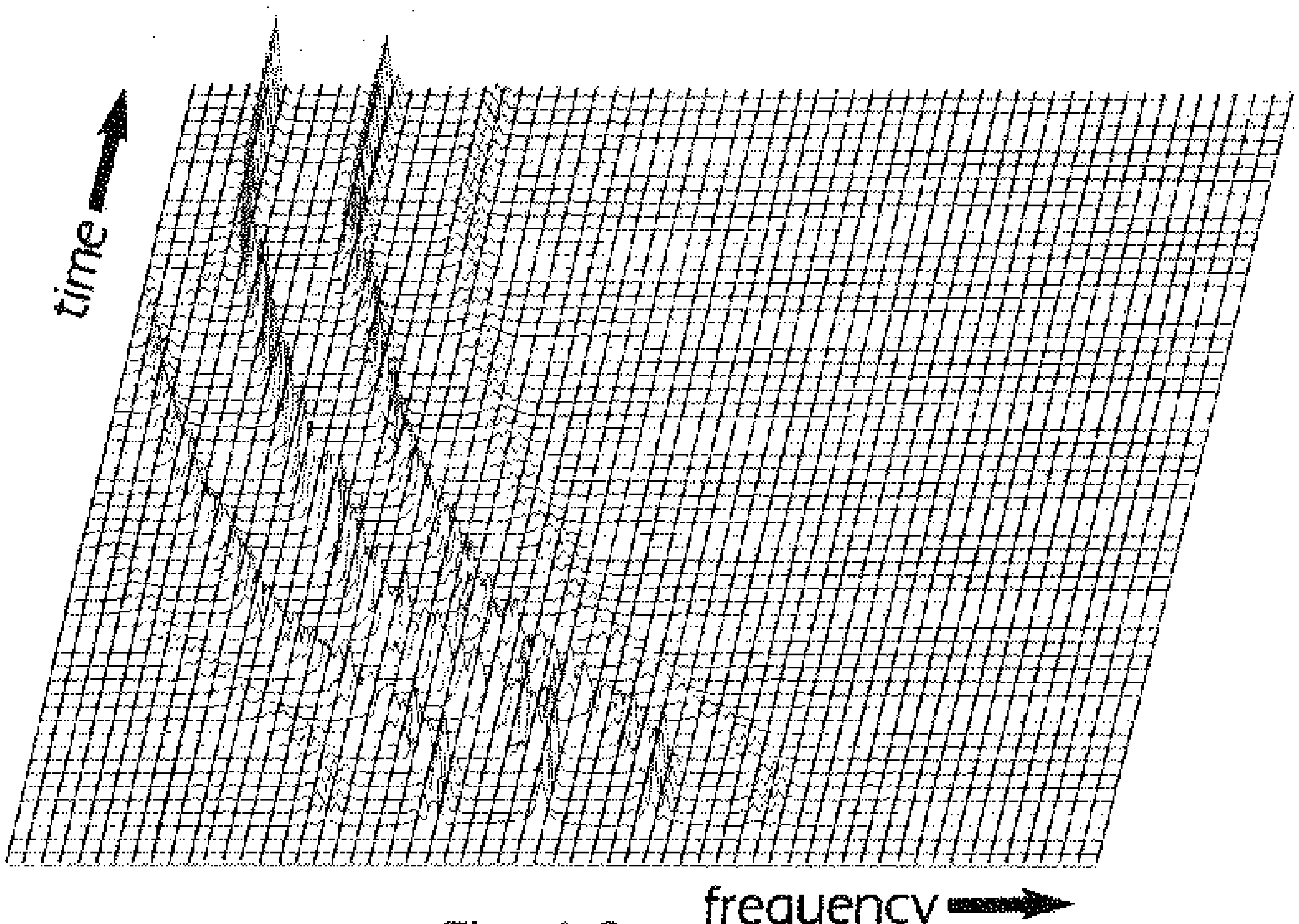


Fig. 6.8

A 3D plot of the spectrum from Fig. 6.7 as the carrier changes through time from 500Hz to 100Hz. The components reflecting around 0Hz can be seen as the carrier approaches 100Hz.

Fig. 6.9 shows the spectrum as the frequency of op. 5, the highest in the stack, changes from 10Hz to 100Hz. Both the carrier and lower modulating frequencies remain at 100Hz. The beginning point of this plot, then, connects with the end point of the previous plot. As all three frequencies will end up at 100Hz the resultant spectrum will consist of components that fall in the harmonic series.

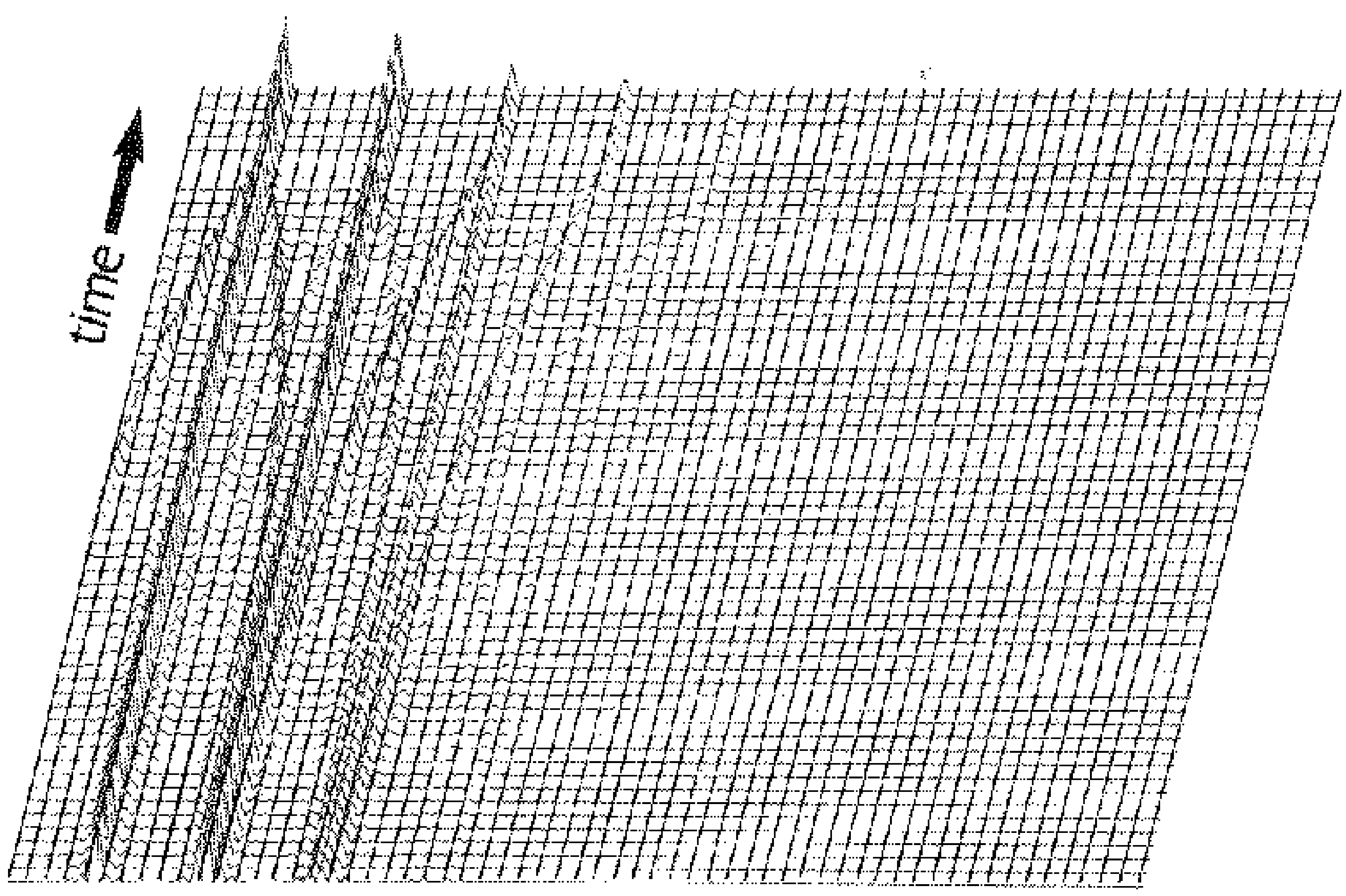


Fig. 6.9

A continuation of the previous plot as the frequency of the highest op in the stack changes from 10Hz to 100Hz. The spreading out of the components can be seen as they approach an integral ratio of 100Hz in relation to the other two frequencies.

To hear what you have just seen in Figs. 6.8 and 6.9, recall "X"-ample 6.1, leaving the settings as given, but change the algorithm to 14, which now puts the working apparatus in a stack or cascade.

Call up Freq. Fine for op. 3 and with the data entry slide slowly change the frequency from 501.2Hz to 100Hz as shown in Fig. 6.8. Now call up op. 5 and again with the data slide, slowly increase its frequency from 10Hz to 97.72Hz as shown in Fig. 6.9. The harmonic series can easily be heard at the conclusion. (That the frequency is not quite 100Hz explains the beating). If you do the "X"-ample again and this time stop at some point between 10Hz and 97.72Hz, you will notice that the sound is not periodic but rather inharmonic. If, however, you stop at 50.12Hz, it is periodic but an octave below the pitch we hear when op. 5's frequency is at 97.72Hz.

The octave difference is explained in terms of the relationship of the intermediate spectrum and the carrier frequency which together determine the final output spectrum. The intermediate spectrum is the result of the modulated modulating wave, ops. 4 and 5. In the case above, the intermediate spectrum is produced by the ratio of $m_1 : m_2$ is 1:.5 or 2:1. Therefore, $2 \pm k_2 \times 1$ will produce a series with 50Hz as the fundamental, an octave below 100Hz. The output spectrum is figured from this intermediate spectrum in relation to the carrier. That is, 100Hz±50Hz then ±100Hz±150Hz etc.

If the desired spectrum is one composed of odd harmonics and cascade (or parallel) modulation is to be used, it is clear that the primary ratio will be odd for the carrier and even for the modulator (see Table 4.3). But what can the secondary ratio be? That is, $c : m_1 : m_2 = 1:2:?$ m_2 must also be even, otherwise the interaction of m_1 and m_2, where m_2 is odd, will produce components in the intermediate spectrum that are odd. With the carrier odd, there would be, then, even components produced in the output spectrum.

Now that we have "mastered" cascade FM with three in a stack, what happens when another is added as in Algorithm 1, ops. 3,4,5,6? We extend the relation for three in cascade to include four as shown in the following.

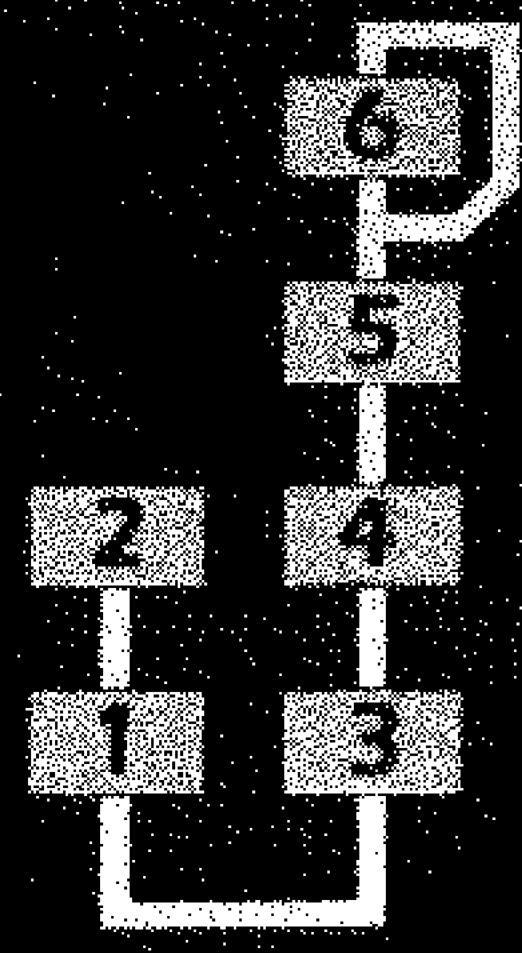
Amplitude	Frequency
$J_{k1}(I_1) \times J_{k2}(k_1 \times I_2) \times J_{k3}(k_1 \times k_2 \times I_3)$	$c \pm k_1 m_1 \pm k_2 m_2 \pm k_3 m_3$

for

$k_1 = 0,1,2,3 \ldots n_1$	where	$n_1 \triangleq I_1 + 2$ and
$k_2 = 0,1,2,3 \ldots n_2$	where	$n_2 \triangleq I_2 + 2.$
$k_3 = 0,1,2,3 \ldots n_3$	where	$n_3 \triangleq I_3 + 2.$

The observations pertaining to three modulating operators in parallel apply as well to three modulating operators in cascade. A small output at the top of the stack will affect the output spectrum noticeably. Because an operator is far from the carrier does not mean that its effect is diminished.

"X"-ample 6.4



	FREQUENCY	OUTPUT
op 1		
op 2		
op 3	1.00	99
op 4	1.00	70
op 5	1.00	63
op 6	0.50	0

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select Algorithm 1, and make the above values, while repeatedly pressing key C3, select output level for op 6 and gradually increase it from zero. With an output of as little as 40 the lower octave can be heard.

Next we will examine the effect of the output of an operator being used to modulate its own frequency. That is where the output "feeds back" to the input.

FEEDBACK FM

A very clever engineer at Nippon Gakki realised several years ago that the basic principle of FM could be applied in a single operator by feeding back its output to its frequency input. While slightly more complicated than an operator in its normal mode, the result of feedback is to provide a simple means of generating a spectrum rich in harmonics. The applications range from sawtooth-like spectra (appropriate as the basis of string sounds) to noise when the feedback operator is in a non-integral frequency ratio to another. Feedback FM is extraordinarly useful, as can be seen in the many uses to which it has been put. Remember as you read the following that while we present feedback in simple cases, it is extensible just as was simple FM.

Algorithm 32 of the synthesizer is the sole example where feedback is implemented on a carrier operator. We shall begin with this simple case as we can hear the result directly.

"X"-ample 6.5

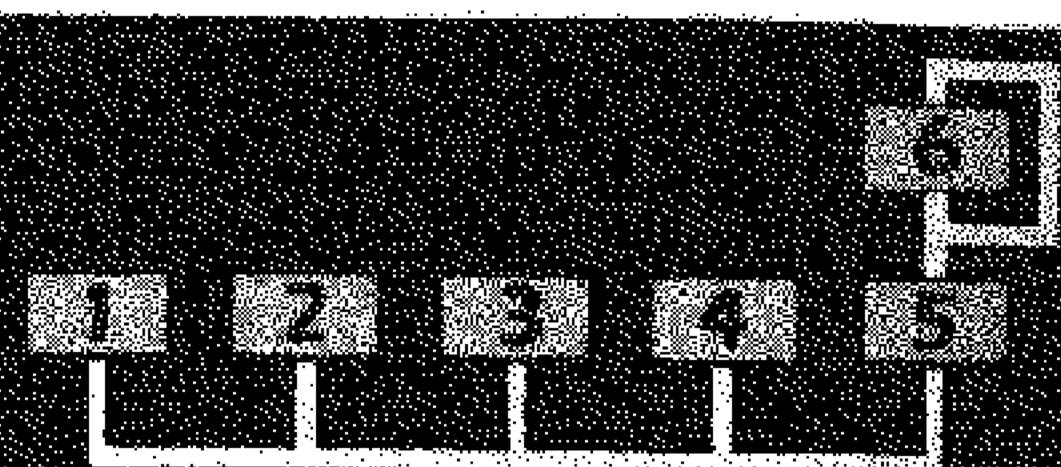
INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select Algorithm 32. Turn of all operators except for op. 6 and set its output level to 99, freq. ratio to 1, and feedback to 7. Press a key and you will hear what sounds like a sawtooth waveform.

The characteristics of feedback FM are such that the ratio of modulation must be 1:1 since the frequency of modulation is at its own frequency. The feedback value, from 0 to 7, is more or less equivalent to index. That the above "X"-ample sounds like a sawtooth wave is because, with a feedback of seven, the spectrum produced contains all harmonics, with amplitudes similar to those of a sawtooth (where the amplitude is equal to 1 over the harmonic number). That is, the relative amplitudes of the harmonics are $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc. With feedback, the amplitude of the higher harmonics falls off a little more steeply as shown in Fig. 6.10c.

For smaller values of feedback there is nearly insignificant harmonic energy. However, as we have already observed, when an operator is a modulator, relatively little energy can have a pronounced effect on the output spectrum, as we see in the "X"-ample.

"X"-ample 6.6



	FREQUENCY	OUTPUT
op 1		
op 2		
op 3		
op 4		
op 5	1.00	99
op 6	1.00	99

INSTRUCTIONS: Starting from the VOICE INIT? position....

Select Algorithm 31 and enter the parameters shown. Select the feedback parameter and while repeatedly pressing the key C2, advance the feedback from a value of 0 to 7. Notice that even with feedback of 1 there is a noticeable change in the sound and with a value of seven there is noise. Why? Because there are so many components at such high frequencies that many have been reflected back from the ½ sampling rate because of aliasing (remember from Chapter 2). How can we visualise the effect of feedback when it is a modulator based upon the theory we already know?

If you count the components that are represented in Fig. 6.10c when the feedback was set to 7, you will find that there are 23. A way to imagine the complexity of feedback when acting as a modulating operator is to think of each of these 23 components as a parallel modulator. That is, instead of 3 operators in parallel we will think of 23 in parallel which would explain the rather extraordinary complexity of the result. The relation would look as follows.

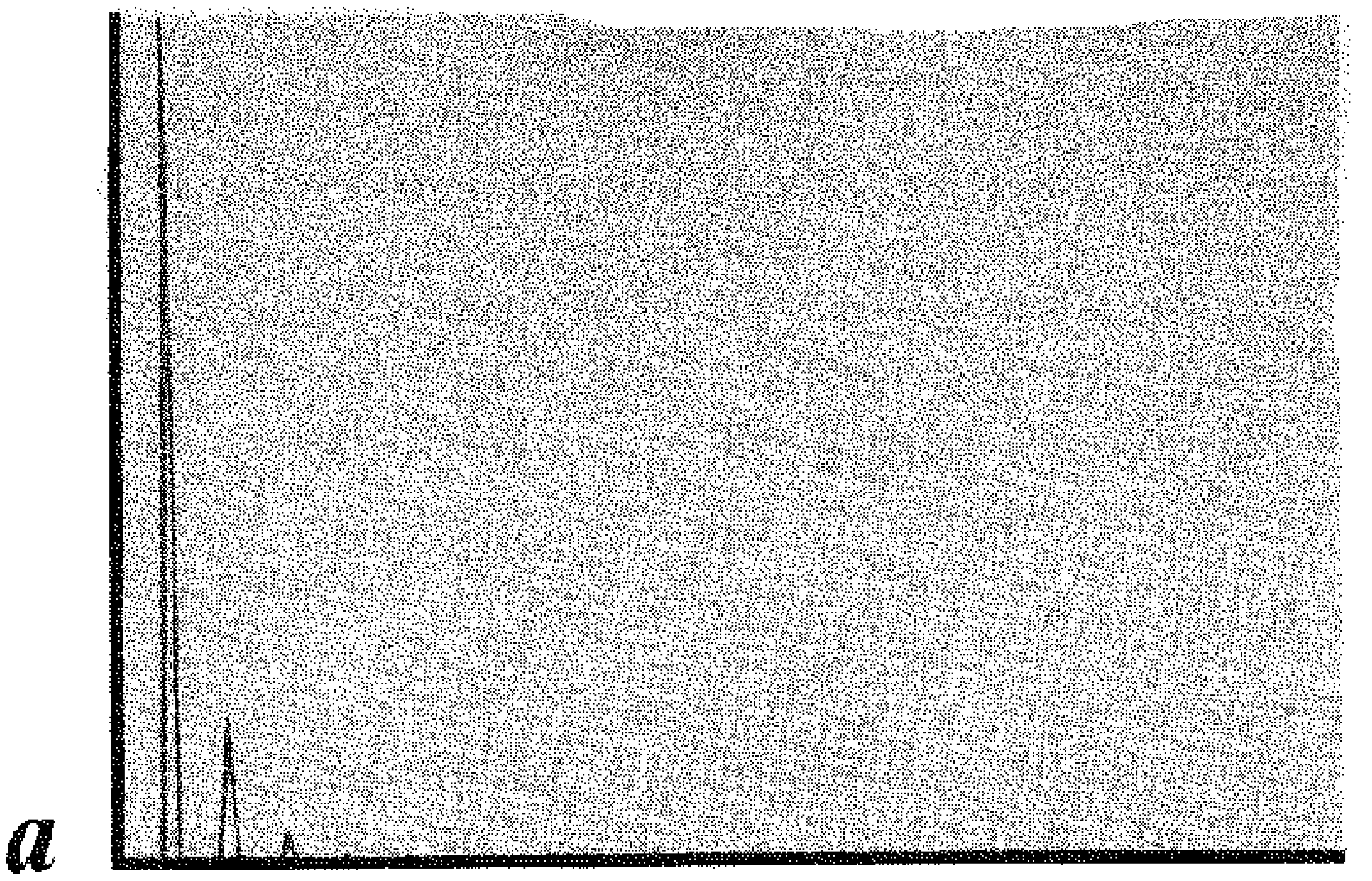


Fig. 6.10a
The output of op. 6, algorithm 32 having a feedback value of 5.

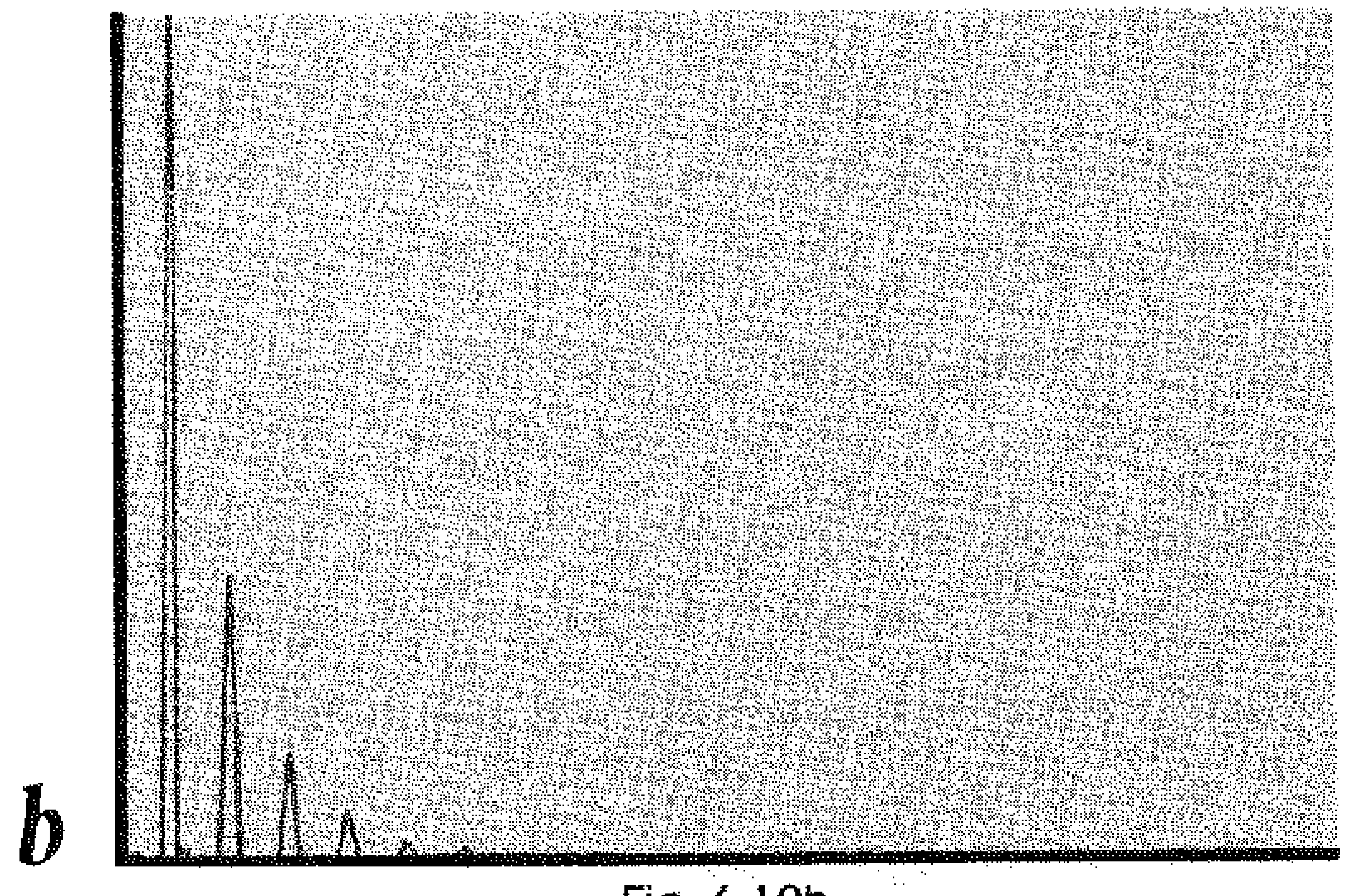


Fig. 6.10b
Feedback value of 6.

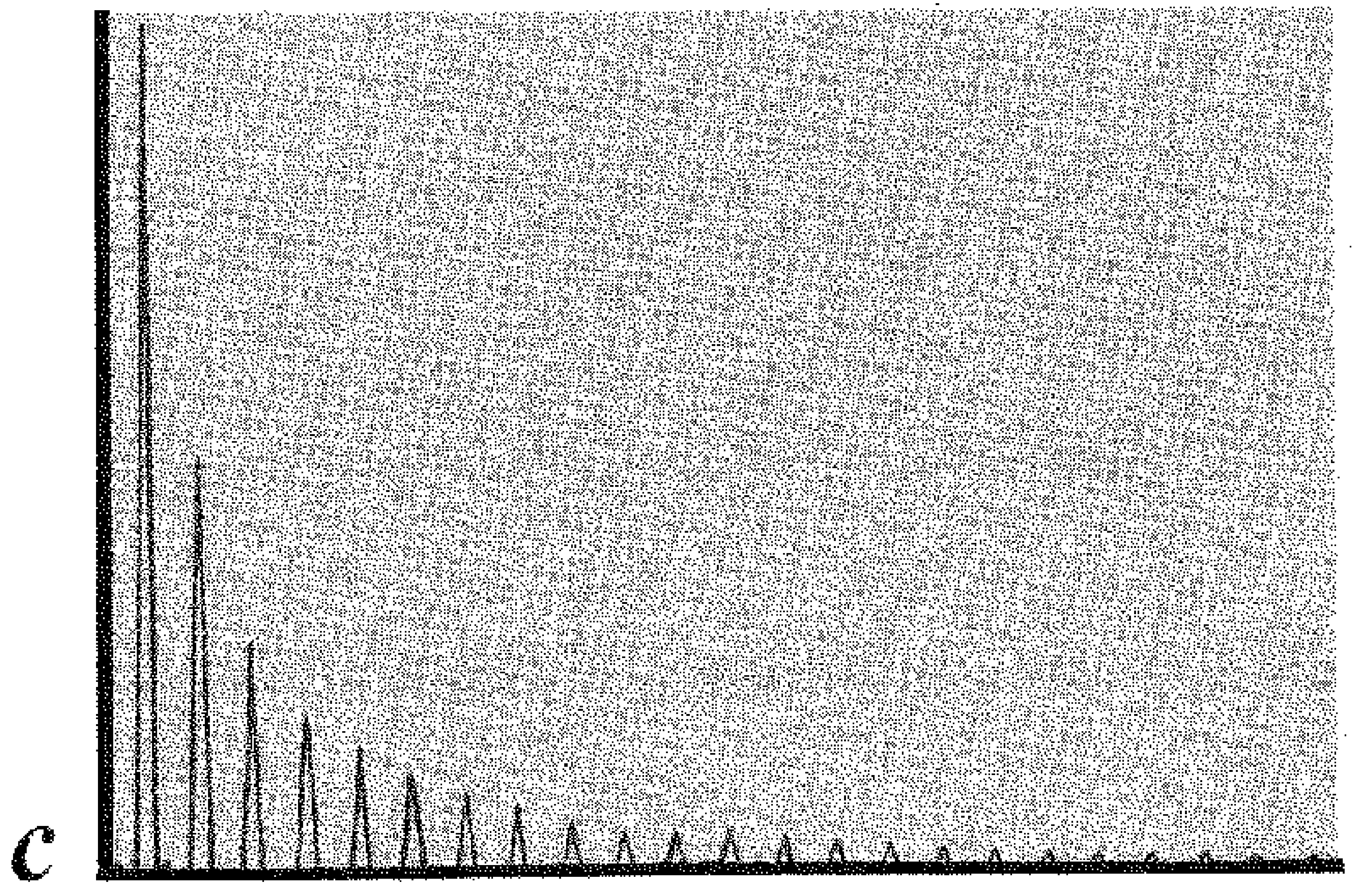


Fig.6.10c
Feedback value of 7.

Amplitude Frequency

$$J_{k1}(I_1)\times J_{k2}(k_1\times I_2)\times J_{k3}(k_1\times k_2\times I_3)\quad (c\pm k_1m_1\pm k_2m_2\pm k_3m_3\ ....\ \pm k_{23}m_{23})$$

for

$k_1 = 0,1,2,3...n_1$ where $n_1 \simeq I_1+2$ and

$k_2 = 0,1,2,3...n_2$ where $n_2 \simeq I_2+2$ and

$k_3 = 0,1,2,3...n_3$ where $n_3 \simeq I_3+2.$

....

....

$k_{23} = 0,1,2,3...n_{23}$ where $n_{23} \simeq I_{23}+2.$

The indices in the above would be related to the relative amplitude of the components. It is clear, then that there will be near astronomical numbers of components that result from full feedback in a modulating operator. The noise spectrum that is produced by the above "X"-ample with feedback at 7b is shown in Fig. 6.11.

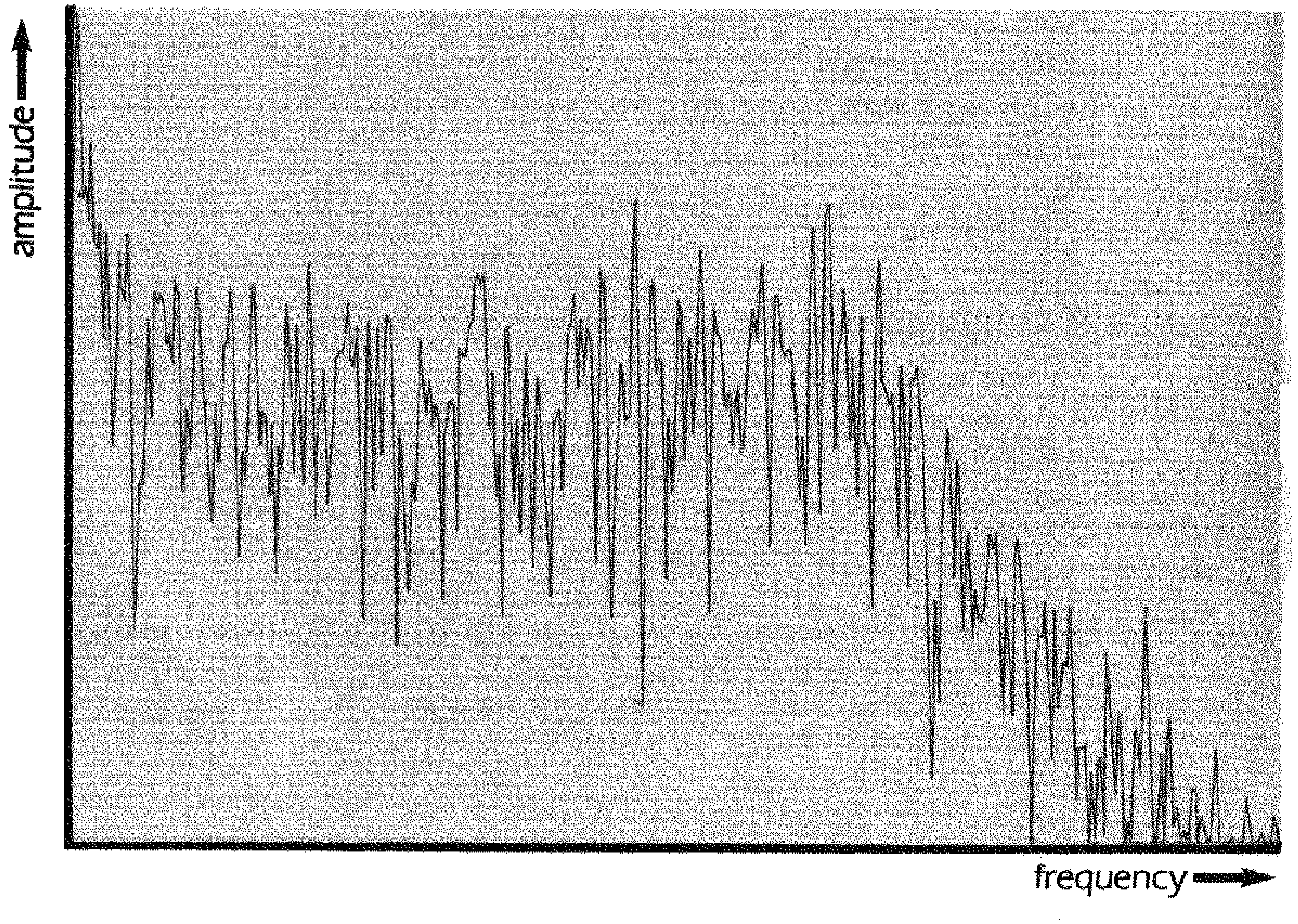


Fig.6.11

The spectrum resulting from a modulating operator having full output and full feedback, where both the carrier and modulating operators have frequency ratios of 1, will be noise. The high frequency components are subjected to aliasing.

After All of This .....

We have been rather detailed in our explanation of the theory of FM. We would be terribly misleading were the reader to believe that without knowledge of the theory one cannot create interesting FM voices. Indeed there are few who have a detailed knowledge of the theory and even they do not go through all that we have here when creating voices. Our intent is to provide the information necessary to allow the user to learn to some level of detail. Since this level will vary widely we are obliged to be more or less complete.

You will notice that nowhere in the theory as we have presented it is there a clue as to how one might draw the waveform with a computer. There is, however, the information necessary to draw the spectrum with a computer. In general, the time domain waveform has little relevance to what we hear and how we manipulate the FM parameters, whereas the spectrum (frequency domain) does have a great deal of relevance. As you read on through the applications be attentive to the general aspects of what has been learned.

CHAPTER 7

APPLICATIONS

Applications

In this chapter we must try to involve our learning and understanding of FM, gained through the study of the previous chapters, into the language and concepts of musicians. Nowhere in this book have we given the impression that, by theory alone, will you achieve all your programming aspirations. The important question is how can the theoretical knowledge be usefully incorporated into an all-arround programming method? The investigation of the theory in itself will have probably inspired some ideas for musical application, but in this chapter we shall take on a variety of existing programming tasks and consider how our new knowledge of FM theory can help to achieve practical results, both in the initial stages of putting a basic sound together and then in the secondary development stage where the "stuff" is added.

In looking at these examples, then, we shall be considering the building blocks of families of sound, and how their component parts can be put together — the chiff of an organ pipe or the transient mechanical "thump" of a piano, for example — all those aspects which can bring a synthesized sound to life: the "stuff" of sounds! And remember that now, we don't have to struggle to describe our sounds. We can use the vocabulary and knowledge we have gained from the previous chapters, such as bandwidth, index, frequency components and so on, which allow us to refer to both sound and synthesis in a direct way. Let's start with something non-musical, but very useful.

Noise, Inharmonic Spectra

We know that noise is an aperiodic waveform with ambiguous pitch, i.e., there are many unrelated components in the spectrum. Yet it has different qualities depending on the bandwidth and the region of the spectrum that it occupies. Let us take for our first example "hiss". The simple, practical exercise of making a "shhhhh" sound with your own voice, then gradually changing it to "ssssss", will tell you that, somehow, we need to remove the low frequencies from the spectrum. So how do we go about forming this spectrum with FM ? Well, because it is noise we want there to be many components in the spectrum, so let's choose an algorithm with a stack of three or four operators — maybe they won't all be needed, but we have the option. We need a high carrier frequency and a low modulator frequency to start with, as you can see that any other combination is bound to give us low frequency components which we do not want.

Think of . . . . $c \pm km$

An index of 2-3 will give a spectrum of about 11 or 12 components.

Think of . . . . ***kn*** *side bands (one upper, one lower), where* $n = I + 2$

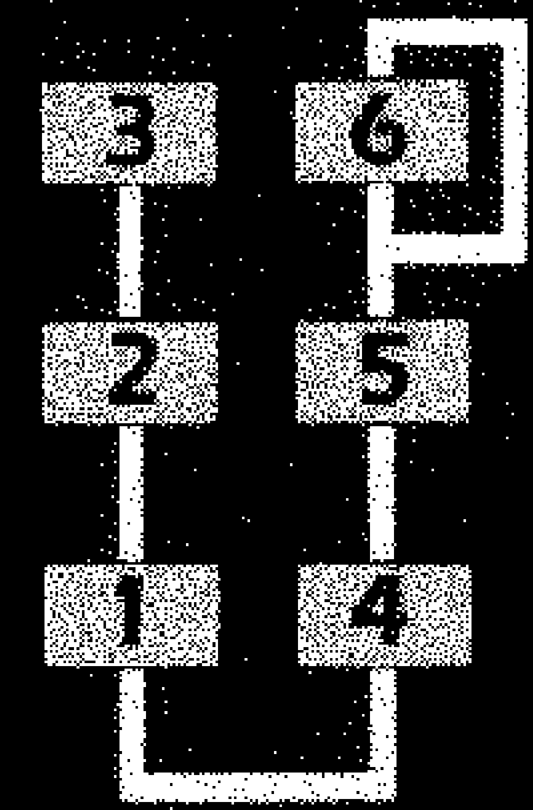
Now, by adding a third modulator in parallel or cascade, we can surround each of these components with their own side bands. As long as the frequency of this third modulator is small, they will not greatly increase the bandwidth, but will thicken the spectrum as we have seen in the previous chapter.

Think of . . . . *complex FM, combinatorial side bands*

We also want to make sure that the ratios are inharmonic so that there is no reinforcing of particular frequencies, and all the components interleave to give as random a spectrum as possible. Set up the following "X"-ample which you can use as a base for experimentation.

"X"-ample 7.1

	FREQUENCY	OUTPUT	R1
op 1	53.63	99	50
op 2	0.72	72	
op 3	0.58	90	
op 4			
op 5			
op 6			



INSTRUCTIONS: Starting from the VOICE INIT ? position.....

Select Algorithm 3, turn off ops. 4—6 and set the values as shown. With the slow attack on the carrier, playing a small cluster of notes just below middle C gives a definite "hiss" sound. Try playing different notes, and try different frequency ratios and indices in order to experiment with this noise model. (Notice the aliasing which occurs in the upper half of the keyboard).

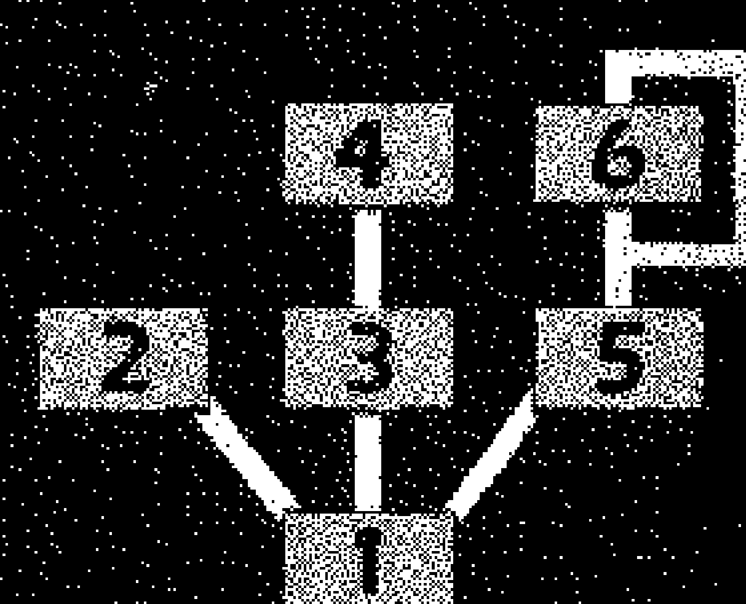
What about a deeper noise such as that of thunder ? It would seem logical to make a start by simply lowering the carrier frequency, in order to sweep the whole spectrum into the lower frequency region. And, indeed, this does work. Starting from the parameters given in the "X"-ample, hold down a cluster of notes just below middle C, and select Frequency Coarse on op. 1, then move the data entry slider down until the frequency of op. 1 is 0.90. Of course a little work on the envelopes will help the "thunder" illusion. Adjust R4 to 20, 10 and 5 on ops. 1, 2 and 3 respectively, and try slightly altering the ratios of ops. 2 and 3. You will soon discover that this simple approach to making "noise" is very flexible.

Another simple adjustment of the envelope will give us the basis for a "gong". Leaving everything the same as for the "thunder", put R1 on op. 1 back up to 99, and on op. 3, make L3 = 0, L4 = 99, R4 = 20, reducing the output level a little to about 80. Let's imagine what is happening here. Ops. 1 and 2, in simple FM, are giving an inharmonic spectrum with a "bell-like" envelope (see chapter 5); then op. 3, with a "reversed" envelope as shown in Fig. 7.1 on p. 131, is interleaving more inharmonic components as the key-off section of the envelope develops and the index of op. 3 rises. Tap some notes in the octave below middle C to hear how this "gong-base" works.

We will investigate another form of noise here, and that is the "chiff" or breath-noise which usually accompanies flute-like sounds as in organs or pan-pipes. In this case, there is a rapid diminishing of the inharmonic components present at the onset of the sound. How does the theory help us to begin to make this effect?

First of all, let's make a basic organ pipe sound with simple FM — not too rich, so we will keep the index low ; and to give it a "hollow" feel, let's ensure that there are some gaps in the spectrum by choosing a ratio of 1:2. Because the organ pipe is a single source of sound, we may as well begin by choosing algorithm 16, 17 or 18, each of which has not only a single carrier but a choice of modulator configurations. The attack time of an organ pipe is not instantaneous, nor the cut-off when the note stops, so we must take care of the carrier (loudness) envelope, reducing R1 and R4. And, because we have read in chapter 5 that the bandwidth of the spectrum from many acoustical sounds develops in relation to the loudness envelope, we should copy this carrier envelope to the modulator so that the index follows the same contour. Now we need to prepare a third operator for the "chiff." Using only ops. 1, 2 and 3, set up the following "X"-ample, and find out how it works.

"X"-ample 7.2



	FREQUENCY	OUTPUT	R1	R4
op 1	1.00	99	60	60
op 2	2.00	60	60	60
op 3	2.82	75	60	60
op 4				
op 5				
op 6				

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select Algorithm 16, and set up the above parameters, turning off op. 3. Play some notes to hear first the plain pipe sound. Now turn on op. 3 and Play some notes to hear the addition of many inharmonic components which will form the basis of the "chiff." We need to make a special envelope for these components which causes them to disappear rapidly. Select op.3 and make L3 — 0. Now select R3, which determines the time from L2 (and that is at maximum) to L3 (zero) and gradually decrease from 99 to about 70, at which point the "chiff" fuses nicely into the overall sound.

You may ask, how did we choose the exact frequency ratio value of operator three? Well, in this case the theory suggests only that it should be inharmonic and not too high (otherwise there will be very high frequency components which may not fuse with the overall sound). The *actual* value is a matter for intuitive research, as are the fine adjustments to the envelopes and indexes. The point is that we can use our intuition and aural skills in a positive way and not in fruitless searching.

We'll talk more about fully developing and refining our sounds in the latter half of this chapter – involving the touch sensitivity of the DX7, the keyboard scaling, what to do with left-over operators and so on. For the moment, let's continue with relating the theory to some more basic building blocks.

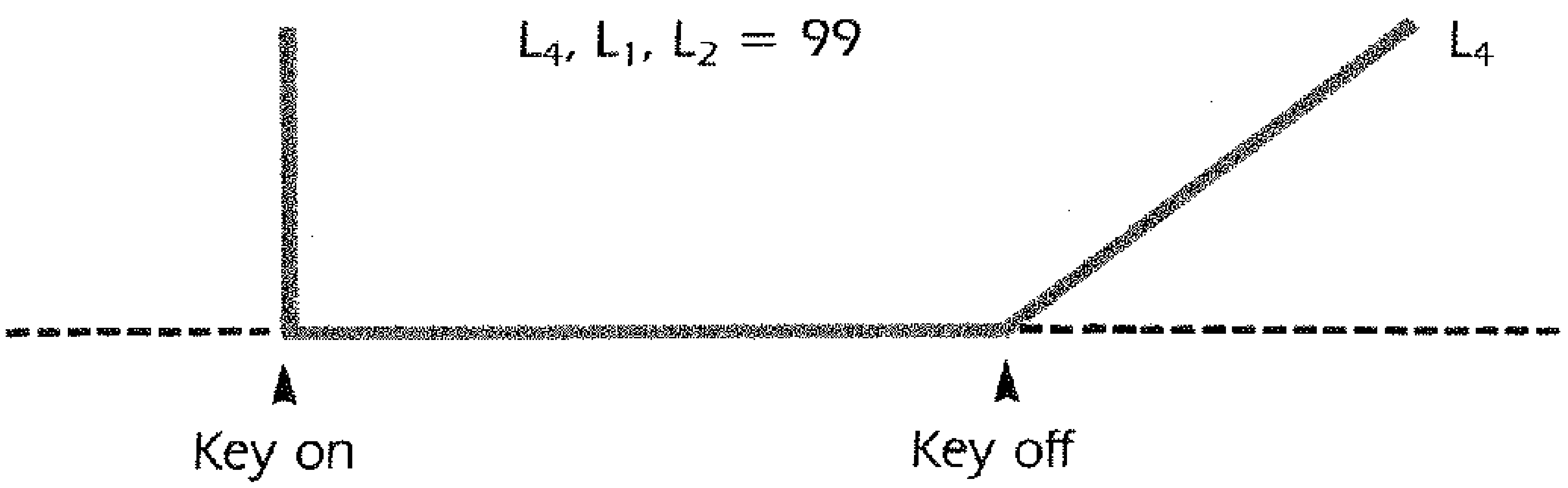


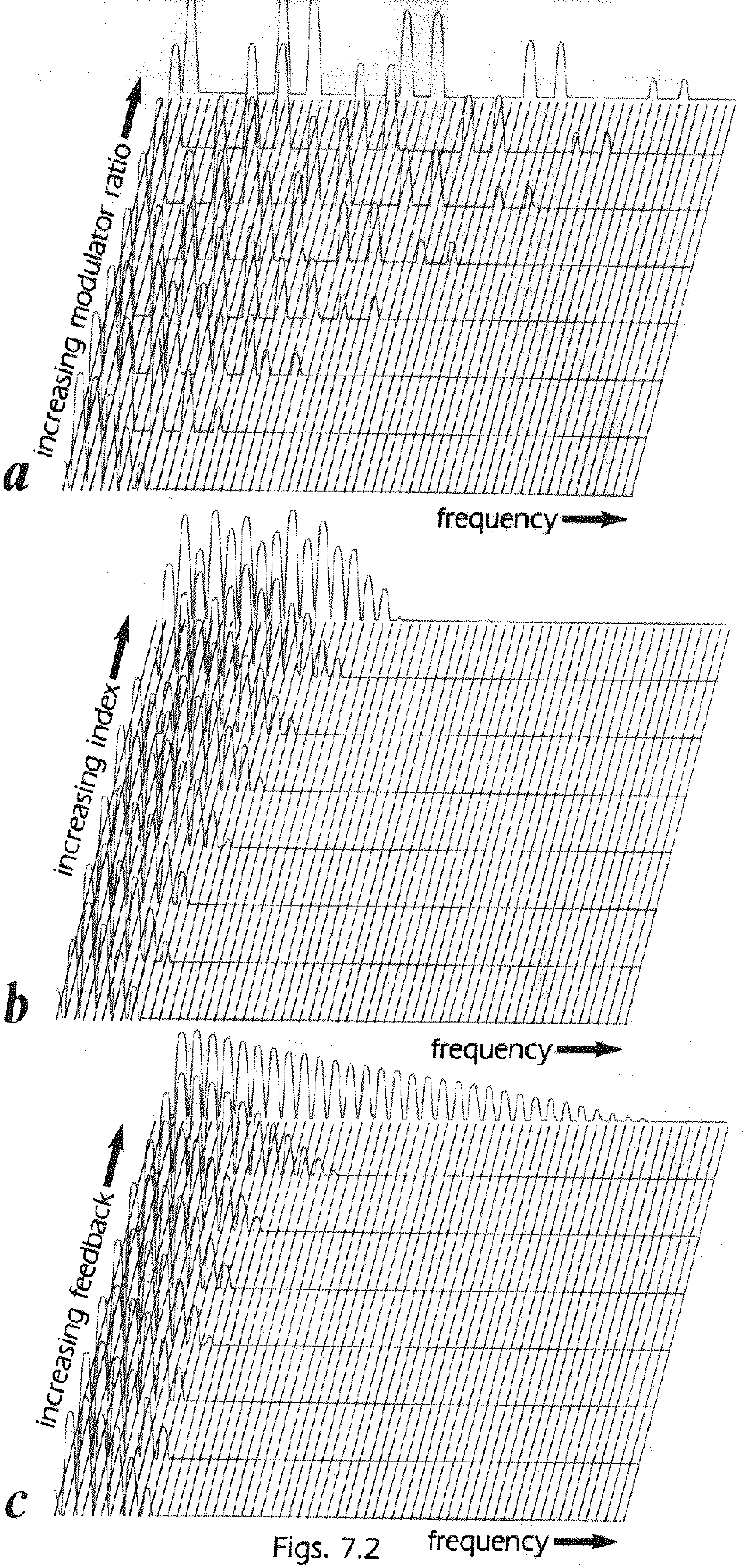
Fig. 7.1
As soon as a key is pressed, the output level drops to zero. When the key is released, as the volume dies away slowly, the output of this op. increases, thus increasing the bandwidth.

Envelopes and Bandwidth

You will already have realised that the envelope generators are very powerful tools in the building of sounds, especially in the way that they control the bandwidth of the spectrum via the output of the modulators. Before we look at another example here are some general guidelines about bandwidth. The three main methods of control are by:-

a) Modulator ratio
b) Modulation index
c) Feedback

However, each of the parameters in a, b and c above operate in a different way. Increasing the modulator ratio will certainly increase the bandwidth, but it will spread out existing components to fill a wider space, and not add any extra components. Increasing the modulation index or the feedback will cause more components to be produced, thus increasing the frequency space occupied. One might say that increasing the modulator ratio thins out the spectrum, and increasing the index makes it grow.



Figs. 7.2

Fig. 7.2 a,b,c show spectral plots of each of these cases.

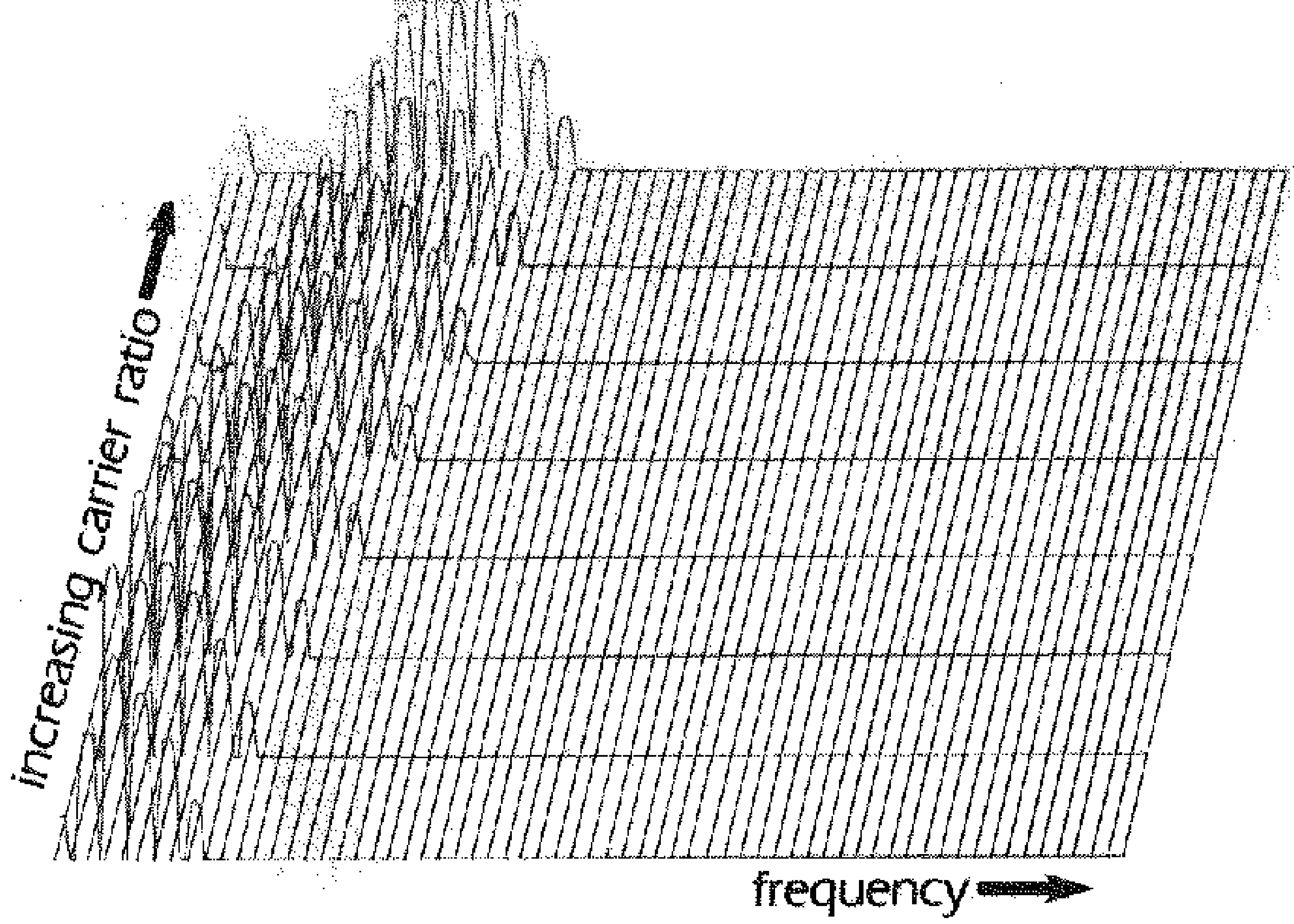
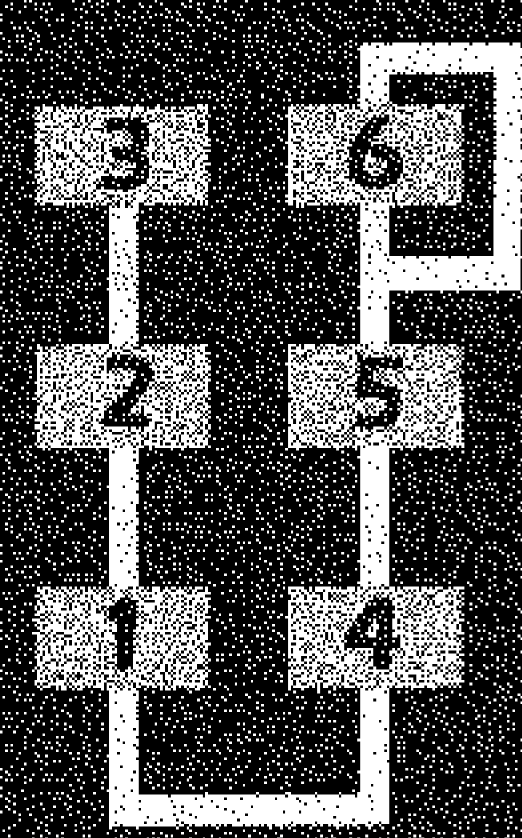


Fig. 7.2d

What happens in the case of an increased carrier ratio? Here, as shown in 7.2d, the whole spectrum is shifted intact – *after the reflected components have "unwrapped" from around 0Hz* – to a higher region of the frequency space, leaving the shape largely unchanged.

Of these three methods for changing bandwidth, only index is controlled dynamically by the envelope generators during the course of a note ; thus it is the method which we can utilise most effectively – as we have done with the previous "X"-amples. In the following "X"-ample, which starts to imitate a "cheap" harpsichord, level 4 is used on the modulators to cause a sudden increase in the bandwidth just after key-off. This is reminiscent of the jack just brushing the string as it falls back into place on a badly made instrument. We'll start this sound very simply and go on to develop it, along with all the others, at the end of this chapter.

"X"-ample 7.3



	FREQUENCY	OUTPUT	R3	R4	L3	L4
op 1	1.00	99	30	55	0	0
op 2	3.00	72	35	99	0	99
op 3	9.00	70	40	99	0	99
op 4						
op 5						
op 6						

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select Algorithm 3, turn off ops. 4-6, and set the parameters as shown. As you play the keys, you hear the "click" at the release of each note. The 3D spectral plot show in Fig. 7.5 is the finished harpsichord ("X"-ample 7.11) and clearly shows the sudden increase in bandwidth at the end of the note.

And what of the ratios in this "X"-ample? By choosing a cascade with ratios of 1:3:9, we can control the spectral shape more easily and with a greater degree of independence in different regions. That is, by using a cascade and juggling the indexes of ops. 2 and 3, we are offering our ears a greater variety of sounds from which to choose than we would by simply using a high index on a simple FM pair of 1:3, which *does* give a metallic stringy sound by the way, but is rather fragile in that a small change of index makes a large change in the sound.

So, in developing the timbre for this "X"-ample, we can use our understanding to choose an algorithm which allows us to make experimental changes and to search aurally for the "right" sound, without moving too far away from the basic area.

Resonances or Formants

Often, certain acoustic instruments and some natural sounds seem to stress certain frequencies regardless of the actual pitch being produced. These regions of emphasis are seen as peaks in the spectrum and are usually called resonances. The relation between these hills and valleys in the spectral envelope (i.e., the overall shape of the spectrum), along with the fact that they stay in one place emphasising different harmonics at different pitches, makes an important contribution to the character of a sound. Indeed, the perception of the spectral envelope, and especially these fixed *formant* regions, is signifficant in the recognition of certain natural sounds such as the human voice. Because there are no filters on "X"-Series synthesizers with which to impose a fixed spectral shape, we have to resort to some cunning FM manipulation in order to obtain the effect of formants or resonances. But also, we can make use of the fact that whole spectral from, i.e., the *relationship* between different emphasised regions of the spectrum, is also very important.

We have seen in chapter 6 how parallel carriers can be used to build up the overall shape of a spectrum, but in this case, the whole spectrum shifts up and down as different pitches are played. Fig. 7.4 shows several traces which have a different form for each note played — the peak of the emphasised region staying at the same frequency regardless of the change in pitch. This effect was obtained using feedback.

Probably the most interesting example to pursue when considering these resonances is that of the human voice, as it involves a realistic blend of theory and intuitive application. In looking at the spectrum of a recorded soprano voice, the authors found a peak around the second and ninth harmonics, as well as a strong fundamental. Other studies of vowel sounds have suggested a fixed formant frequency for an "aahh" sound at around 2,000Hz.

Knowing this then, we might consider using an algorithm like No.22, for example, where we could use the three parallel carriers to provide the resonant qualities, and the simple FM pair to provide the basic fundamental. The modulating ratio in both cases will be 1.00, in order to ensure that the components all fall in the harmonic series, but we can experiment here with some detune or Frequency Fine adjustment to introduce some "roughness" into the voice.

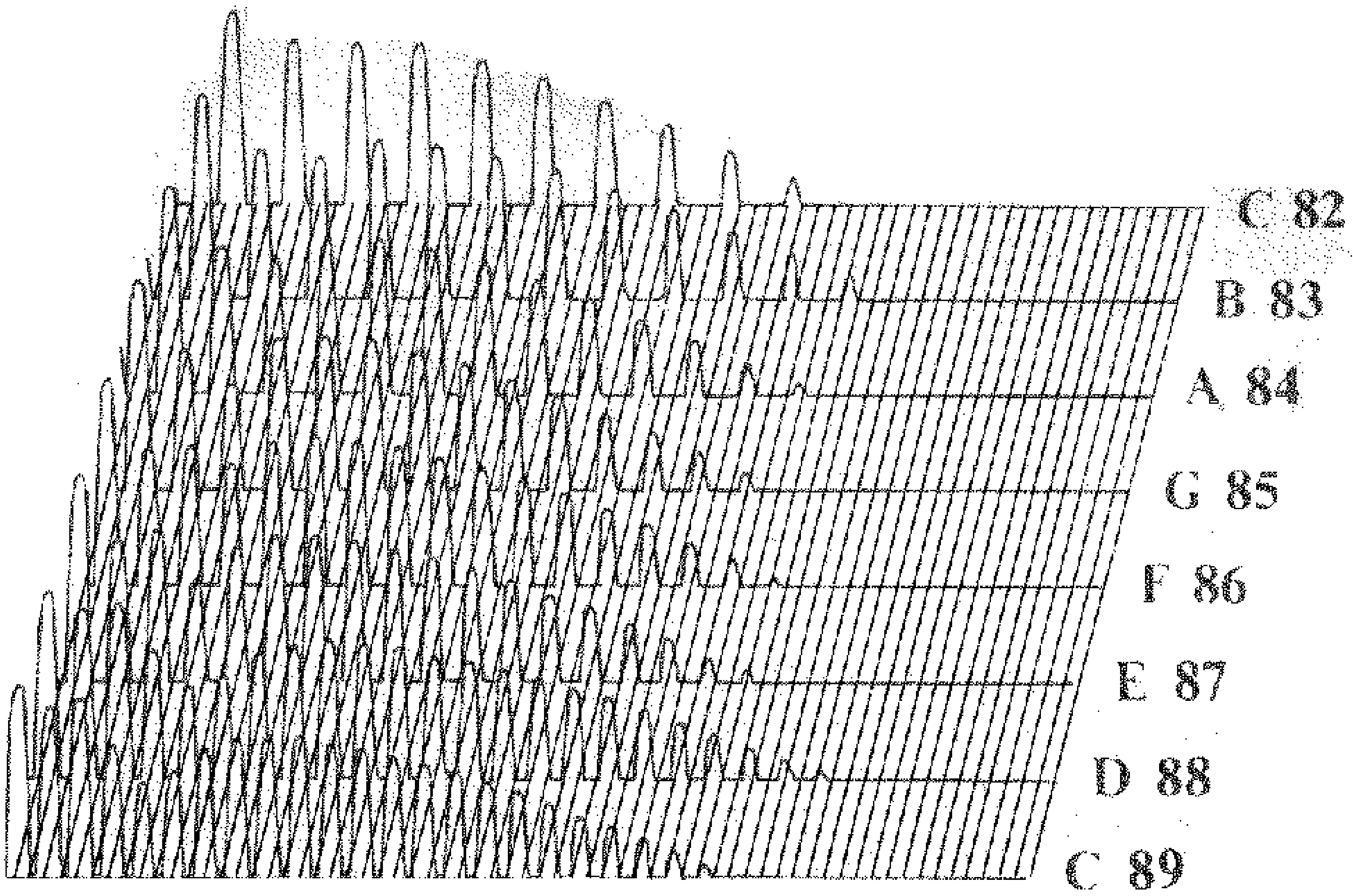
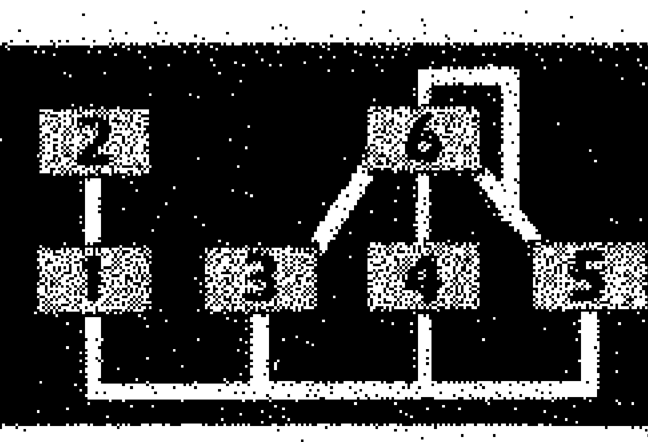


Fig. 7.4

The consistent position of the formant peak was obtained by reducing the output level, as shown, as successive notes were played. A similar effect can be obtained by using the keyboard scaling, which automatically reduces or increases the output of an operator according to pitch.

Forming the envelopes may prove to be the most difficult task, but when imitating an acoustic or natural sound, a good hint is first to form the volume envelope for the carrier, then copy this to the modulating operators, carrying out finer adjustments later by ear. The following "X"-ample is that of a soprano voice, most effective in the region around A880Hz. As you can see, the envelopes are simplified in order to test the theory. However, you might like to experiment with these to get a more satisfactory result.

"X"-ample 7.4



	Frequency	Output	R1 R2 R3 R4	L1 L2 L3 L4
op 1	1.00	99	45 60 70 65	50 75 99 0
op 2	1.00	67		
op 3	2455.0 fixed	45		
op 4	9.09	42		
op 5	2.00	80		
op 6	1.02	60		

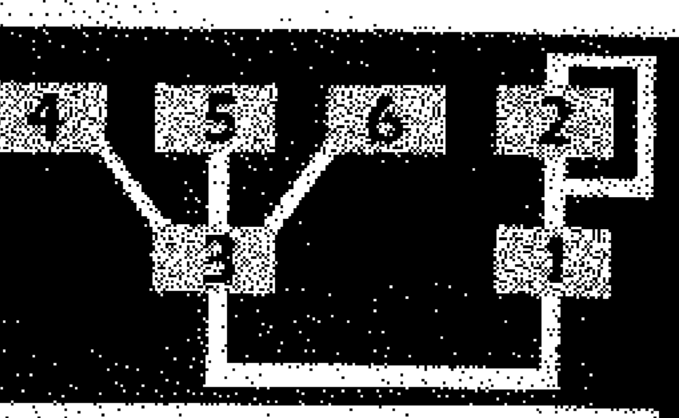
INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select Algorithm 22 and set up the parameters shown. In addition, an LFO PM Depth of 35 with an LFO speed of 45 should be set. A slight pitch envelope will also help the voice, so set Pitch EG level 1 to 47, rate 1 and 2 to about 20. Velocity sensitivity on each operator will also help the expression – op.1 = 1, op.2–6 = 2.

Feedback

Feedback, as we have seen, is a very strong feature of "X"-Series Synths, and its most general use is to provide spectra which have a linear character (refer to Fig. 7.2c) rich in all harmonics, as is typical of brass and string sounds. Also, the linear form which it is able to impose on spectra makes it effective in producing filtered type sounds. We can use feedback, then, when we want to brighten a sound without changing its quality. A simple experiment will make this clear, and maybe we could use this for developing an "analogue-synth", or "filter-synth" base:-

"X"-ample 7.5



	FREQUENCY	OUTPUT	AMS	Feedback
op 1	1.00	99	0	
op 2	1.00	80	3	7
op 3	1.00	99	0	
op 4	1.00	99	3	
op 5				
op 6				

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select Algorithm 12, and set the parameters as shown. Note that in this "X"ample we shall be using the modulation wheel to control index, so be sure the function controls are correctly set (Mod Wheel Range = 99, EG BIAS = ON)

Now play a chord and compare the effect of moving the modulation wheel on each of the simple FM pairs — notice how the pair with feedback (ops. 1-2) seems to have a smoother change from mellow to bright. With the pair 3-4, you can hear the changing amplitude relationship of certain components if you listen carefully. When this "X"-ample is developed later in the chapter, you will see how a little detune can alleviate this roughness by offsetting reflected components, thus preventing them from cancelling in certain cases.

Building Finished Sounds

We shall now develop all the "X"-amples in this book into useful finished sounds. But, we should be sure before starting that we understand all the tools at our disposal, both practical and theoretical, for building sounds with FM synthesis on "X"-Series synthesizers. Consider first one single operator. This produces a sinusoid whose output level can be controlled automatically by an envelope generator and

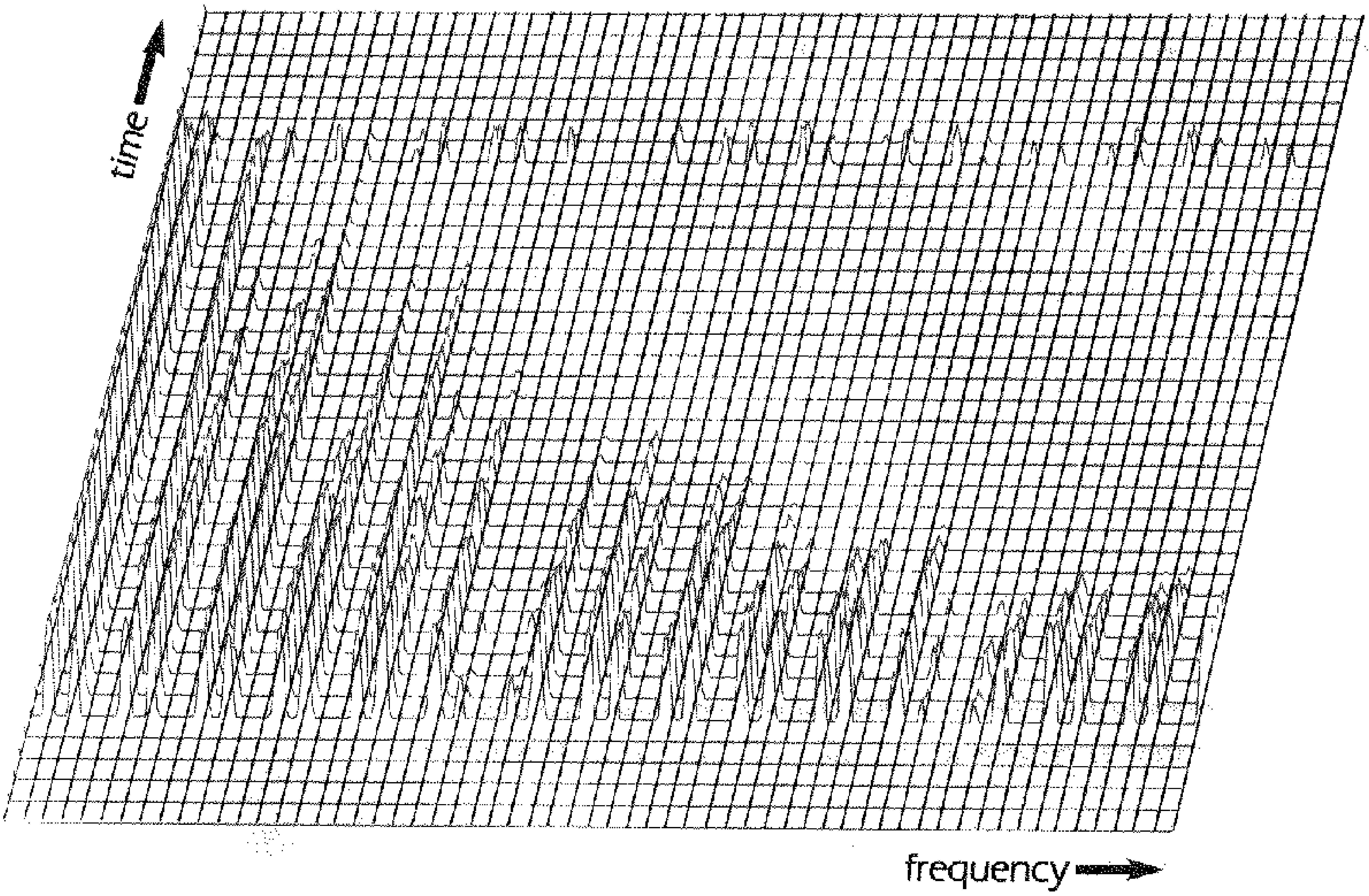


Fig. 7.5

The harpsichord spectrum of "X"-ample 7.11.

keyboard (pitch related) scaling or manually by various input devices such as Modulation Wheel, Foot Pedal, Breath Control, Key Pressure and, perhaps most imporrtant from a musical performance point of view, Key Velocity.

The overall time of the envelope itself can also be pitch-related by rate scaling, which shortens the time as higher notes are played. The frequency of this sinusoid can be related by ratio to a note on the keyboard or, on certain "X"-Series synthesizers, can be fixed at a given frequency and also made to change automatically according to an envelope generator. In the "X"-Series synthesizers, however, there is more than one operator, and these can then be configured so as to produce algorithms of both simple and complex FM in different additive combinations. One of these operators is capable of supporting the feedback function.

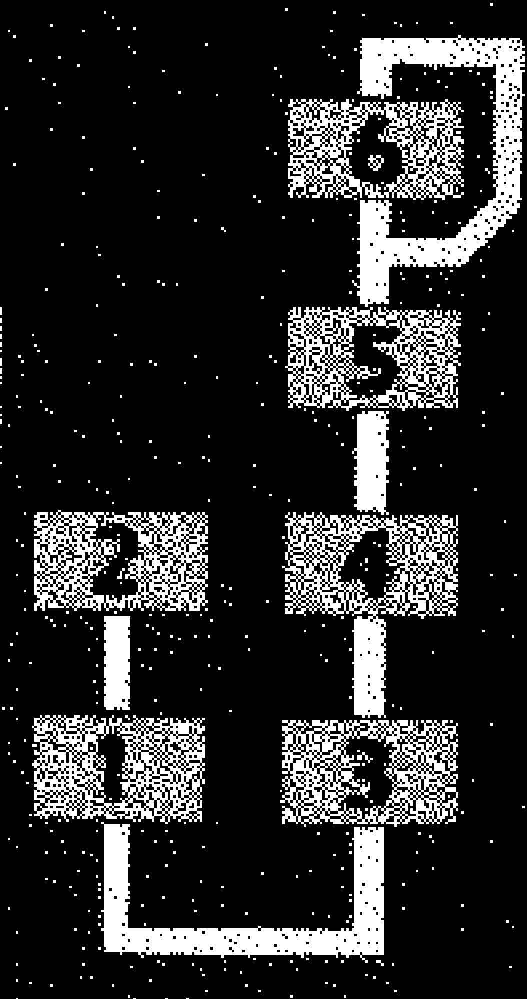
In addition to the operators, there are possibilities to add portamento and vibrato effects to the final "patch" or "voice". This, then, is a brief summary of the practical hardware of the "X"-Series synth. A good working knowledge of the control of these facilities is invaluable to good programming.

Now let's summarise the theory. One operator will produce a sine wave. Two or more in an FM configuration will produce a rich spectrum whose character depends on the frequency ratio of the operators and the modulation index. The frequency ratio and the basic output levels and envelopes of the operators (including the keyboard scaling factors) are fixed in the programming of a sound. However, the modulation index can be further controlled dynamically by the external input devices and, on certain "X"-Series synths, key velocity.

If you are a keyboard player you will realise the importance of this latter facility of expression. By making the modulation index dependent on key velocity, not only volume, but timbre too can be made to change depending on how hard, fast or forcefully a note is played. The increased bandwidth seems to have an agreeable correlation with the increased effort of the musical gesture, therefore the velocity sensitivity parameter should be used with some care in voice programming — and not simply used on the carrier to give increased volume with harder playing. Just as timbre seems to develop over time with many natural sounds (as we have seen when working with the envelopes), so it also seems to develop with an increase in effort by the instrumentalist.

"X"-ample 7.6

	FREQUENCY	OUTPUT	R1	L1	Velocity Sens.
op 1	1.00	99	99	99	0
op 2	1.00	85	20	50	0
op 3					
op 4					
op 5					
op 6					



INSTRUCTIONS: Starting from the VOICE INIT ? position.....

Select Algorithm 1 and set up the above parameters. Play a note and notice that, due to R1 and L1 on op. 2, there is a delay before the bandwidth increases sharply. Now select velocity sensitivity for op. 2 and set it to =3. You will see that the delay depends on how hard you play the note.

Test yourself with a chord. Is each note being played with the same force? If not, you will hear an arpeggio effect as the bandwidths increase for each note after different delays. Perhaps you can imagine how this extension of the velocity function could be a useful instrumental control factor in voice programming.

One final word about velocity sensitivity has to do with its hardware application, which involves its relation to the envelope generator. As velocity sensitivity affects levels of the *envelope* rather than overall output levels, rates — or rather, envelope times — are also affected by the velocity sensitivity parameter. The above simple "X"-ample (7.6) shows this effect clearly. You may like to use this as a playing exercise to test that, when you play a chord, each of the notes are being played with equal force.

Having said this final word about the velocity sensitivity function, we are now ready to refine and finish all the sound "X"-amples that we have started during the course of this book.

In chapter 5 we looked at the bases for three sounds — a bell, hand drum and trumpet. The most direct way of developing a sound made with a simple FM pair is simply by repeating the sound additively — maybe adding a little detune to get a beating or chorus-like effect. To this end, algorithm No. 5 will prove very useful. As the pairs are completely independent however, we could, in the case of the bell, use this fact to put the remaining pairs at different pitches, to make the bell sound much richer. As the pitches chosen in the "X"-ample are higher than the original, we can shorten their envelopes with rate scaling so that these higher harmonics fade faster than the fundamental. Build up "X"-ample 7.7 and play this extended bell sound whilst having a look at the 3D spectral plot shown in Fig. 7.6.

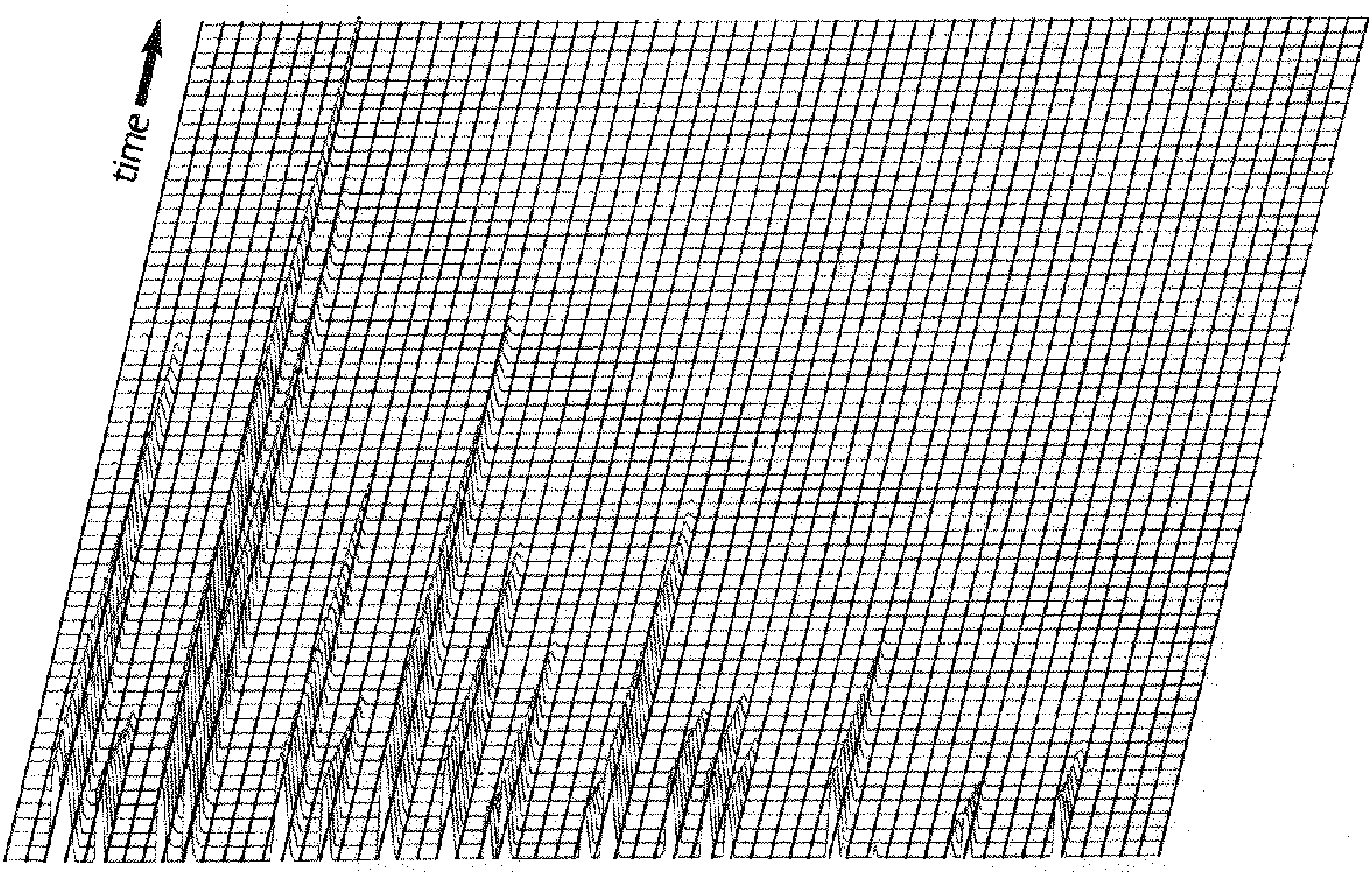
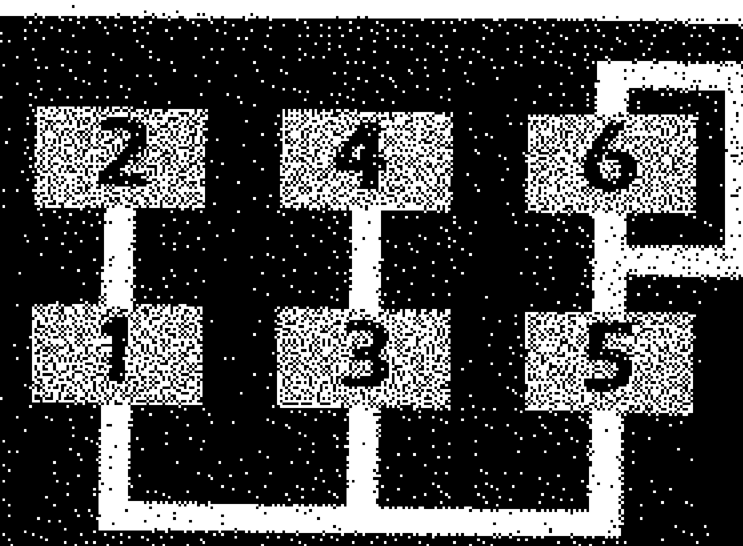


Fig. 7.6

The bell spectrum of "X"-ample 7.7.

"X"-ample 7.7



	FREQUENCY	OUTPUT	L1	L2	L3	L4	R1	R2	R3	R4
op 1	0.50	99	99	99	0	0	99	99	30	20
op 2	0.68	80	99	99	0	0	99	99	30	20
op 3	0.58	95	99	99	0	0	99	99	30	25
op 4	0.79	77	99	99	0	0	99	99	30	25
op 5	0.87	90	99	99	0	0	99	99	30	30
op 6	1.18	74	99	99	0	0	99	99	30	30

INSTRUCTIONS: Starting from the VOICE INIT? position.....

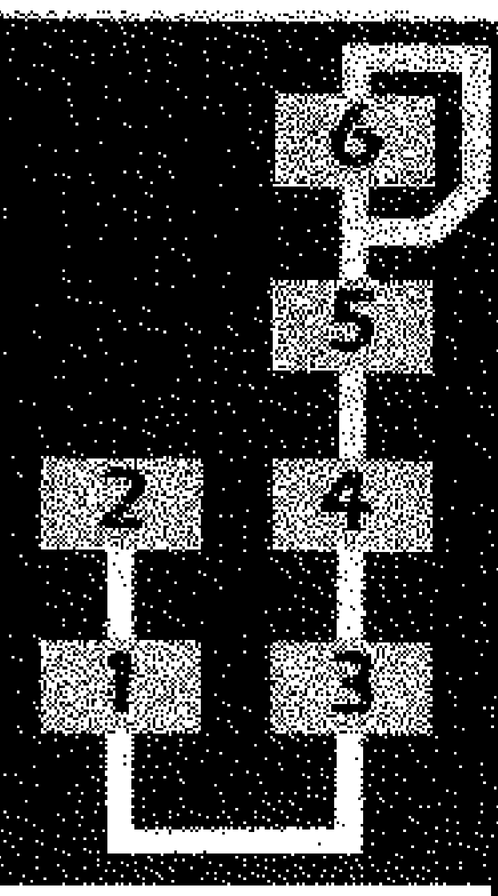
Select Algorithm 5 and set the above parameters. You will note that ops. 1 and 2 are set as for "X"-ample 5.10 in chapter 5, and the sound has been developed in a simple, logical way. Playing only ops. 1,3 and 5 will show the fundamental pitches which were chosen for each of the pairs. But notice that the ratios between the operators of each pair were kept the same. Try experimenting with the modulation indexes and velocity sensitivity now.

If you remember, the hand drum base developed out of the experimentation with the envelopes of op. 2 in the previous "X"-ample in chapter 5. To make this sound more musical and expressive, we should consider other components of the sound of similar instruments in this family, eg. the "hand slap" associated with conga drums. Well, that is noise – a sharp period of increased bandwidth with many inharmonic components.

One possible way to extend our simple FM drum would be to repeat the sound with a second FM pair, mistuned so that the drum does not have a definite pitch. But this pair can be at the base of a cascade which can be used to introduce more and more inharmonic components, linked to velocity, in order to build the slapping sound. Try the following "X"-ample and observe how it is gradually built up from the original simple FM pair.

We are now left with one final, simple-FM starting point from chapter 5, and that is the trumpet base. As you will remember from the "X"-ample, it was a very simple exercise so we must think of improving the envelopes to make it more realistic – no trumpet player can play a constant tone without beginning to lose his breath! Some velocity sensitivity is needed in order to express notes differently, and maybe we should use the remaining operators in this case to make some breath noise.

"X"-ample 7.8



	FREQUENCY	OUTPUT	L1	L2	L3	L4	R1	R2	R3	R4	Rsc	Vs
op 1	0.50	99	99	99	0	0	99	99	50	50	1	1
op 2	0.68	74	99	99	0	0	99	99	60	60	1	1
op 3	0.51	99	99	99	0	0	99	99	50	50	1	1
op 4	0.69	77	99	99	0	0	99	99	60	60	1	2
op 5	0.58	90	99	99	0	0	99	99	70	50	1	3
op 6	3.48	95	99	99	0	0	99	99	80	50	2	4

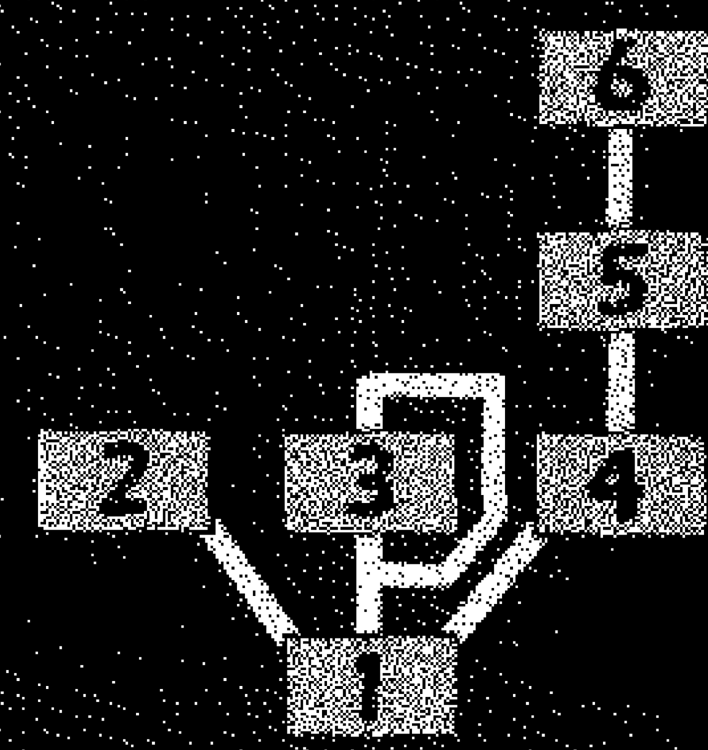
INSTRUCTIONS: Starting from the VOICE INIT? position.....

Leave Algorithm 1 selected and set up the above parameters. Play notes in the fashion of a hand drum, adding the operators one by one from 1-6 in order to observe the contribution of each. Notice that there are still some fundamental similarities to the parameters of the bell in the previous "X"-ample, and also how simple the envelopes are. Now that you know how different operators contribute to the sound, you could begin to be a little more adventurous in your experimentation with the envelopes.

When this example was introduced in chapter 5, it was to demonstrate the important perceptual effects of the envelopes. Now, you should also appreciate the role of the feedback function in the making of this brass sound. Rate scaling has been used on some operators in order to slow the attack in the low registers and make it faster in the higher registers; also some keyboard level scaling on the modulators to reduce the index in the high registers, copying the reduction of bandwidth which tends to occur in some acoustic instruments as the pitch is increased.

As algorithm 18 has been chosen for this sound, the basic pair upon which the trumpet is built becomes ops. 1 and 3; op. 2 "thickens" the spectrum around the fundamental (this is a subtle efect, as it is placing harmonics in the same region as those caused by the main modulator, 3; therefore it is useful to use this operator to introduce minor effects like a little detune which causes a pleasing small variation in timbre without destroying the overall sound). Ops. 4-6 are making the rasp of this particular trumpet – rather like the chiff technique we used earlier for the organ pipe. All the details for this sound are given in "X"-ample 7.9, but you must study the sound carefully to find the blend of logic and intuition which leads to its construction.

"X"-ample 7.9



	FREQUENCY	OUTPUT	L1	L2	L3	L4	R1	R2	R3	R4	Rsc	Vs
op 1	1.00	99	99	95	80	0	60	24	12	50	4	1
op 2	1.00	78	75	99	61	0	50	43	18	50	5	2
op 3	1.00	87	70	97	92	0	60	39	12	50	5	2
op 4	1.00	68	75	99	90	0	66	33	18	50	5	3
op 5	3.00	78	99	61	61	0	75	12	22	50	5	2
op 6	0.74	82	99	50	0	0	70	56	40	70	3	4

INSTRUCTIONS: Starting from the VOICE INIT? position.....

Select Algorithm 18 and set the above parameters, building up with ops 1 and 3, setting the feedback to 6, then continuing with the other operators. Add this little bit of keyboard scaling (index) to ops 3 and 4 with a-LIN curve.

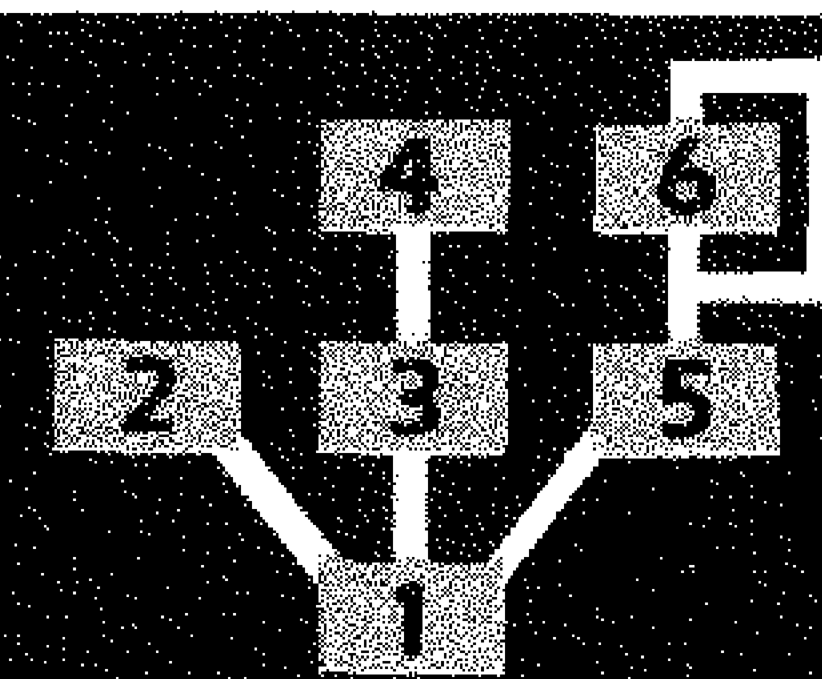
	Break point	*L depth*	*R depth*
op 3	*D4*	*3*	*6*
op 4	*E3*	*0*	*17*

A very slight pitch envelope also helps the trumpet character. Set L1 to 48 and R2 to 70 leaving the others as they are.

When you look at "patch" diagrams which are complete, as in the above "X"-ample, it is important to extract and separate that information which is fundamental to the sound, that which is subtle, and that which is completely extraneous! For instance, in the above "X"-ample, you may ask "Why a frequency ratio of exactly 0.74 for operator 6?" The correct answer should be, "It sounded good to us when we made the patch", with the proviso, however, that what was wanted for this operator was a small non-integer ratio which would give inharmonic components in between and around the components of the spectrum already created by the other operators (*think of . . . . the effect of cascade modulators*). In other words the *exact* value was determined by aural judgement — and don't be surprised if this changes slightly day by day. The approximate value which supports the fundamental idea (in this case an initial roughness to the sound) is indicated by the theory. Let's finish off the sounds we started at the beginning of this chapter.

A possibility with the organ pipe would be to give it more character — to make it sound older perhaps, by introducing some inharmonic components, a small amount of detune to get a slow movement of the timbre, increasing the chiff at the beginning of the note, and maybe causing a little noise at the end of the note (key-off) to correspond to shutting off the air to the pipe. Reset "X"-ample 7.2, not forgetting to adjust the rates and levels of op 3 as indicated. Then add the following values:-

"X"-ample 7.10



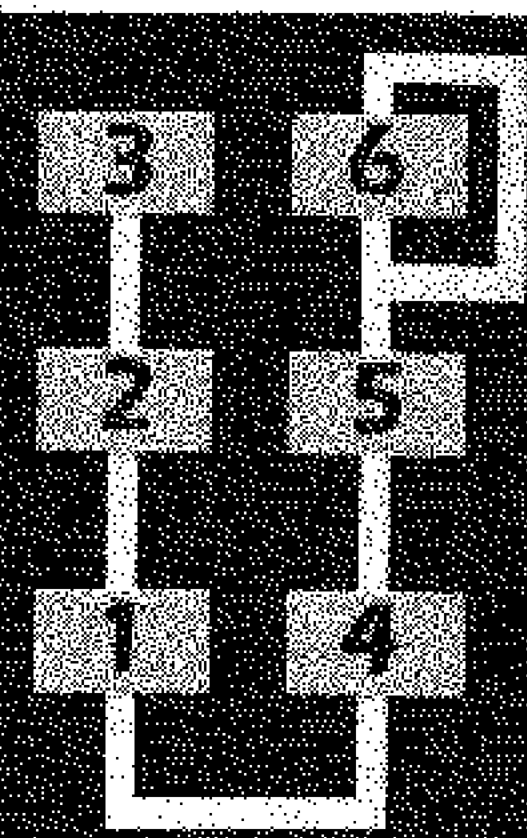
	Frequency	Output	L3	L4	R3	R4
op 1						
op 2						
op 3						
op 4	1.00	72	copy envelope op 1			
op 5	2.00	65	70	99	60	90
op 6	0.89	80	60	0	25	99

INSTRUCTIONS:

Start from "X"-ample 7.2. Add the above parameters, together with some velocity sensitivity (1 is enough just to give a bit of "feel" to the keyboard) to ops 1,3 and 5. Add a little detune to op 5 (+3 or 4). Slowing R4 of op 5 (ie. reducing it from 99) will, of course, delay the fading of the output of that operator after the key is released, thus increasing the pipe-closing effect at key-off.

Having made an "old" pipe organ, let's now complete our "cheap" harpsichord! We need to improve the envelope of the carrier a little, just to give a more "plucked" feel to the sound, and, as harpsichords often play notes in two octaves for each key, the other half of Algorithm 3 can be used to provide the "upper string". This can be done by simply multiplying all the original frequency ratios by two (remember chapters 3 and 4), but watch out for aliasing! We may need to repair this with keyboard scaling. Also remember that the envelope character of the shorter "upper strings" will be different from our original. Here is the completed harpsichord, which corresponds to the 3D spectral plot shown in Fig. 7.5.

"X"-ample 7.11



	Frequency	Output	L1	L2	L3	L4	R1	R2	R3	R4	Rsc	Vsn
op 1	1.00	99	99	90	0	0	90	30	20	48	3	4
op 2	3.00	84	99	90	0	0	99	55	25	70	1	1
op 3	9.00	80	99	85	0	99	76	30	30	90	1	1
op 4	2.00	99	99	90	0	0	90	30	20	48	6	3
op 5	6.00	88	99	90	0	0	99	55	25	70	4	1
op 6	18.00	80	99	85	0	99	76	30	30	90	6	0

INSTRUCTIONS:

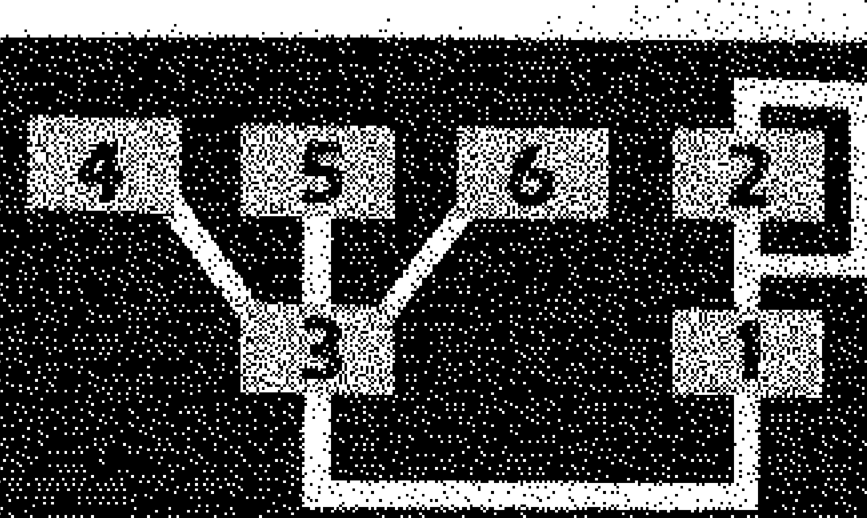
Start from "X"-ample 7.3. Make the above modifications to the envelopes of ops 1-3, then copy them to ops 4-6. Notice that the shortening of the envelopes of the second stack is done simply by rate scaling. There is keyboard scaling on four operators. On the two carriers, this is to obtain a good volume balance across the keyboard. On the two upper modulators its main purpose is to cure the aliasing which occurs at the higher frequencies due to the very large bandwidth. The left curve function in each case is +LIN and the right curve function is –LIN.

	Break point	*L depth*	*R depth*
op 1/4	*C2*	*20*	*6*
op 3/6	*G3*	*10*	*60*

In the "X"-ample No. 7.5 we looked at the sound of feedback and suggested that it could be developed as a "filter-synth" exercise. Notice in "X"-ample 7.12 how much movement and change of timbre can be achieved by detune of the parallel modulators, operators 4,5 and 6. The feedback has been reduced to 6 in this "X"-ample in order to reduce slightly the bandwidth, so that it blends more with the second half of the algorithm. The carrier frequency of the feedback pair has now been fixed at a very low rate – about 2 Hz – in order to cause beating (chapter5).

You should do your own experimentation with the envelope in this case to find some effective sounds. As the parameters are set, the modulation wheel controls the effect of the feedback pair and the whole sound has been transposed down one octave.

"X"-ample 7.12



	Frequency	Output	AMS	Feedback	Detune
op 1	2.08Hz	99	0		
op 2	1.00	85	3	6	
op 3	1.00	99	0		
op 4	1.00	85	0		−2
op 5	1.00	75	0		
op 6	1.00	75	0		+2

INSTRUCTIONS:

Start from "X"-ample 7.5. Make the above modifications and add ops 5 and 6. Turn oscillator sync. OFF, as this will increase the variety of timbres due to an "out-of-phaseness" of the operators. Use the key transpose function to make middle C = C2. The modulation wheel will be effective in bringing in the whole of the spectrum due to the FM pair with feedback. Although there is no AMS on op 1, it is not heard when there is no index (i.e. the modulation wheel is at min. position) because it is at a frequency, 2Hz, which is simply too low for our ears to hear even though its amplitude is still maximum.

Choosing the Algorithm

What processes have we used in making the sounds in all the examples in this chapter? It begins, of course, with a careful analysis of what we want to hear, and how we want to play it. We are dealing mainly with a piano-style keyboard, but that is not the only means of triggering a synthesizer, and the "interface" should be considered when we are programming. Having decided what we want to hear, and how it is to be played – by computer, keyboard or other musical instrument fitted with an appropriate signal conversion device – we then proceed to arrange the operators in our FM system to best produce the results we want. Does the sound have clearly distinct component parts? If yes, then we should choose an algorithm which itself is separated into additive components.

What sort of timbres do we need from each "mini-algorithm"? Is a simple FM pair adequate, or do we need the broader bandwidth and control offered by a stack or cascade? Can the algorithm be arranged so that we leave ourselves a variety of methods by which to change the character of the spectrum, for example choosing between increasing the feedback level and raising the modulation index?

These are the sort of questions that need to be answered if programming is to be logical and well structured, so you should satisfy yourself that you are familiar with the use of a single operator, a simple FM pair, two parallel modulators, a cascade, the effect of feedback and how the modulation index and frequency ratios affect the sound in each case. We'll build a piano sound with these thoughts in mind. Three areas of the sound can be considered (of course this is not the only approach) – the more brilliant timbres and long sustain of the bass strings, the slightly mellower and shorter sustain of the treble strings and the mechanical "bonk" or "thunk" of the piano mechanism. A cascade of three operators would seem a likely choice for the bass strings, as this will allow us close control of different regions of a broad bandwidth.

This leaves us – on the DX7 – three operators for the treble strings and the "bonk". For both these sounds, single sine waves will not do, therefore we are left with a choice of parallel carriers and a single modulator, or a single modulator with parallel carriers. As the sound of the treble strings can probably be produced with a simple FM pair, either of these two options will suit the requirements. We have only to consider the best method of introducing the mechanical noise – either based around a carrier or a modulator. In the case of parallel carriers, the index would be the same for both carriers. Therefore, having obtained a good sound for the treble strings we would be forced to accept that level of index for the "bonk". Let's opt for parallel modulators where we will have more control.

That leaves us with a choice of two algorithms; 10 and 11. Where will the feedback option be most useful? Probably we already have enough control over bandwidth in the cascade, so that suggests we start working with algorithm 11. The next "X"-ample offers no parameter values, but instead contains a series of hints for building this sound yourself from scratch.

"X"-ample 7.13 (YOUR OWN!)

INSTRUCTIONS: Starting from the VOICE INIT? position.....

The first part of this process is to approximate the sound that you are building. We have already decided to use algorithm 11. Begin with op.1 and develop an envelope for the bass strings. (Don't forget to add some rate scaling – about 3 or 4 – before working on the envelope.) Copy this envelope to op.2 using ratios of 1:1. It may be useful to increase level 4 on op.2 to about 50, so that the harmonics do not decay totally as the volume decays. Copy the op.2 envelope to op.3 and try ratios of 5,6,7,8, or 9 for op.3 with some detune. All the time, juggle with the output levels. Begin with small indexes and work up. Use velocity sensitivity sparingly with the modulators (1 or 2). Use a little more on the carriers. Use keyboard level scaling to remove inappropriate effects of the cascade modulation in the middle to higher octaves (start by scaling out op.3, then 2, then 1).
Now begin work on the treble strings. Maybe the same envelope you used on op.1 will suffice to begin with, so copy this to op.4 and op.6, which will from the basic FM pair for the treble strings – be sure to remove (or readjust) any keyboard level scaling which may have been copied from op.1. (Perhaps an increased rate scaling will be effective here.) Try testing the index in this pair, first with ratios of 1:1 and then with a low fixed frequency on the carrier, op.4 (about 1Hz), which will give beating effects that you may like. (This beating effect would be expected from the three strings per key in the treble range of the piano.) Use feedback to increase the brilliance, as well as simply increasing the index of op.6. Now, by putting a short

percussive envelope on op.5 (just a fraction of a second; for example, L1=99, L2=0, L3=0, L4=0, R1=80, R2=70, R3=99, R4=99) and using a low frequency on this operator (try fixing the frequency around 100-200 Hz), the output of this operator can be introduced to produce a mechanical type of "thump" to the tone.
Increase this output level (op.5) slowly after you have set up the basic pair of 4 and 6.
Now you are ready to "fine-tune" your sound. Turn on all operators and begin the adjustments necessary. The envelopes will perhaps be most important, so take care as to how you develop them. Listen for velocity sensitivity effects and make good use of keyboard level scaling and rate scaling — this is now your "X"-ample!

Well, at the conclusion of a whole chapter of sound-making there has been no evidence of mathematics, psychoacoustics, sine functions, exponential relationships, 180° phasing and so on .... or has there? We hope that through reading this book, you will not only have improved your abilities as practical FM programmers, but also that you will have gained an impression of the "inside" of sounds and how they work. Sound synthesis is a developing art and is worthy of a little scientific understanding, which not only supports the art but is interesting and inspiring in itself.

Now YOU work out what is happening! Enjoy your music!

APPENDIX 1

Logarithmic Representation and "Pitch Frequency"

This appendix should explain some of the mystery which surrounds the idea of a **logarithmic** representation. Lets start by looking at the table of frequencies arising from the discussion on page 33 in chapter 2:

Freq coarse parameter			freq of key pressed		freq of harmonic
op 1	1	×	220	=	220 (harmonic No. 1)
op 2	2	×	220	=	440 (harmonic No. 2)
op 3	4	×	220	=	880 (harmonic No. 4)
op 4	8	×	220	=	1760 (harmonic No. 8)
op 5	16	×	220	=	3520 (harmonic No. 16)

We can see that while we hear a constant pitch distance of an octave from operator to operator, the frequency distance always doubles. Or we can say that *to change the pitch a constant distance the frequency must change by a constant factor* (in this case of an octave, by a factor of 2). In Fig. A.1 we see a line representing frequency in hertz. The line is divided into equal units of frequency on which is marked the points which we have heard to be successive octaves (as shown in chapter two). This representation of frequency does not preserve the equally perceived pitch distance.

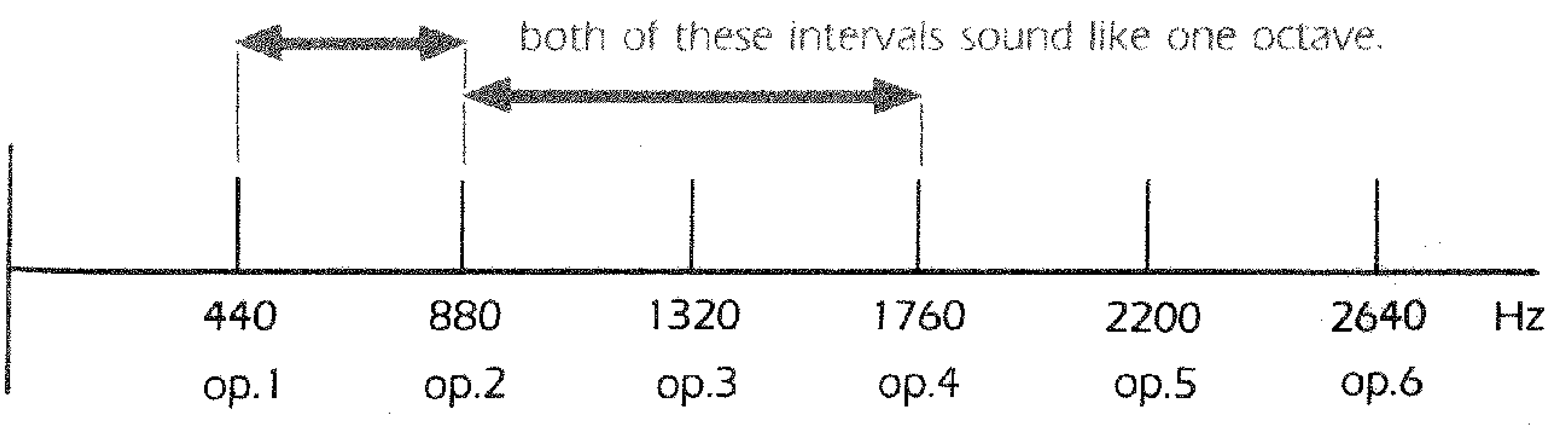


Fig. A.1

Plotting the frequencies of successive octaves in linear frequency does not show equal perceptual distances, but it does allow us to see the mathematical construction clearly.

If equal distances between octaves are to be preserved, how should the frequency scale change? In the "X" experiment in chapter two, it was observed that successive octaves were always two times the previous; well, then, we can set out the table of frequencies again but with a change. The frequency coarse parameters which are required to produce the octaves are this time seen as multiples of 2; first once, then twice, three times, and four times. Presumably, if we were to continue there would be yet more octaves.

Freq. coarse parameter	Freq. of key pressed		Freq. of octave
1	x 220	=	220 (fundamental)
1 x 2	x 220	=	440 (octave #1)
1 x 2 x 2	x 220	=	880 (octave #2)
1 x 2 x 2 x 2	x 220	=	1760 (octave #3)
1 x 2 x 2 x 2 x 2	x 220	=	3520 (octave #4)

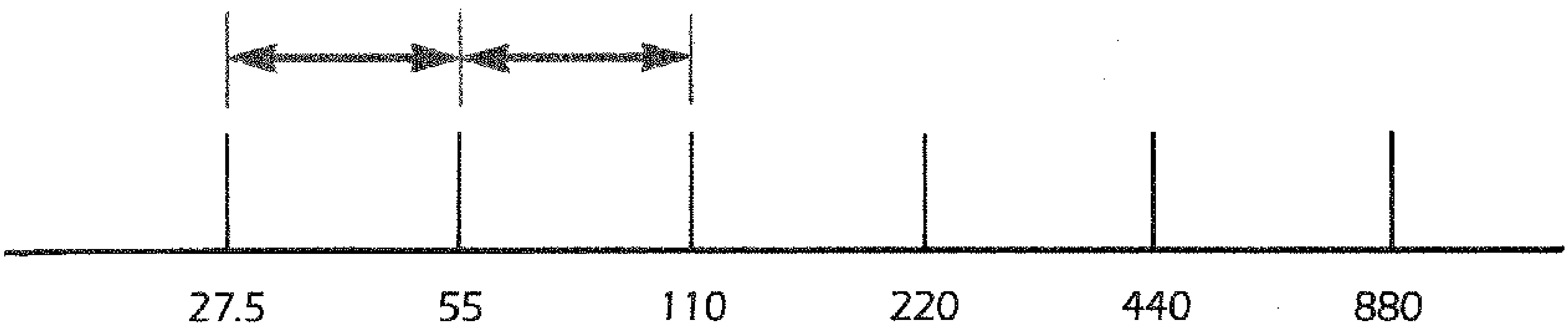
Any number **n** multiplied by itself a certain number of times (**m**) can be represented in the form n^m, or "n to the mth power." In other words, 2 times 2 times 2 is called 2^3 or 2 all alone is called 2^1, so we can simplify all of the above table in the following way. The only thing that you have to take on good faith, as it is perhaps less evidents, is that any number to the power 0 is equal to 1. So now our table can be written as follows:

Freq coarse parameter		Freq. of key pressed		Freq. of octave
2^0	times	220	=	220 (fundamental)
2^1	times	220	=	440 (octave #1)
2^2	times	220	=	880 (octave #2)
2^3	times	220	=	1760 (octave #3)
2^4	times	220	=	3520 (octave #4)

In Fig. A.2 we now plot frequency from 27.5Hz (piano lowest A), but instead of plotting the actual frequency as a number or Hertz or cycles per second, or even as simple multiples of a given frequency, as we would in a linear representation, the equal distances 1,2,3,4 and so on are "powers" (or exponents) of the number 2. This in turn gives us the number by which the reference frequency must be multiplied in order to ensure that we get equal distances for "octaves"

This scale is called a **LOG** scale, and it allows us to deal with variables which do not relate to each other in a simple way but have an "exponential" relation, such as perceived musical intervals and actual physical frequency. The mathematical meaning of logarithm and the way that it relates to the power to which a number is raised is as follows:

if $n^m = x$ (n to the mth power equals x)
then $\log_n x = m$ (log to the base of n of x equals m)
or, the log of a number is the power to which the base must be raised to equal that number. Therefore,
$2^4 = 16$ (2 to the 4th power equals 16)
and $\log_2 16 = 4$ (log to the base 2 of 16 equals 4).
(At the beginning of the book you were assured that only the four basic arithmetic operations would be required to understand the contents. Remember that exponents and logarithms are simply a special form of multiplication).



now the octaves are given equal intervals, which is how we perceive them.

1	2	4	8	16	factors which give successive octaves
2^0	2^1	2^2	2^3	2^4	powers of two which give successive octaves

or $\log_2 1 = 0$ $\log_2 2 = 1$ $\log_2 4 = 2$ $\log_2 8 = 3$ $\log_2 16 = 4$

Fig. A.2

What can we say about the frequency scale now? While it closely reflects the way in which we perceive pitch, it is certainly not linear — in fact it is "exponential". If we call 27.5Hz (bottom A) frequency 1, then our next equidistant point, the next octave, is at 2 times 1 (55Hz), the next at 4 times 1 (110Hz), and so on, giving us equal octaves, as in our previous table. The next line of the graph shows these factors clearly: 1, 2,4,8,16,and so on. The final line re-writes these numbers as powers of 2. As these factors are all powers of 2, log to the base 2 of the factors yields a scale where distance on the line correlates with perceived distance in pitch.

Having understood that 27.5Hz x 2^n will give the frequency of the nth octave, we can extend this relation to include the scale steps within the octaves as well.

In Fig. A.2 we have seen how the frequency space falling between 27.5Hz and 880 Hz can be divided into equally *perceived* intervals by using log of frequency. Fig. A.3 is the same as Fig. A.2, except that we have changed the form of the exponents from integers to fractions. That is, instead of $440 = 27.5 \times 2^4$ we have $440 = 27.5 \times 2^{48/12}$ and for the octave above $880 = 27.5 \times 2^{60/12}$. As the difference between 48 and 60 is twelve you might correctly suspect that the intervening half steps can be figured out by increasing the numerator of the exponent. The B flat above A440 then is 466.16 or $27.5 \times 2^{49/12}$ and B natural is $493.88 = 27.5 \times 2^{50/12}$, etc. Since a piano has 88 keys, the lowest note, A, is $27.5 \times 2^{0/12} = 27.5$Hz, and the highest is $27.5 \times 2^{87/12} =$ 4186Hz (remember that from 0 to 87 is 88 steps).

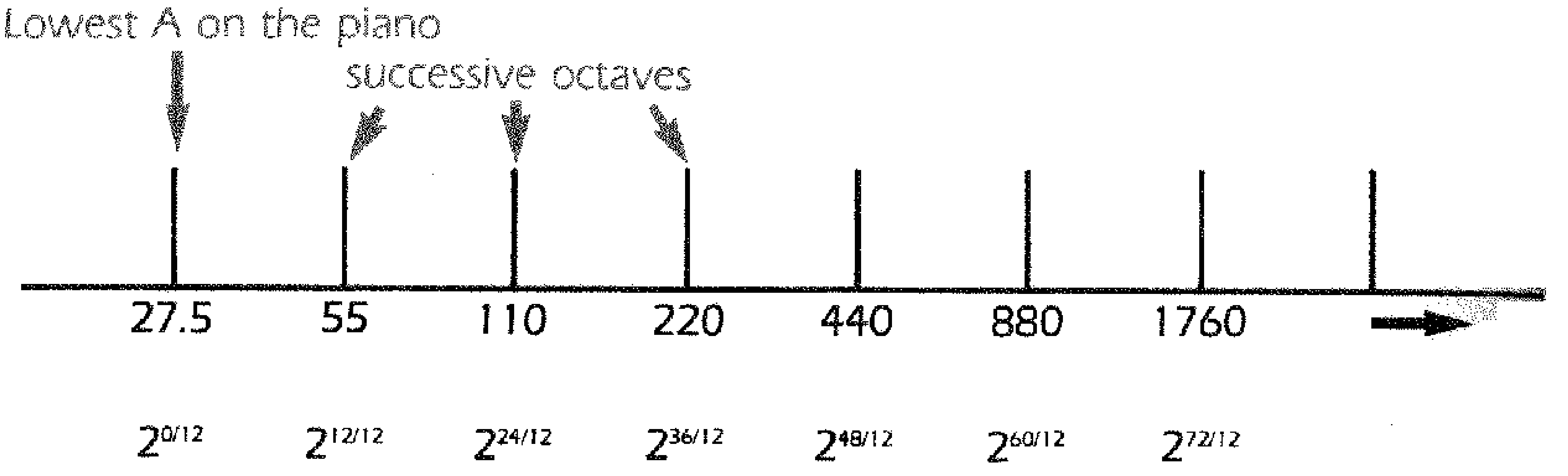


Fig. A.3

By expressing integer powers of two as fractional powers, where the denominator is 12 as above, we see how each octave can be divided into equal steps by increments of 1 in the numerator. Therefore the octave between A440 and A880 is divided into twelve equal scale steps by incrementing the numerator of the exponent by 1 for each successive semitone. To determine the semitones starting at A220, then, the numerator would again increment by 1 (36, 37, 38, 39 ... to 47), and so on for other octaves.

Given the frequency f of any note, the octave above will be at two times that frequency, $2f$, the first semi-tone above will be at $2^{1/12}$ times f, and so on. The following table will be useful for calculating "pitch frequencies" in the equal-tempered scale. Remembering that the ear is very sensitive to pitch, it is useful to give values of the powers of two to three decimal places, if this table is to be used for real pitch determination. For quick estimates of frequencies, however, simply round them down.

$2^{0/12} =$	1	$2^{7/12} =$	1.498
$2^{1/12} =$	1.059	$2^{8/12} =$	1.587
$2^{2/12} =$	1.122	$2^{9/12} =$	1.682
$2^{3/12} =$	1.189	$2^{10/12} =$	1.781
$2^{4/12} =$	1.260	$2^{11/12} =$	1.910
$2^{5/12} =$	1.335	$2^{12/12} =$	2.000
$2^{6/12} =$	1.414		

APPENDIX 2

"X" Synth Comparisons by Index vs. Op. Output Level

The information contained here allows an estimation of modulator output values on other "X"-Series instruments, given values for a DX7. (DX9 values are the same as DX7.)

In order to have the possibilities of using and comparing different "X"-synths for exercises and sound making, the following tables and graphs have been prepared, based upon the internal workings of the "X"'s. These graphs should give you adequate information to make practical comparisons between DX7, DX21 and CX5.

A selection of output levels have been converted to index by the formulas shown. TL is simply a value that the "X" reads for any given output level shown on the LCD. If you want to calculate an index accurately for output levels not shown here, then convert the output level to a TL value by the table given below (TL v output level), then apply the appropriate formula for index according to the model of your synth. DX5, 1, TX7 are the same as DX7; DX27, 100 are the same as DX21; and CX11, 7m are the same as CX5.

TL vs. OUTPUT LEVEL (For DX7 and DX21)

Level table

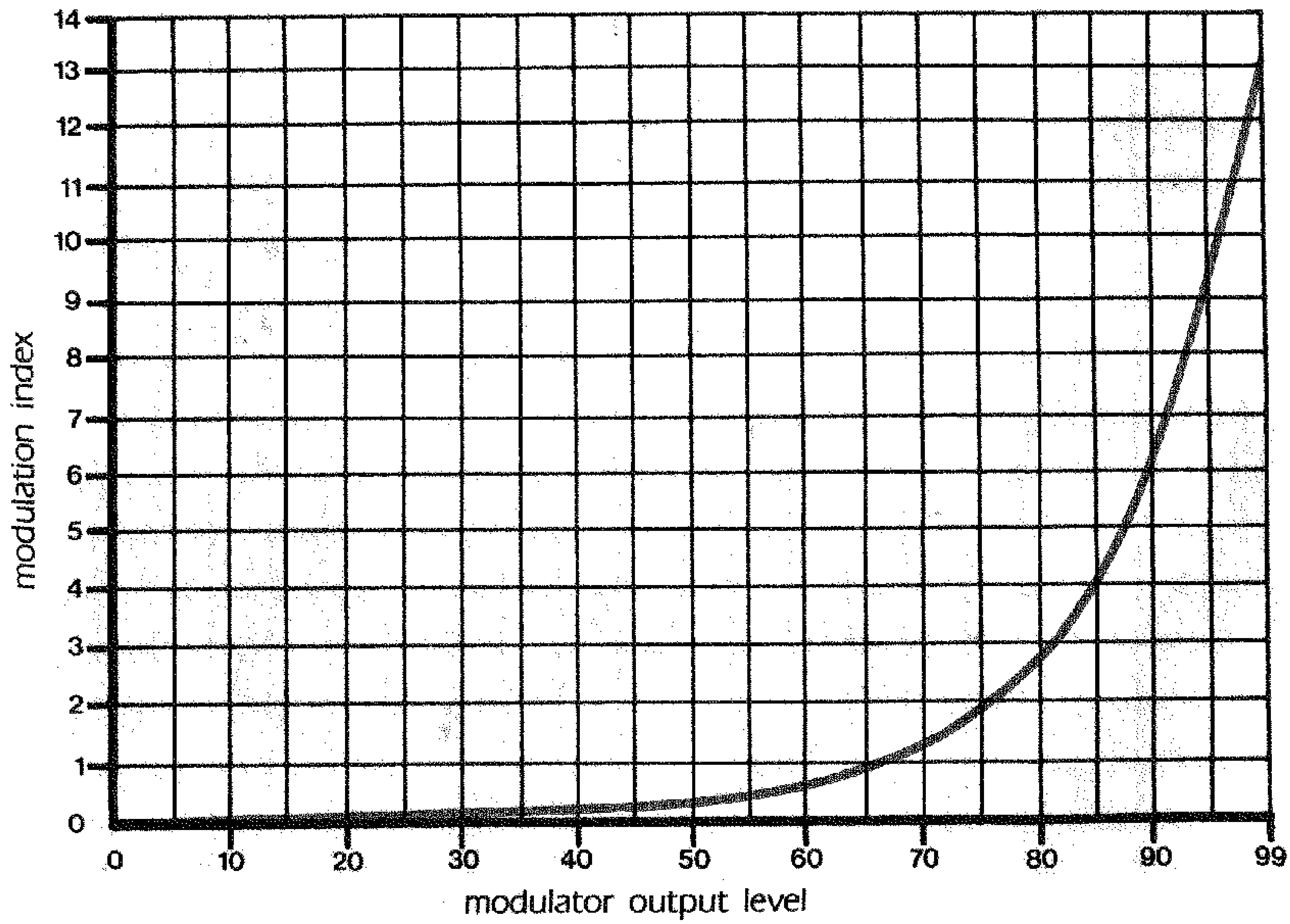
TL values	0	1	2	3	4	5	6	7	8	9
0	127	122	118	114	110	107	104	102	100	98
10	96	94	92	90	88	86	85	84	82	81
20	79	78	77	76	75	74	73	72	71	70
30	69	68	67	66	65	64	63	62	61	60
40	59	58	57	56	55	54	53	52	51	50
50	49	48	47	46	45	44	43	42	41	40
60	39	38	37	36	35	34	33	32	31	30
70	29	28	27	26	25	24	23	22	21	20
80	19	18	17	16	15	14	13	12	11	10
90	9	8	7	6	5	4	3	2	1	0

(For example, for an output level of 67 on the synth, look up 60 in left hand column, then across to 7, to make 67, and take the "TL" value from the box, in this case 32.)

DX7 modulation index table for values: $I = \Pi \times 2^x$ $\quad x = (33/16) - TL/8$

Output	TL	TL/8	X	2^x	Index
10	96	12	–9.9375	0.00102	0.003
20	79	9.875	–7.8125	0.00445	0.013
30	69	8.625	–6.5625	0.01058	0.031
40	59	7.375	–5.3125	0.02516	0.079
50	49	6.125	–4.0625	0.05985	0.188
60	39	4.875	–2.8125	0.14235	0.446
65	34	4.25	–2.1875	0.21953	0.690
70	29	3.625	–1.5625	0.33856	1.068
75	24	3.0	–0.9375	0.52214	1.639
80	19	2.375	–0.3125	0.80525	2.512
85	14	1.75	+0.3125	1.24186	3.894
90	9	1.125	+0.9375	1.91521	6.029
95	4	0.5	+1.5625	2.95365	9.263
99	0	0	+2.0625	4.17710	13.119

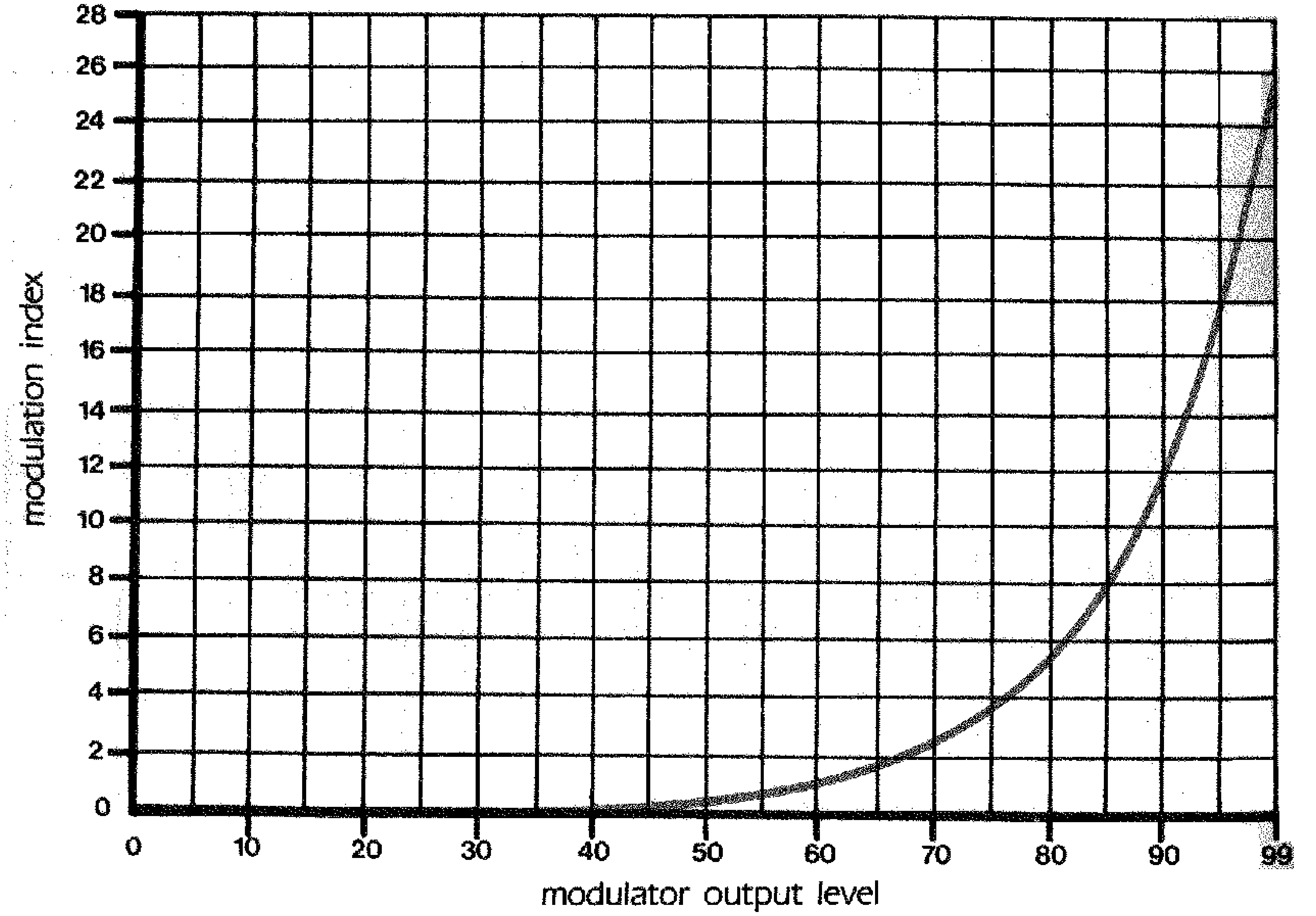
DX7——Index vs. Output Level



DX21 modulation index table for values: $I = 8\Pi \times 2^y$ $y = -TL/8$

Output	TL	TL/8 (= –y)	2^y	Index
10	96	12	0.00024	0.004
20	79	9.875	0.00107	0.025
30	69	8.625	0.00253	0.063
40	59	7.375	0.00602	0.151
50	49	6.125	0.01433	0.360
60	39	4.875	0.03408	0.855
65	34	4.25	0.05256	1.320
70	29	3.625	0.08105	2.037
75	24	3.0	0.12500	3.143
80	19	2.375	0.19278	4.850
85	14	1.75	0.29730	7.466
90	9	1.125	0.45850	11.527
95	4	0.5	0.70711	17.773
99	0	0	1.00000	25.136

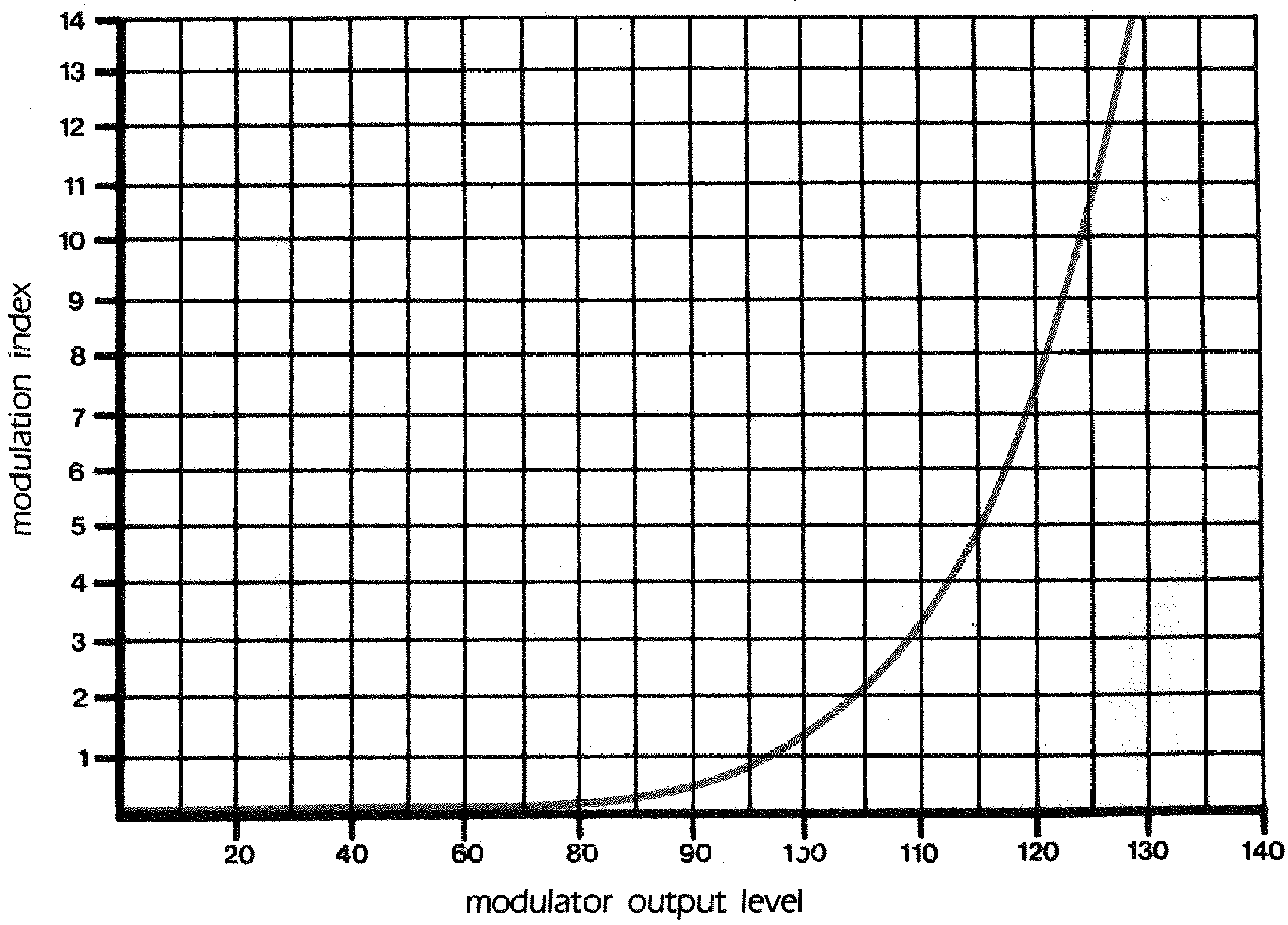
DX21——Index vs. Output Level



CX5 modulation index table for values: $I = 8\Pi \times 2^z$ $z = -(135\text{-output})/8$

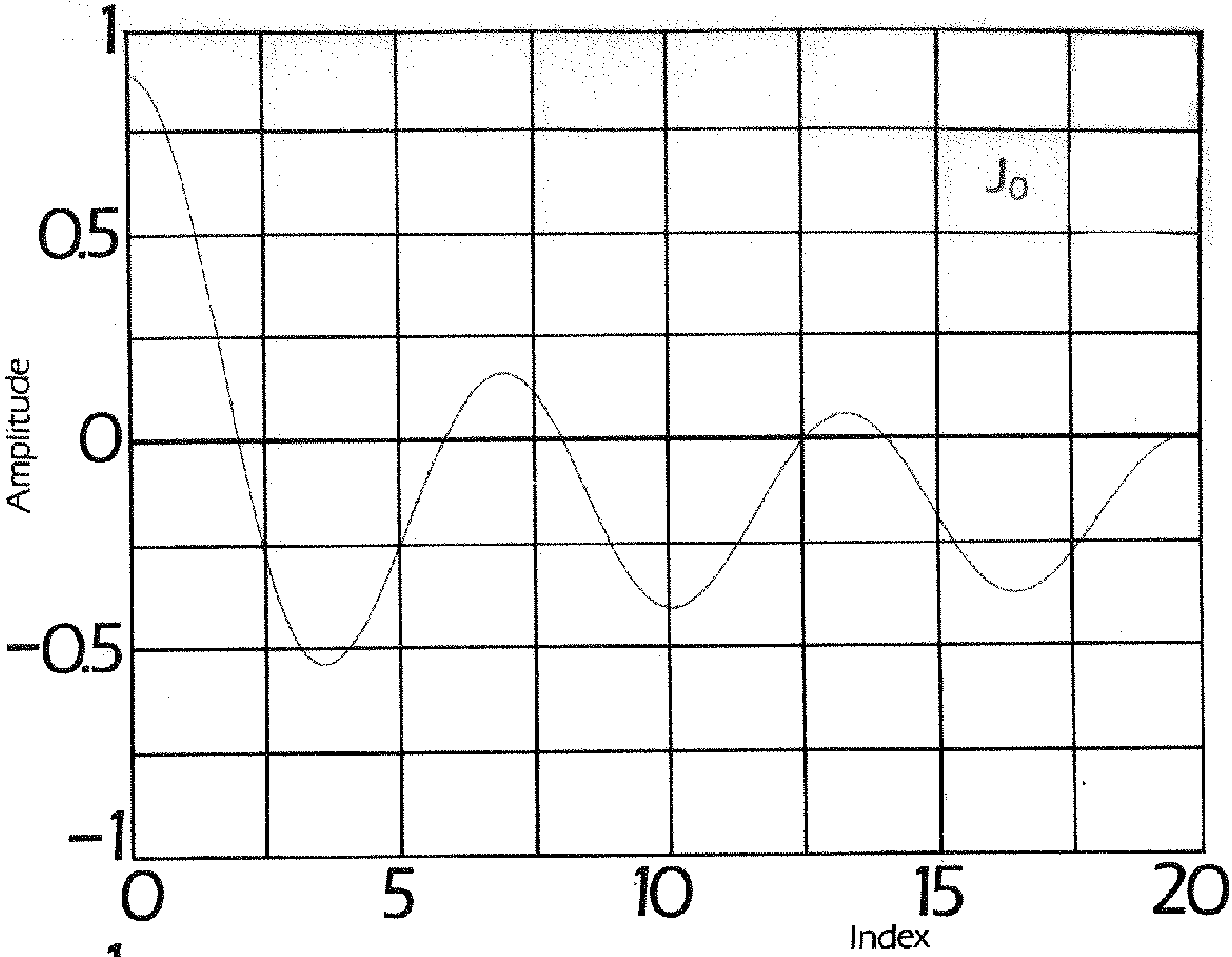
Output	z	2^z	Index
20	−14.375	0.00005	0.003
40	−11.875	0.00027	0.005
60	−9.375	0.00151	0.025
80	−6.875	0.00852	0.037
90	−5.625	0.02026	0.218
95	−5.000	0.03125	0.508
100	−4.375	0.04819	1.207
105	−3.75	0.07433	1.869
110	−3.125	0.11463	2.891
115	−2.5	0.17678	4.425
120	−1.875	0.27263	6.853
127	−1.0	0.50000	12.570

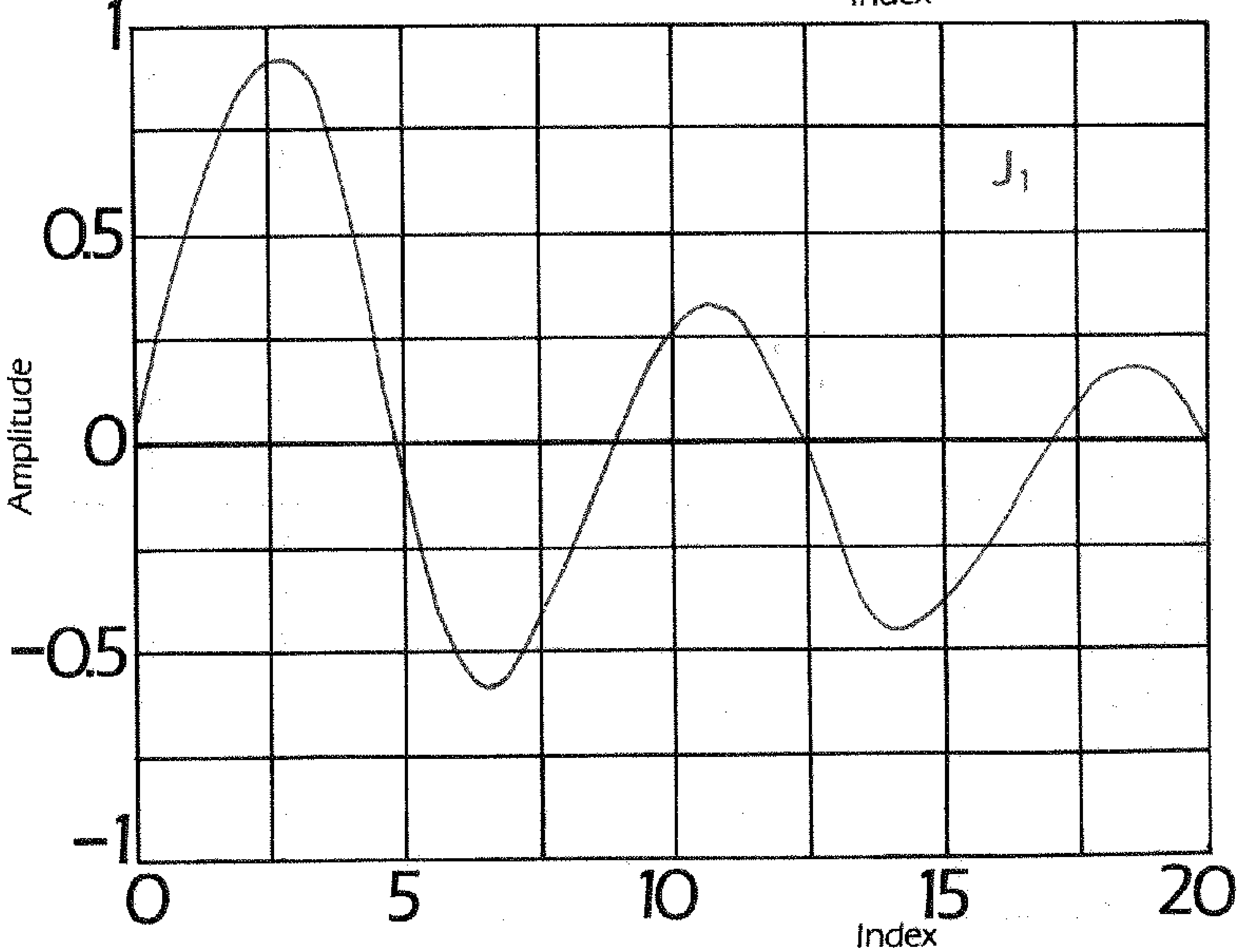
CX5——Index vs. Output Level

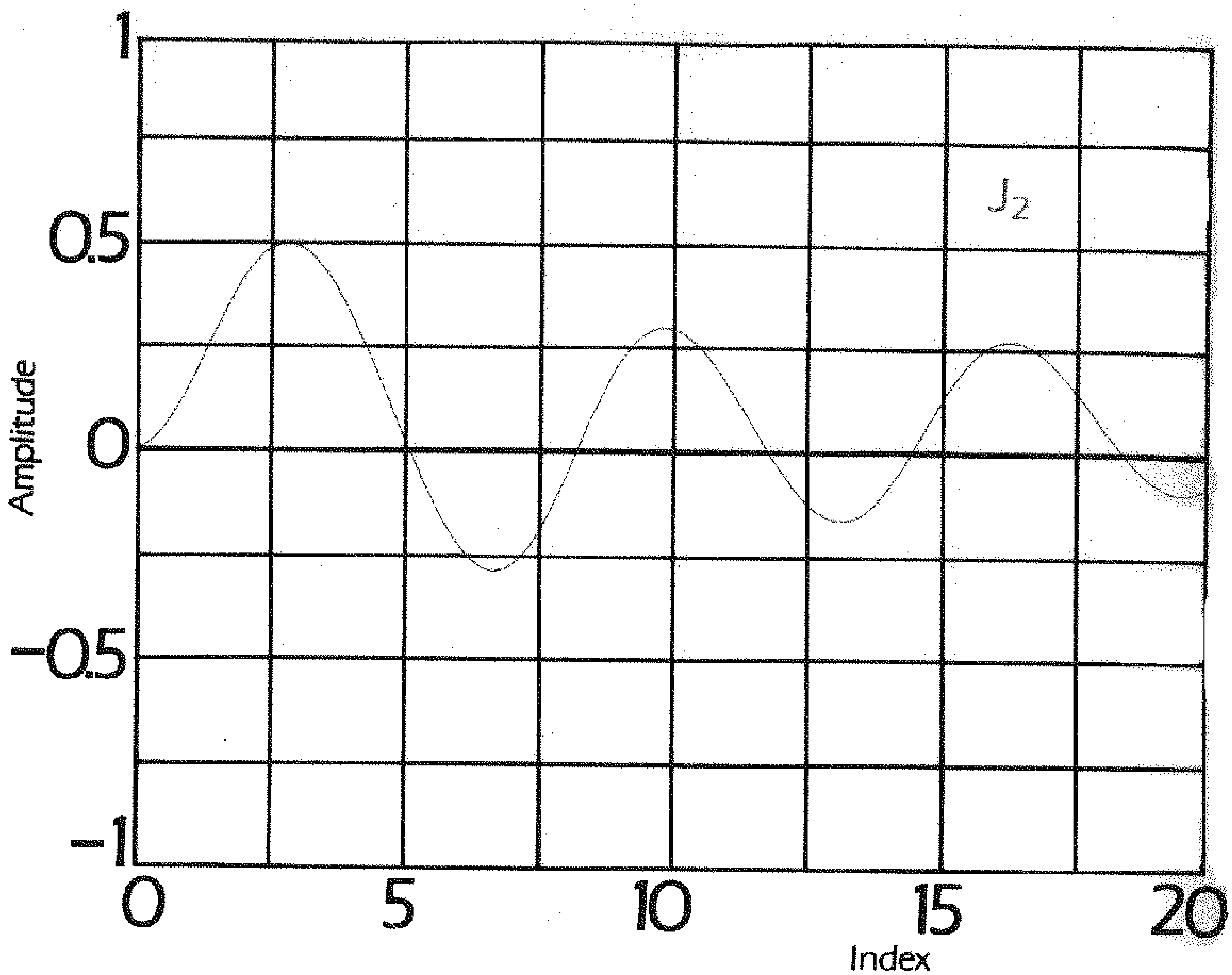


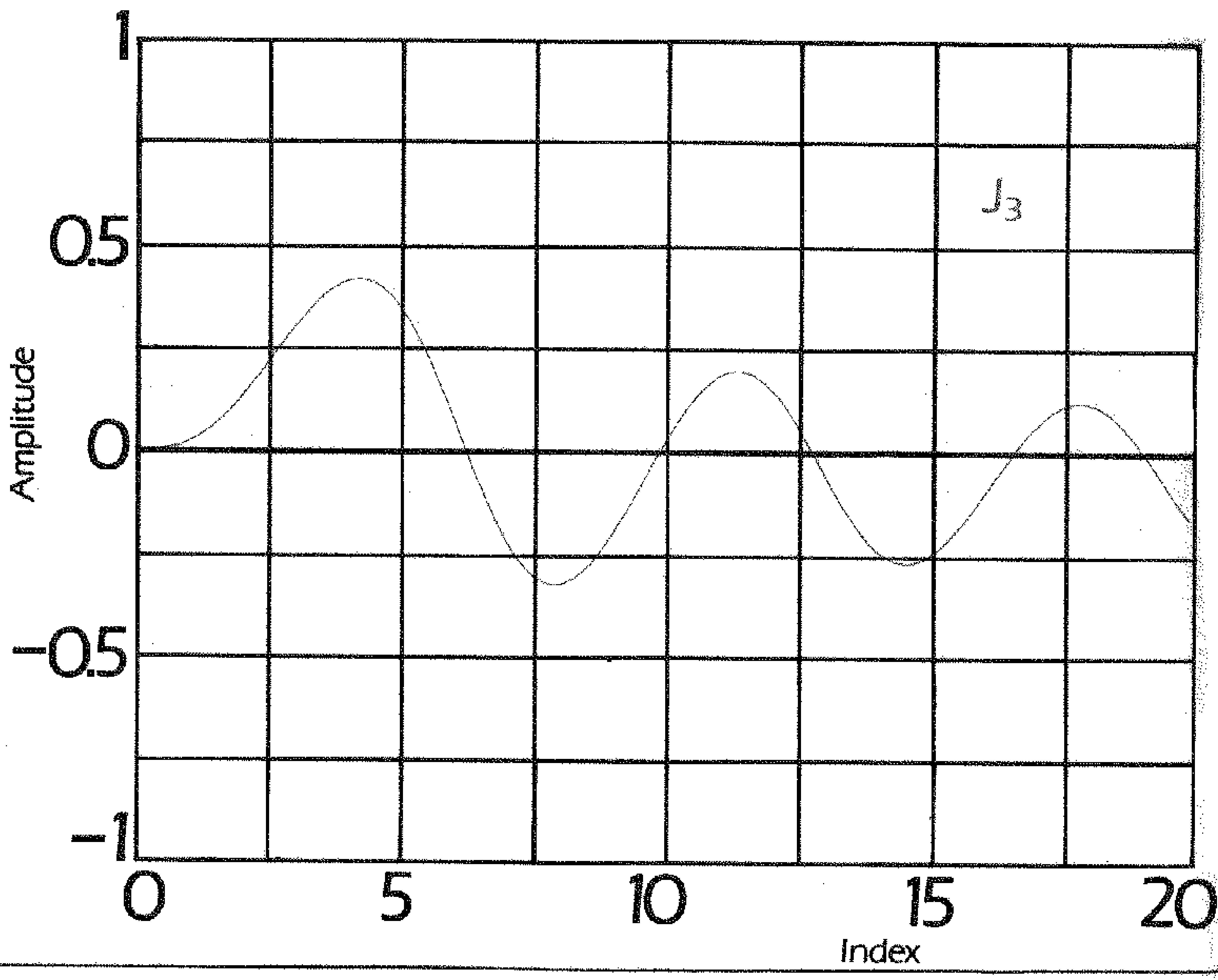
APPENDIX 3

Bessel Function Graphs


1
0.5
0
−0.5
−1
Amplitude
0
5
10
15
20
Index
J_0


1
0.5
0
−0.5
−1
Amplitude
0
5
10
15
20
Index
J_1


J2
1
0.5
0
−0.5
−1
Amplitude
0
5
10
15
20
Index


J3
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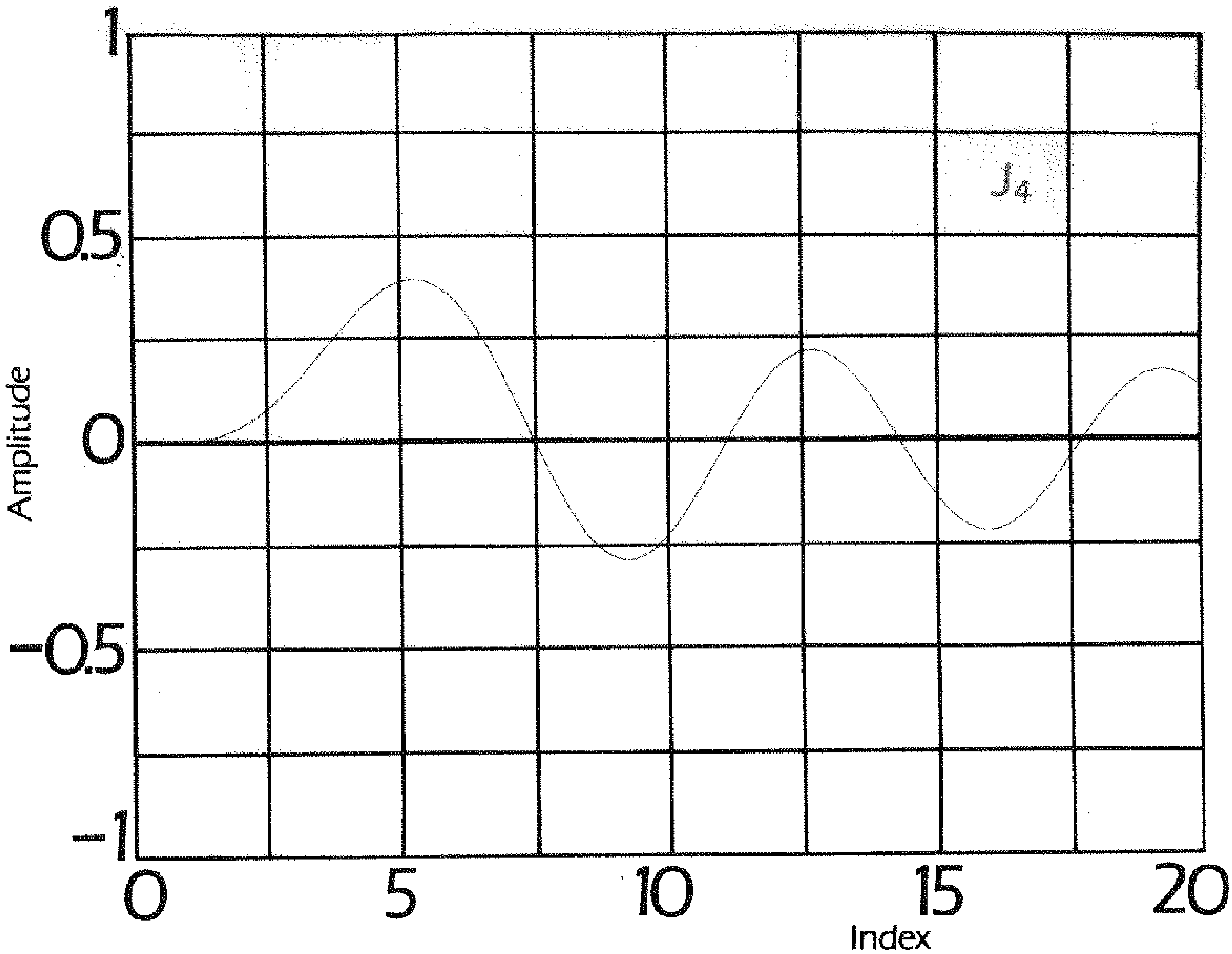
0
−0.5
−1
Amplitude
0
5
10
15
20
Index



J_4
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0.5
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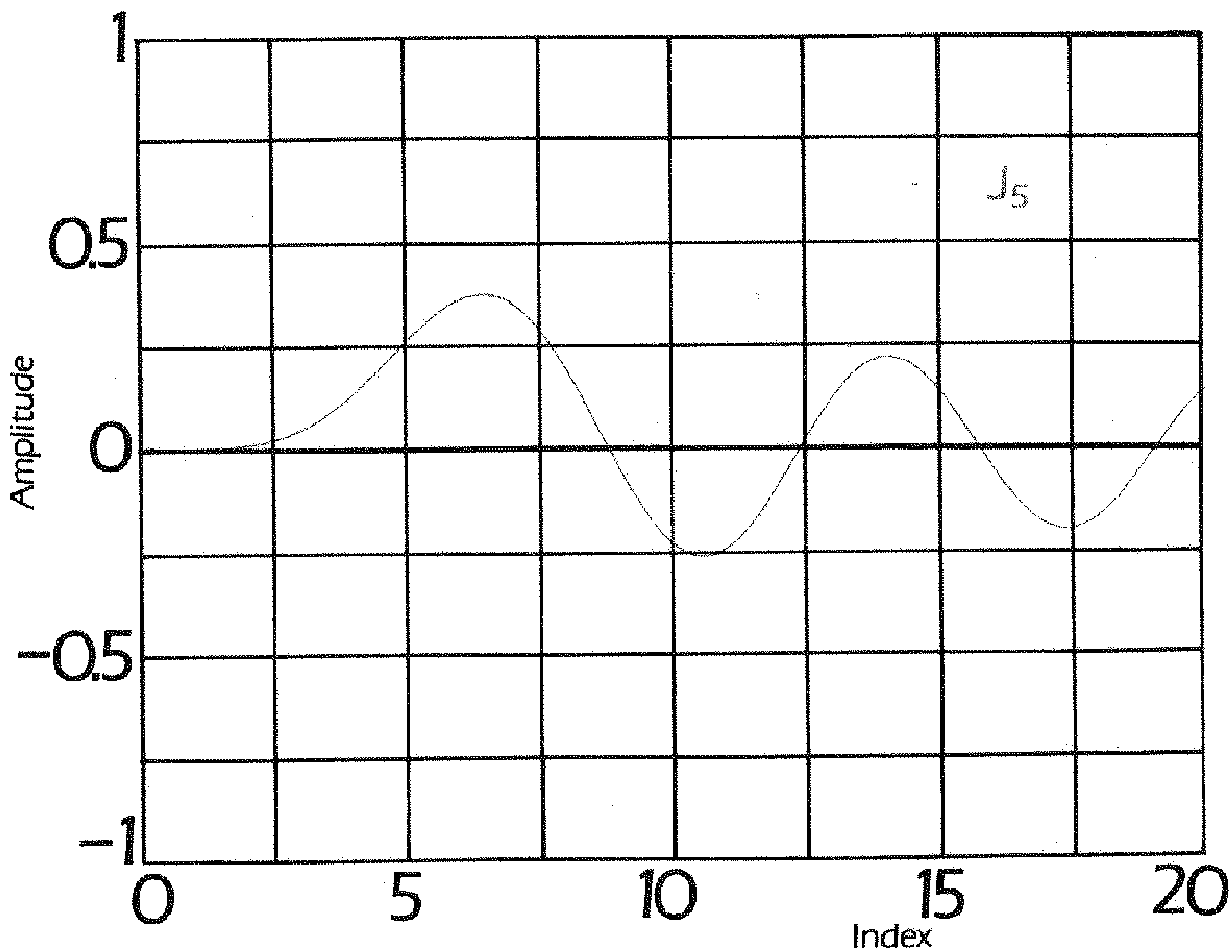
-1
Amplitude
0
5
10
15
20
Index



J_5
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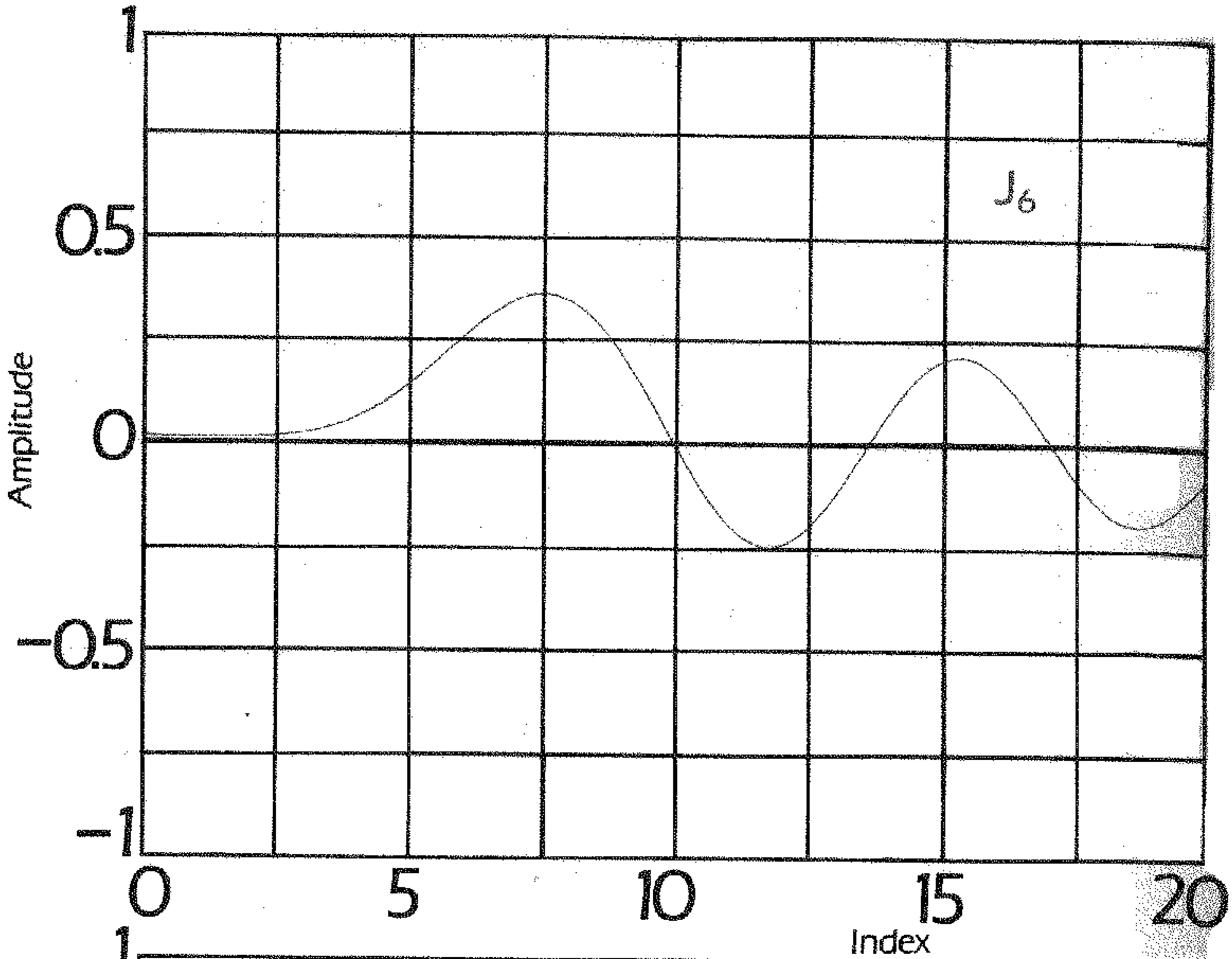
0.5
0
-0.5
-1
Amplitude
0
5
10
15
20
Index



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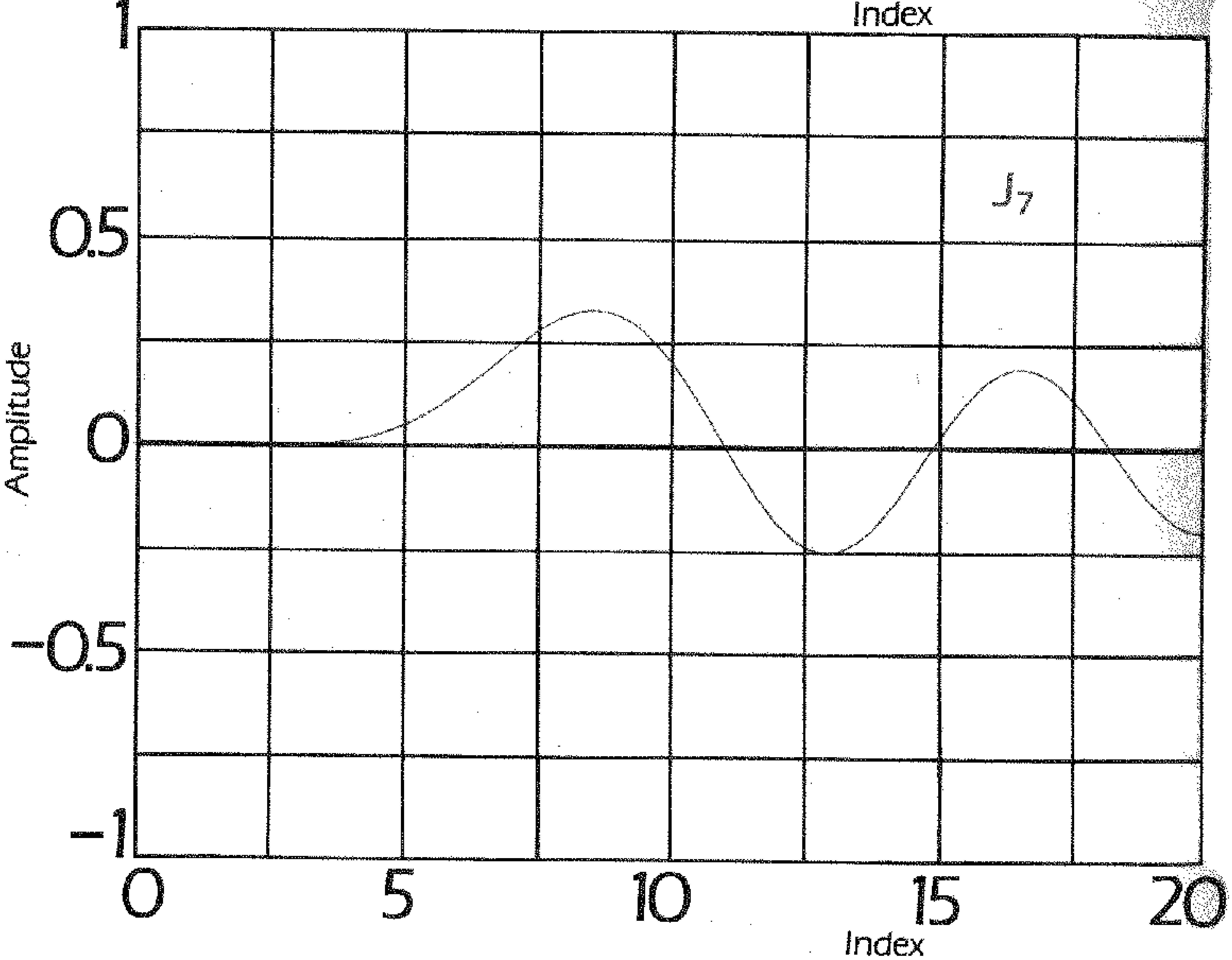
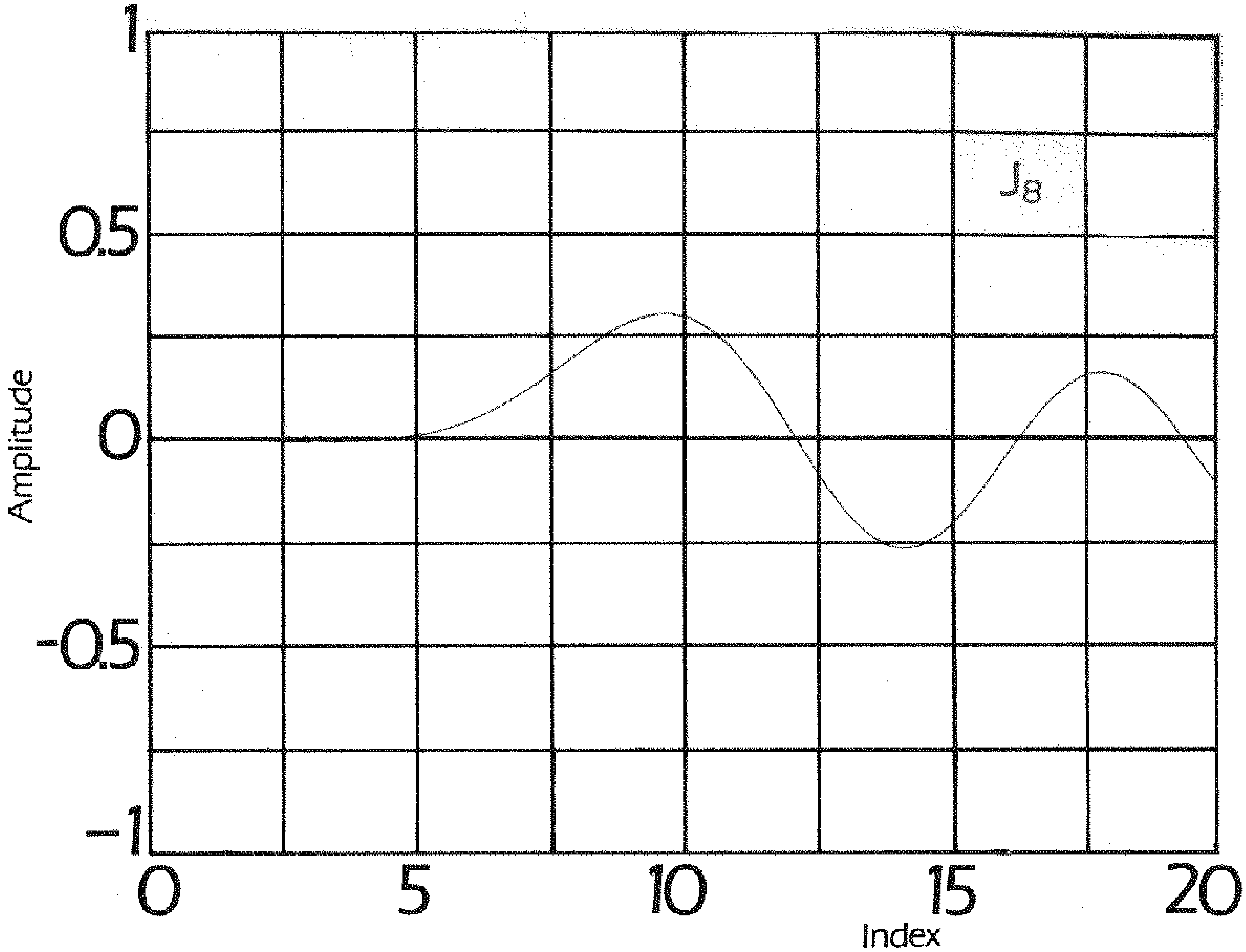
0.5
0
−0.5
−1
Amplitude
J6
0
5
10
15
20
Index



1
0.5
0
−0.5
−1
Amplitude
J7
0
5
10
15
20
Index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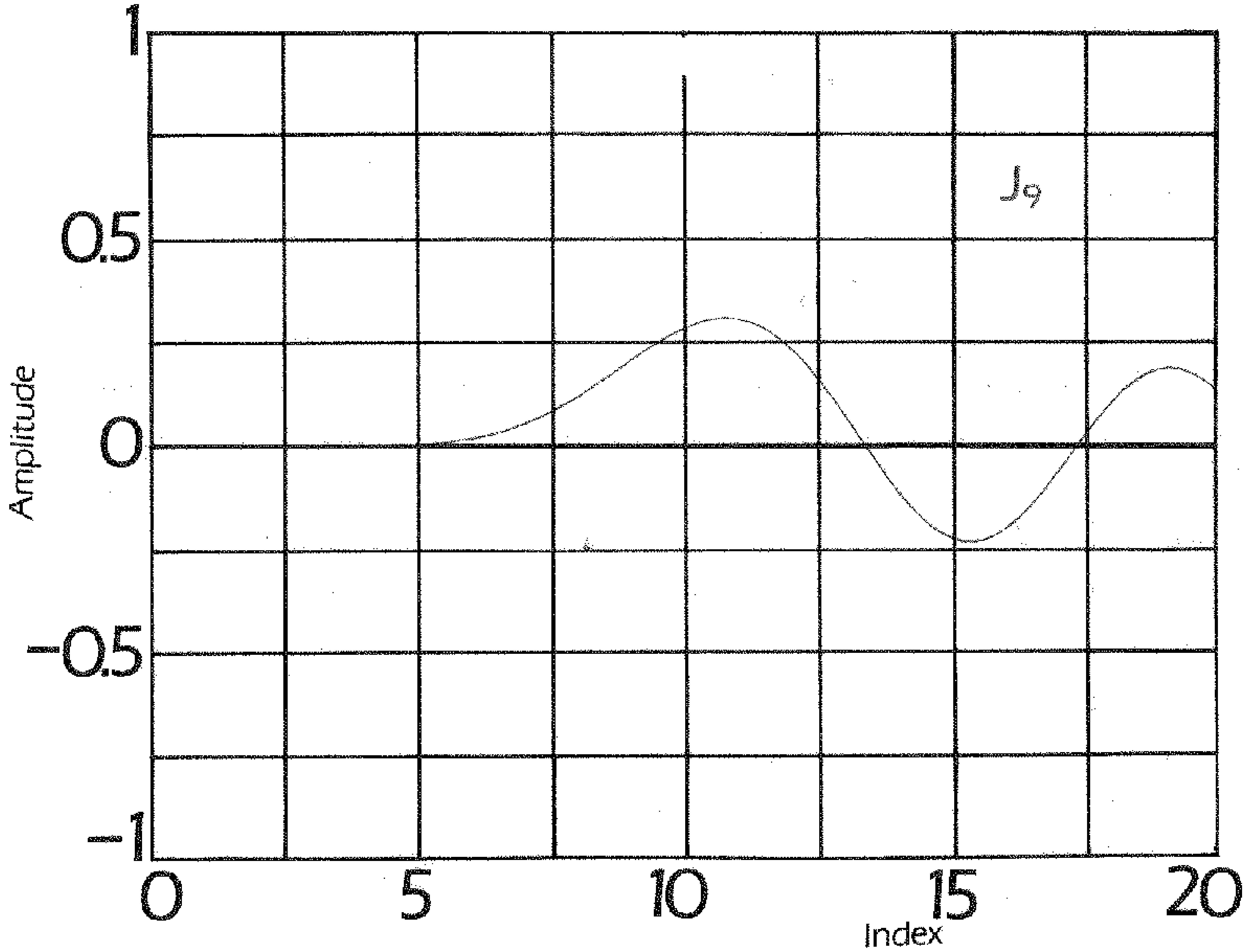
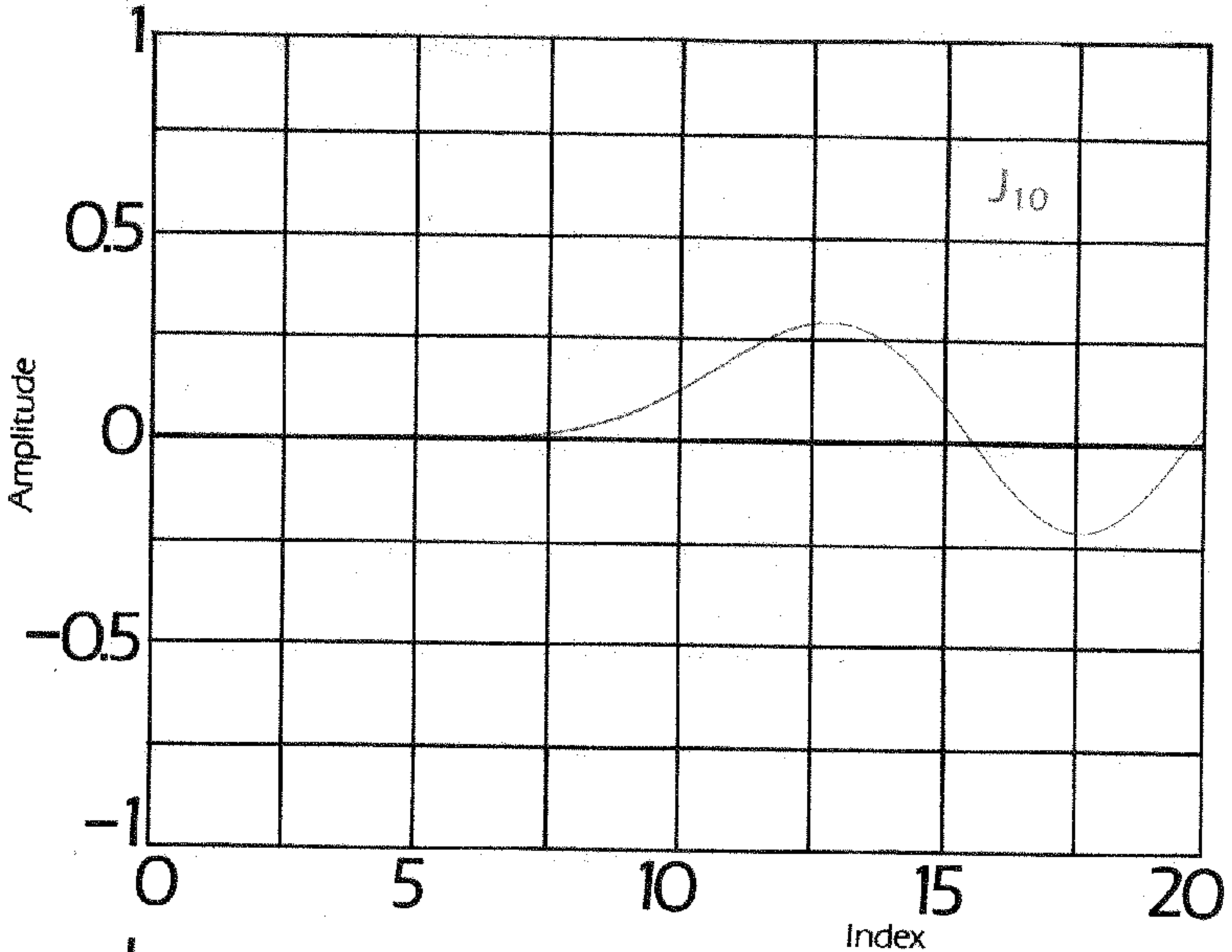
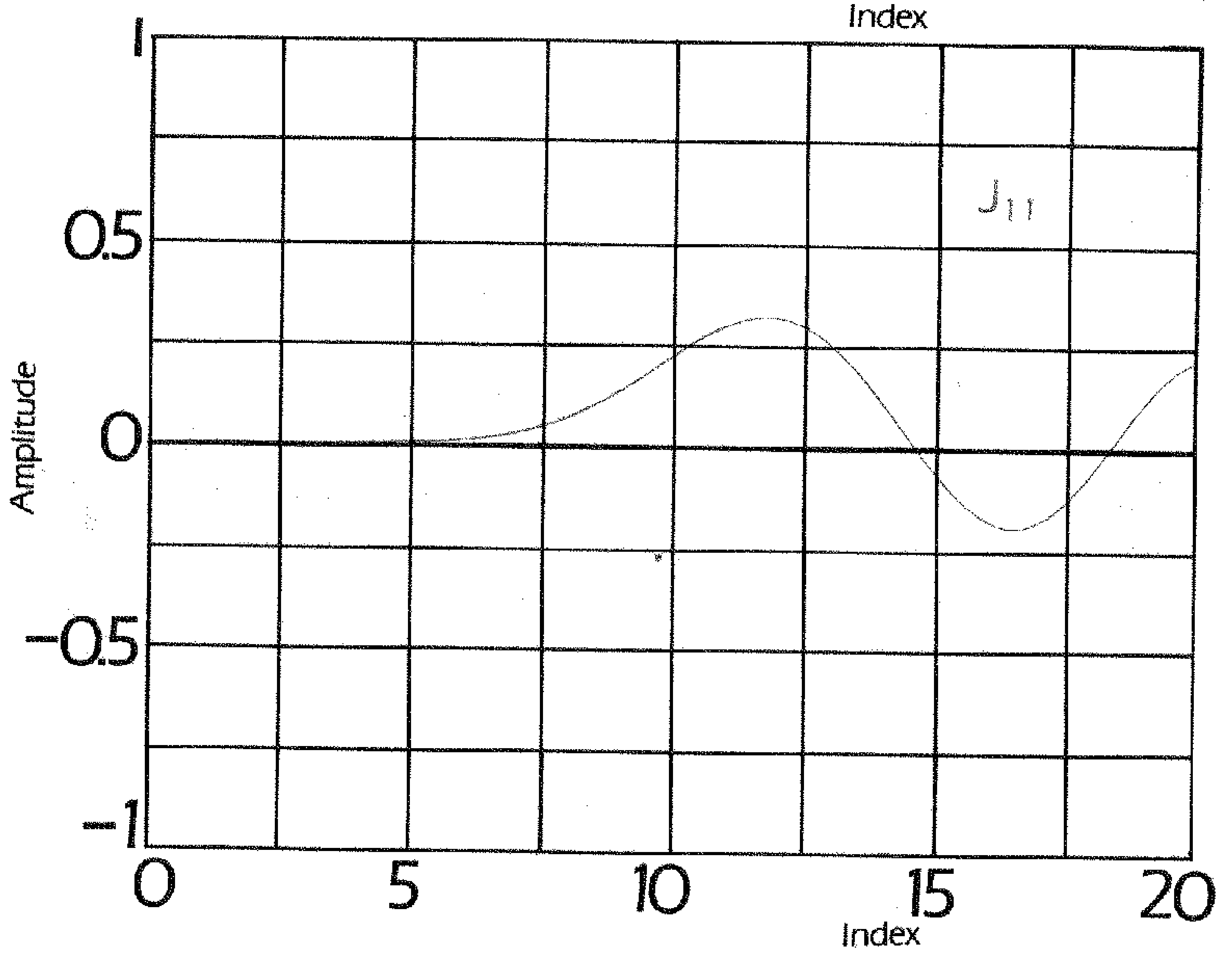
0.5
0
-0.5
-1
0
5
10
15
20
Index
Amplitude
J_8

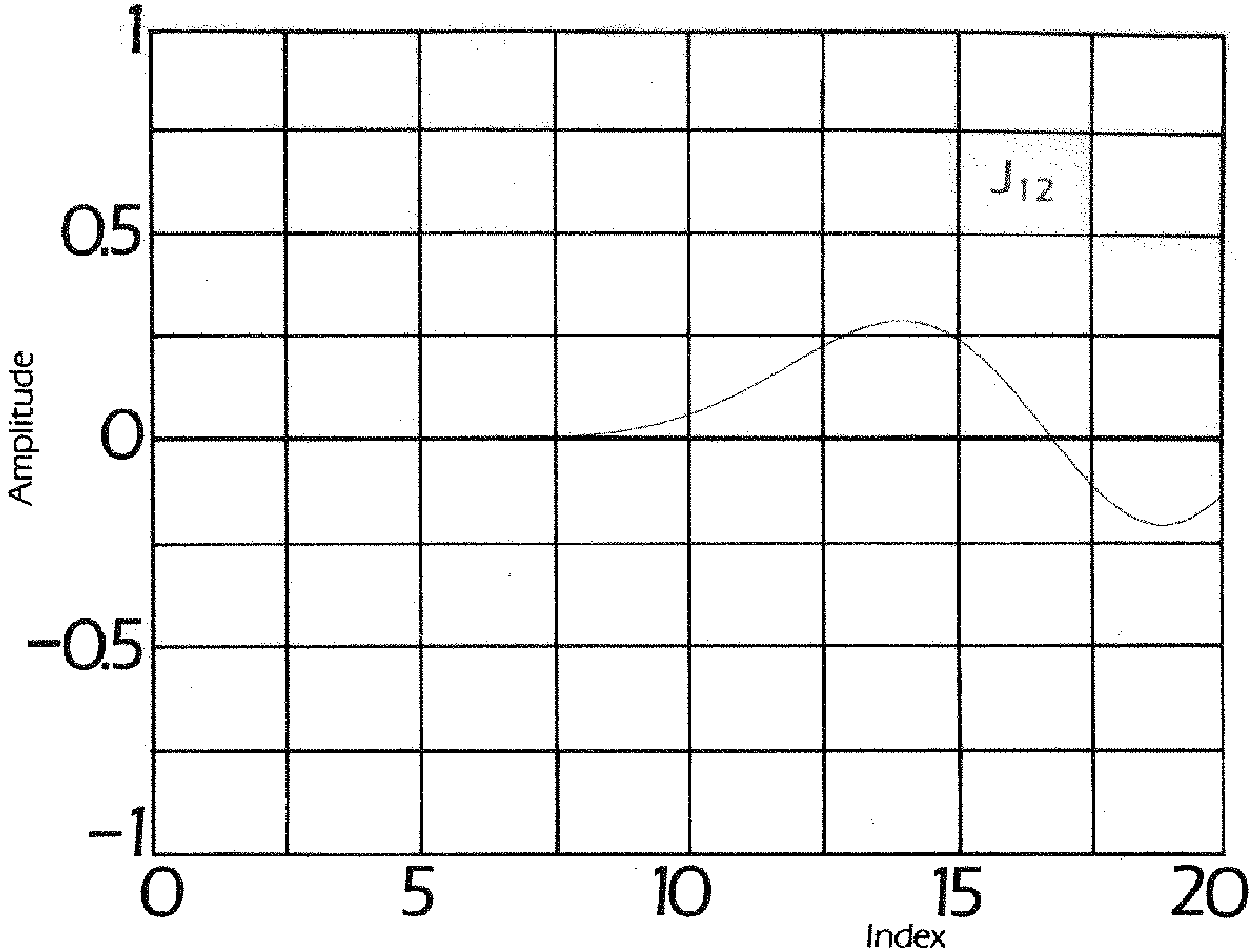

1
0.5
0
-0.5
-1
0
5
10
15
20
Index
Amplitude
J_9

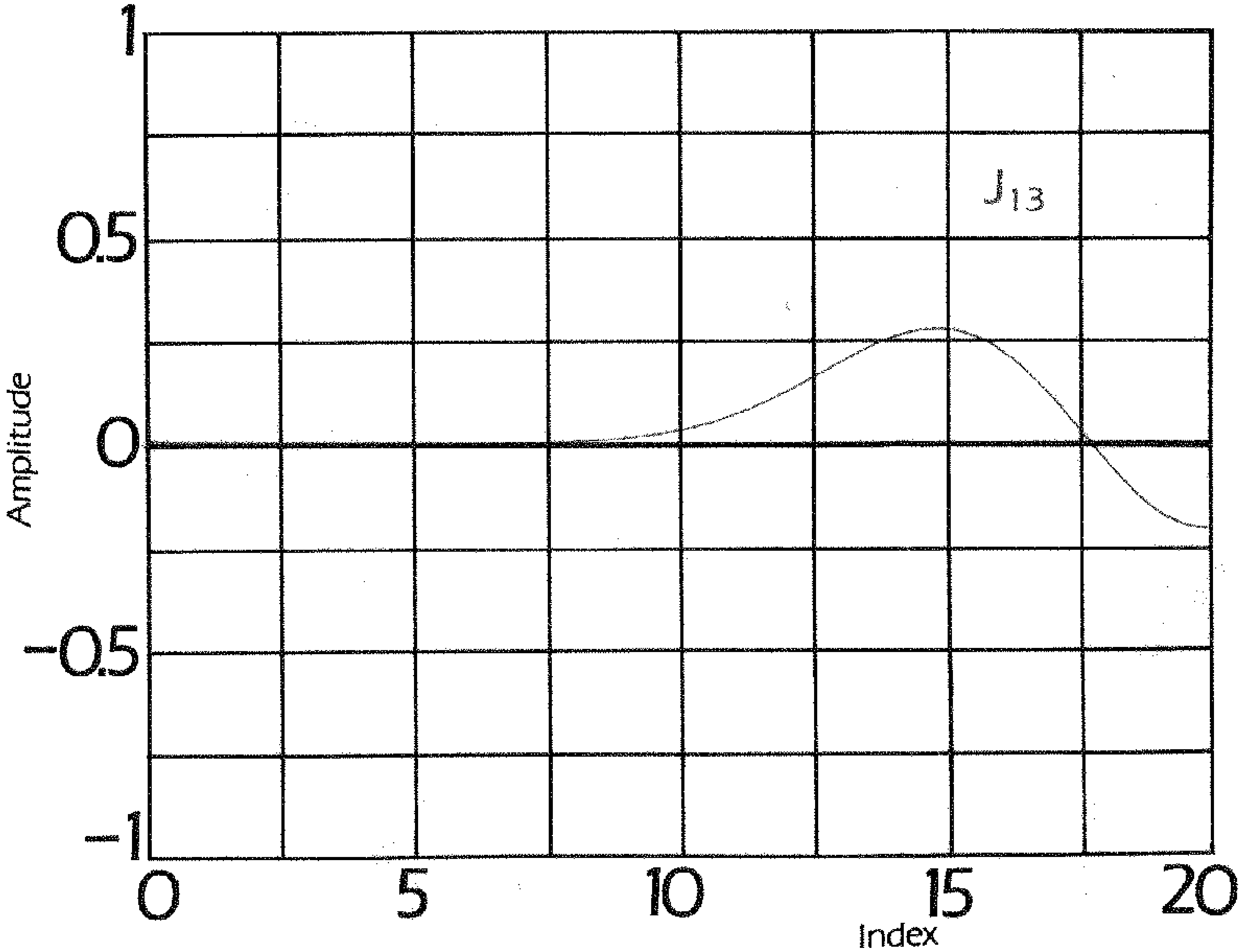


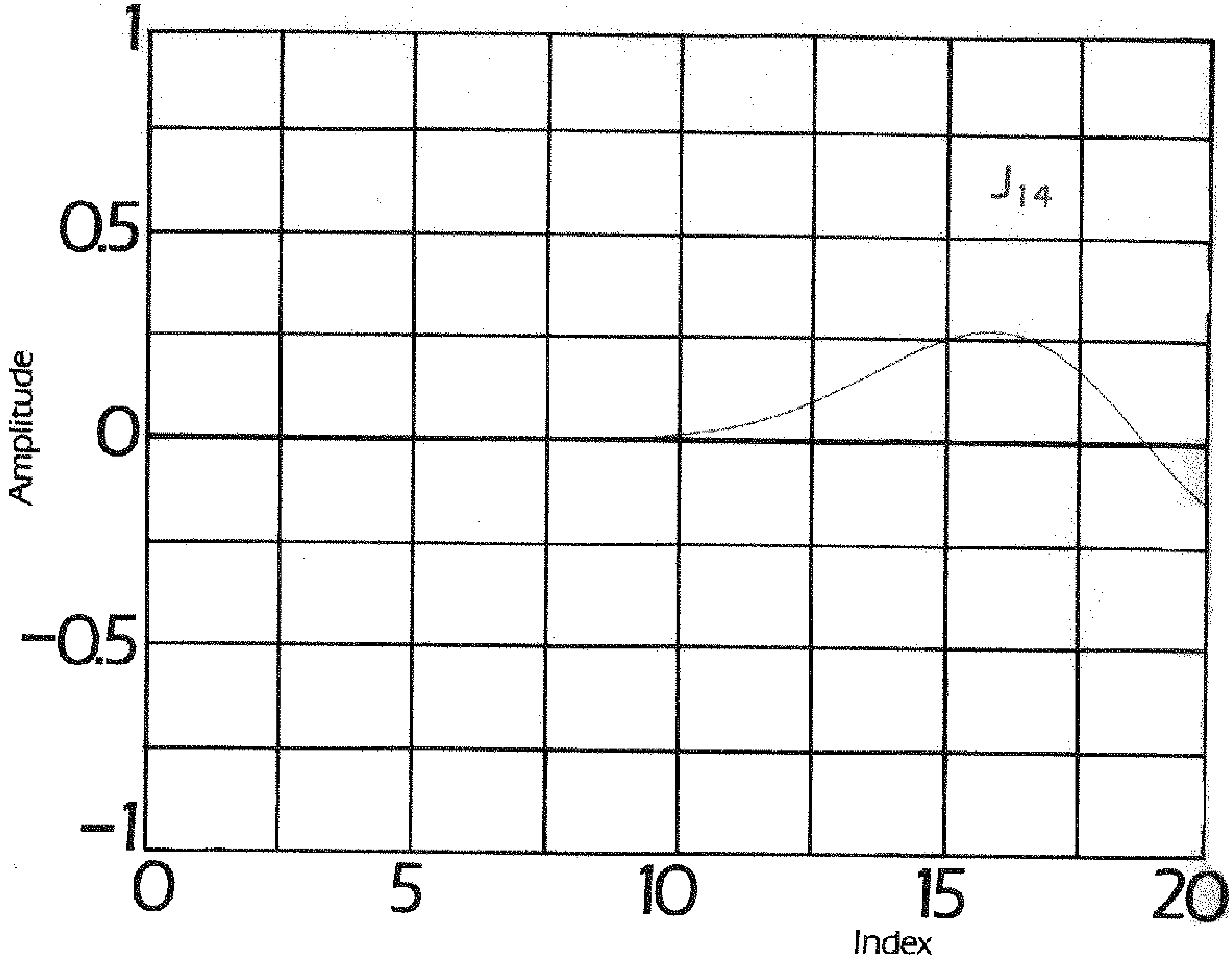
J10
Amplitude
Index

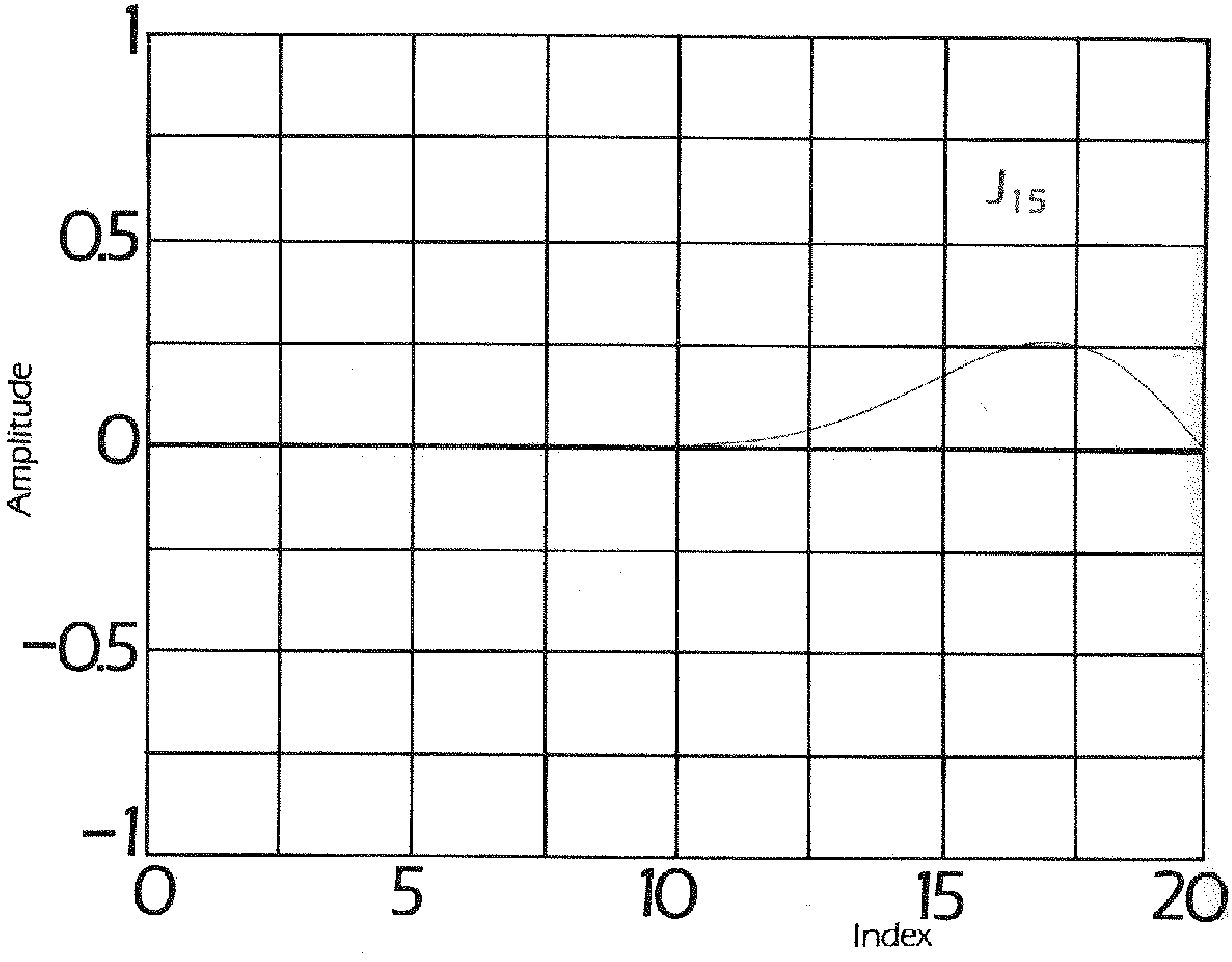


J11
Amplitude
Index


J_{12}
Amplitude
Index
1
0.5
0
−0.5
−1
0
5
10
15
20


J_{13}
Amplitude
Index
1
0.5
0
−0.5
−1
0
5
10
15
20


1
0.5
0
−0.5
−1
Amplitude
J_{14}
0
5
10
15
20
Index


1
0.5
0
−0.5
−1
Amplitude
J_{15}
0
5
10
15
20
Index

APPENDIX 4

Bessel Function Tables

The numbers along the top of each table refer to a Bessel function order J_0 .... 15.
The vertical tables on the left-hand side are Modulation Index from 0 to 19.75 in steps of 0.25.

For all but the most rigorous applications, an estimation from the graphs in Appendix 3 will suffice, as we have done in calculating spectra.

Bessel Function Values (amplitude scaling factor)

Index	Order J_0	J_1	J_2	J_3
0.000000	1.000000	0.000000	0.000000	0.000000
0.250000	0.984436	0.124026	0.007772	0.000324
0.500000	0.938470	0.242268	0.030604	0.002564
0.750000	0.864242	0.349244	0.067074	0.008484
1.000000	0.765198	0.440051	0.114903	0.019563
1.250000	0.645906	0.510623	0.171091	0.036868
1.500000	0.511828	0.557936	0.232088	0.060964
1.750000	0.369033	0.580156	0.294003	0.091851
2.000000	0.223891	0.576725	0.352834	0.128943
2.250000	0.082750	0.548378	0.404698	0.171084
2.500000	–0.048384	0.497094	0.446059	0.216600
2.750000	–0.164141	0.425972	0.473939	0.263394
3.000000	–0.260052	0.339059	0.486091	0.309063
3.250000	–0.332751	0.241120	0.481132	0.351043
3.500000	–0.380128	0.137378	0.458629	0.386770
3.750000	–0.401406	0.033229	0.419128	0.413841
4.000000	–0.397150	–0.066043	0.364128	0.430171
4.250000	–0.369200	–0.155553	0.295998	0.434140
4.500000	–0.320543	–0.231060	0.217849	0.424704
4.750000	–0.255121	–0.289187	0.133358	0.401488
5.000000	–0.177597	–0.327579	0.046565	0.364831
5.250000	–0.093081	–0.345014	–0.038353	0.315793
5.500000	–0.006844	–0.341438	–0.117315	0.256118
5.750000	0.075975	–0.317945	–0.186565	0.188160
6.000000	0.150645	–0.276684	–0.242873	0.114768
6.250000	0.213090	–0.220721	–0.283721	0.039140
6.500000	0.260095	–0.153841	–0.307430	–0.035347
6.750000	0.289457	–0.080323	–0.313256	–0.105310
7.000000	0.300079	–0.004683	–0.301417	–0.167556
7.250000	0.291997	0.068582	–0.273078	–0.219245
7.500000	0.266340	0.135248	–0.230273	–0.258061
7.750000	0.225234	0.191603	–0.175788	–0.282332
8.000000	0.171651	0.234636	–0.112992	–0.291132
8.250000	0.109207	0.262204	–0.045643	–0.284333
8.500000	0.041939	0.273122	0.022325	–0.262616
8.750000	–0.025949	0.267218	0.087027	–0.227434
9.000000	–0.090334	0.245312	0.144847	–0.180935
9.250000	–0.147414	0.209147	0.192635	–0.125845
9.500000	–0.193929	0.161264	0.227879	–0.065315
9.750000	–0.227333	0.104838	0.248839	–0.002751
10.000000	–0.245936	0.043473	0.254630	0.058379
10.250000	–0.248976	–0.019020	0.245264	0.114733
10.500000	–0.236648	–0.078850	0.221629	0.163280
10.750000	–0.210069	–0.132470	0.185424	0.201465
11.000000	–0.171190	–0.176785	0.139048	0.227348
11.250000	–0.122660	–0.209325	0.085447	0.239706
11.500000	–0.067654	–0.228379	0.027936	0.238095

Index	Order J_0	J_1	J_2	J_3
11.750000	–0.009669	–0.233081	–0.030004	0.222866
12.000000	0.047689	–0.223447	–0.084930	0.195137
12.250000	0.100931	–0.200357	–0.133642	0.156719
12.500000	0.146884	–0.165484	–0.173361	0.110008
12.750000	0.182885	–0.121179	–0.201893	0.057839
13.000000	0.206926	–0.070318	–0.217744	0.003320
13.250000	0.217766	–0.016121	–0.220199	–0.050354
13.500000	0.214989	0.038049	–0.209352	–0.100080
13.750000	0.199019	0.088895	–0.186089	–0.143030
14.000000	0.171073	0.133375	–0.152020	–0.176809
14.250000	0.133080	0.168891	–0.109376	–0.199593
14.500000	0.087545	0.193429	–0.060865	–0.210220
14.750000	0.037391	0.205681	–0.009502	–0.208258
15.000000	–0.014224	0.205104	0.041572	–0.194018
15.250000	–0.064110	0.191945	0.089283	–0.168526
15.500000	–0.109231	0.167213	0.130807	–0.133457
15.750000	–0.146892	0.132606	0.163731	–0.091024
16.000000	–0.174899	0.090397	0.186199	–0.043847
16.250000	–0.191682	0.043287	0.197009	0.005207
16.500000	–0.196381	–0.005764	0.195682	0.053202
16.750000	–0.188891	–0.053724	0.182476	0.087300
17.000000	–0.169854	–0.097668	0.158364	0.134931
17.250000	–0.140612	–0.134964	0.124964	0.163941
17.500000	–0.103110	–0.163420	0.084434	0.182719
17.750000	–0.059775	–0.181416	0.039334	0.190280
18.000000	–0.013356	–0.187995	–0.007533	0.186321
18.250000	0.033248	–0.182910	–0.053293	0.171230
18.500000	0.077165	–0.166634	–0.095179	0.146054
18.750000	0.115726	–0.140314	–0.130693	0.112433
19.000000	0.146629	–0.105701	–0.157756	0.072490
19.250000	0.168071	–0.065030	–0.174828	0.028702
19.500000	0.178854	–0.020877	–0.180995	–0.016250
19.750000	0.178449	0.024000	–0.176019	–0.059649

Index	Order J_4	J_5	J_6	J_7
0.000000	0.000000	0.000000	0.000000	0.000000
0.250000	0.000010	0.000000	0.000000	0.000000
0.500000	0.000161	0.000008	0.000000	0.000000
0.750000	0.000801	0.000060	0.000004	0.000000
1.000000	0.002477	0.000250	0.000021	0.000002
1.250000	0.005877	0.000744	0.000078	0.000007
1.500000	0.011768	0.001799	0.000228	0.000025
1.750000	0.020914	0.003758	0.000558	0.000071
2.000000	0.033996	0.007040	0.001202	0.000175
2.250000	0.051526	0.012121	0.002345	0.000386
2.500000	0.073782	0.019502	0.004225	0.000777
2.750000	0.100739	0.029664	0.007131	0.001451
3.000000	0.132034	0.043028	0.011394	0.002547
3.250000	0.166947	0.059904	0.017373	0.004241
3.500000	0.204405	0.080442	0.025429	0.006743
3.750000	0.243017	0.104596	0.035904	0.010298
4.000000	0.281129	0.132087	0.049088	0.015176

Index	Order J_4	J_5	J_6	J_7
4.250000	0.316905	0.162387	0.065183	0.021658
4.500000	0.348423	0.194715	0.084276	0.030022
4.750000	0.373785	0.228045	0.106309	0.040525
5.000000	0.391232	0.261141	0.131049	0.053376
5.250000	0.399259	0.292602	0.158078	0.068719
5.500000	0.396717	0.320925	0.186783	0.086601
5.750000	0.382906	0.344578	0.216361	0.106957
6.000000	0.357642	0.3620878	0.245837	0.129587
6.250000	0.321295	0.372118	0.274094	0.154142
6.500000	0.274803	0.373565	0.299913	0.180121
6.750000	0.219647	0.365633	0.322031	0.206867
7.000000	0.157798	0.347896	0.339197	0.233584
7.250000	0.091633	0.320358	0.350240	0.259349
7.500000	0.023825	0.283474	0.354141	0.283151
7.750000	–0.042791	0.238160	0.350095	0.303922
8.000000	–0.105357	0.185775	0.337576	0.320589
8.250000	–0.161145	0.128072	0.316383	0.332122
8.500000	–0.207701	0.067133	0.286681	0.337593
8.750000	–0.242982	0.005279	0.249015	0.336228
9.000000	–0.265471	–0.055039	0.204317	0.327461
9.250000	–0.274264	–0.111357	0.153879	0.310983
9.500000	–0.269131	–0.161321	0.099319	0.286777
9.750000	–0.250531	–0.202813	0.042518	0.255143
10.000000	–0.219603	–0.234062	–0.014459	0.216711
10.250000	–0.178103	–0.253741	–0.069449	0.172435
10.500000	–0.128326	–0.261052	–0.120295	0.123572
10.750000	–0.072978	–0.255774	–0.164951	0.071643
11.000000	–0.015040	–0.238286	–0.201584	0.018376
11.250000	0.042396	–0.209558	–0.228670	–0.034357
11.500000	0.096288	–0.171113	–0.245081	–0.084624
11.750000	0.143808	–0.124955	–0.250152	–0.130520
12.000000	0.182499	–0.073471	–0.243725	–0.170254
12.250000	0.210402	–0.019313	–0.226168	–0.202239
12.500000	0.226165	0.034738	–0.198375	–0.225178
12.750000	0.229112	0.085917	–0.161726	–0.238130
13.000000	0.219276	0.131620	–0.118031	–0.240571
13.250000	0.197397	0.169537	–0.069445	–0.232431
13.500000	0.164872	0.197782	–0.018367	–0.214108
13.750000	0.123676	0.214986	0.032678	–0.186468
14.000000	0.076244	0.220378	0.081168	–0.150805
14.250000	0.025337	0.213817	0.124710	–0.108798
14.500000	–0.026123	0.195807	0.161162	–0.062432
14.750000	–0.075213	0.167465	0.188748	–0.013907
15.000000	–0.119179	0.130456	0.206150	0.034464
15.250000	–0.155589	0.086906	0.212576	0.080367
15.500000	–0.182467	0.039280	0.207809	0.121604
15.750000	–0.198407	–0.009754	0.192214	0.156203
16.000000	–0.202642	–0.057473	0.166721	0.182514
16.250000	–0.195086	–0.101250	0.132779	0.199303
16.500000	–0.176336	–0.138698	0.092276	0.205808
16.750000	–0.147622	–0.167806	0.047440	0.201793
17.000000	–0.110741	–0.187044	0.000715	0.187549
17.250000	–0.067941	–0.195450	–0.045364	0.163893
17.500000	–0.021787	–0.192679	–0.088315	0.132120
17.750000	0.024986	–0.179019	–0.125842	0.093943

Index	Order J_4	J_5	J_6	J_7
18.000000	0.069640	–0.155370	–0.155956	0.051399
18.250000	0.109588	–0.123191	–0.177090	0.006749
18.500000	0.142548	–0.084412	–0.188176	–0.037648
18.750000	0.166672	–0.041320	–0.188709	–0.079454
19.000000	0.180647	0.003572	–0.178767	–0.116478
19.250000	0.183774	0.047671	–0.159009	–0.146794
19.500000	0.175995	0.088453	–0.130634	–0.168844
19.750000	0.157898	0.123608	–0.095312	–0.181519

Index	Order J_8	J_9	J_{10}	J_{11}
0.000000	0.000000	0.000000	0.000000	0.000000
0.250000	0.000000	0.000000	0.000000	0.000000
0.500000	0.000000	0.000000	0.000000	0.000000
0.750000	0.000000	0.000000	0.000000	0.000000
1.000000	0.000000	0.000000	0.000000	0.000000
1.250000	0.000001	0.000000	0.000000	0.000000
1.500000	0.000002	0.000000	0.000000	0.000000
1.750000	0.000008	0.000001	0.000000	0.000000
2.000000	0.000022	0.000002	0.000000	0.000000
2.250000	0.000055	0.000007	0.000001	0.000000
2.500000	0.000124	0.000018	0.000002	0.000000
2.750000	0.000256	0.000040	0.000006	0.000001
3.000000	0.000493	0.000084	0.000013	0.000002
3.250000	0.000895	0.000167	0.000028	0.000004
3.500000	0.001543	0.000311	0.000056	0.000009
3.750000	0.002543	0.000552	0.000107	0.000019
4.000000	0.004029	0.000939	0.000195	0.000037
4.250000	0.006161	0.001535	0.000341	0.000068
4.500000	0.009126	0.002425	0.000573	0.000122
4.750000	0.013133	0.003711	0.000931	0.000210
5.000000	0.018405	0.005520	0.001468	0.000351
5.250000	0.025173	0.007998	0.002248	0.000568
5.500000	0.033657	0.011309	0.003356	0.000893
5.750000	0.044057	0.015635	0.004889	0.001368
6.000000	0.056532	0.021165	0.006964	0.002048
6.250000	0.071184	0.028090	0.009715	0.002997
6.500000	0.088039	0.036590	0.013288	0.004297
6.750000	0.107027	0.046826	0.017842	0.006040
7.000000	0.127971	0.058921	0.023539	0.008335
7.250000	0.150572	0.072949	0.030541	0.011303
7.500000	0.174408	0.088919	0.038998	0.015076
7.750000	0.198926	0.106763	0.049041	0.019793
8.000000	0.223455	0.126321	0.060767	0.025597
8.250000	0.247218	0.147331	0.074231	0.032624
8.500000	0.269355	0.169427	0.089433	0.041003
8.750000	0.288949	0.192136	0.106302	0.050841
9.000000	0.305067	0.214881	0.124694	0.062217
9.250000	0.316798	0.236992	0.144376	0.075172
9.500000	0.323300	0.257728	0.165026	0.089696
9.750000	0.323841	0.276289	0.186231	0.105723
10.000000	0.317854	0.291856	0.207486	0.123117
10.250000	0.304970	0.303615	0.228208	0.141669

Index	Order J_8	J_9	J_{10}	J_{11}
10.500000	0.285058	0.310802	0.247746	0.161094
10.750000	0.258253	0.312735	0.265395	0.181024
11.000000	0.224972	0.308856	0.280428	0.201014
11.250000	0.185915	0.298769	0.292116	0.220548
11.500000	0.142060	0.282274	0.299759	0.239047
11.750000	0.094639	0.259390	0.302725	0.255886
12.000000	0.045095	0.230381	0.300476	0.270412
12.250000	−0.004962	0.195758	0.292607	0.281967
12.500000	−0.053824	0.156283	0.2788872	0.289912
12.750000	−0.099750	0.112954	0.259214	0.293656
13.000000	−0.141046	0.066976	0.233782	0.292688
13.250000	−0.1766142	0.019731	0.202946	0.286603
13.500000	−0.203671	−0.027279	0.167298	0.275129
13.750000	−0.222536	−0.072483	0.127649	0.258154
14.000000	−0.231973	−0.114307	0.085007	0.235745
14.250000	−0.231599	−0.151243	0.040555	0.208163
14.500000	−0.221441	−0.181917	−0.004387	0.175866
14.750000	−0.201948	−0.205155	−0.048411	0.139513
15.000000	−0.173984	−0.220046	−0.090072	0.099950
15.250000	−0.138797	−0.225990	−0.127945	0.058193
15.500000	−0.097973	−0.222738	−0.160690	0.015395
15.750000	−0.053367	−0.210416	−0.187109	−0.027183
16.000000	−0.007021	−0.189535	−0.206206	−0.068222
16.250000	0.038927	−0.160973	−0.217236	−0.106395
16.500000	0.082349	−0.125955	−0.219754	−0.140414
16.750000	0.121223	−0.085998	−0.213638	−0.169093
17.000000	0.153737	−0.042856	−0.199113	−0.191395
17.250000	0.178378	0.001559	−0.176751	−0.206488
17.500000	0.194011	0.045261	−0.147456	−0.213783
17.750000	0.199937	0.086282	−0.112440	−0.212975
18.000000	0.195933	0.122764	−0.073170	−0.204063
18.250000	0.182267	0.153047	−0.031316	−0.187366
18.500000	0.159686	0.175755	0.011319	−0.163518
18.750000	0.129383	0.189861	0.052883	−0.133452
19.000000	0.092941	0.194744	0.091553	−0.098372
19.250000	0.052250	0.190223	0.125620	−0.059708
19.500000	0.009413	0.176567	0.153572	−0.019058
19.750000	−0.033360	0.154493	0.174164	0.021875

Index	Order J_{12}	J_{13}	J_{14}	J_{15}
0.000000	0.000000	0.000000	0.000000	0.000000
0.250000	0.000000	0.000000	0.000000	0.000000
0.500000	0.000000	0.000000	0.000000	0.000000
0.750000	0.000000	0.000000	0.000000	0.000000
1.000000	0.000000	0.000000	0.000000	0.000000
1.250000	0.000000	0.000000	0.000000	0.000000
1.500000	0.000000	0.000000	0.000000	0.000000
1.750000	0.000000	0.000000	0.000000	0.000000
2.000000	0.000000	0.000000	0.000000	0.000000
2.250000	0.000000	0.000000	0.000000	0.000000
2.500000	0.000000	0.000000	0.000000	0.000000
2.750000	0.000000	0.000000	0.000000	0.000000

Index	Order J_{12}	J_{13}	J_{14}	J_{15}
3.000000	0.000000	0.000000	0.000000	0.000000
3.250000	0.000001	0.000000	0.000000	0.000000
3.500000	0.000001	0.000000	0.000000	0.000000
3.750000	0.000003	0.000000	0.000000	0.000000
4.000000	0.000006	0.000001	0.000000	0.000000
4.250000	0.000012	0.000002	0.000000	0.000000
4.500000	0.000024	0.000004	0.000001	0.000000
4.750000	0.000043	0.000008	0.000001	0.000000
5.000000	0.000076	0.000015	0.000003	0.000000
5.250000	0.000130	0.000027	0.000005	0.000001
5.500000	0.000216	0.000048	0.000010	0.000002
5.750000	0.000347	0.000081	0.000017	0.000003
6.000000	0.000545	0.000133	0.000030	0.000006
6.250000	0.000836	0.000213	0.000050	0.000011
6.500000	0.001254	0.000334	0.000082	0.000019
6.750000	0.001843	0.000512	0.000131	0.000031
7.000000	0.002656	0.000770	0.000205	0.000051
7.250000	0.003757	0.001136	0.000315	0.000081
7.500000	0.005225	0.001644	0.000474	0.000126
7.750000	0.007147	0.002339	0.000701	0.000194
8.000000	0.009624	0.003275	0.001019	0.000293
8.250000	0.012766	0.004513	0.001458	0.000434
8.500000	0.016692	0.006128	0.002052	0.000633
8.750000	0.021526	0.008203	0.002847	0.000909
9.000000	0.027393	0.010830	0.003895	0.001286
9.250000	0.034412	0.014112	0.005255	0.001795
9.500000	0.042692	0.018156	0.006999	0.002472
9.750000	0.052323	0.023073	0.009204	0.003359
10.000000	0.063370	0.028972	0.011957	0.004508
10.250000	0.075862	0.035959	0.015352	0.005977
10.500000	0.089785	0.044129	0.019486	0.007834
10.750000	0.105072	0.053556	0.024459	0.010151
11.000000	0.121600	0.064295	0.030369	0.013009
11.250000	0.139178	0.076365	0.037310	0.016495
11.500000	0.157548	0.089748	0.045362	0.020698
11.750000	0.176381	0.104382	0.054591	0.025707
12.000000	0.195280	0.120148	0.065040	0.031613
12.250000	0.213783	0.136874	0.076724	0.038496
12.500000	0.231373	0.154324	0.089621	0.046428
12.750000	0.247487	0.172202	0.103670	0.055465
13.000000	0.261537	0.190149	0.118761	0.065644
13.250000	0.272923	0.207748	0.134734	0.076972
13.500000	0.281060	0.224533	0.151374	0.089428
13.750000	0.285398	0.239995	0.168410	0.102950
14.000000	0.285450	0.253598	0.185517	0.117437
14.250000	0.280819	0.264795	0.202316	0.132739
14.500000	0.271218	0.273047	0.218383	0.148658
14.750000	0.256499	0.277840	0.233254	0.164947
15.000000	0.236666	0.278715	0.246440	0.181306
15.250000	0.211895	0.275282	0.257438	0.197391
15.500000	0.182542	0.267250	0.265749	0.212812
15.750000	0.149140	0.254443	0.270894	0.227145
16.000000	0.112400	0.236823	0.272436	0.239941
16.250000	0.073195	0.214497	0.270001	0.250735
16.500000	0.032535	0.187738	0.263295	0.259065

Index	Order J_{12}	J_{13}	J_{14}	J_{15}
16.750000	–0.008454	0.156980	0.252124	0.264481
17.000000	–0.048575	0.122819	0.236416	0.266572
17.250000	–0.086596	0.086006	0.216229	0.264974
17.500000	–0.121300	0.047430	0.191766	0.259396
17.750000	–0.151529	0.008090	0.163380	0.249636
18.000000	–0.176241	–0.030925	0.131572	0.235592
18.250000	–0.194550	–0.068480	0.096989	0.217285
18.500000	–0.205773	–0.103431	0.060411	0.194863
18.750000	–0.209467	–0.134666	0.022730	0.168610
19.000000	–0.205458	–0.161154	–0.015068	0.138948
19.250000	–0.193858	–0.181985	–0.051940	0.106436
19.500000	–0.175073	–0.196417	–0.086816	0.071758
19.750000	–0.149796	–0.203906	–0.118637	0.035712

APPENDIX 5

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APPENDIX 6

A Glossary of Terms

The entries in this glossary contain practical descriptions for musicians, rather than rigorous scientific definitions. See the page cited at the end of each entry for fuller descriptions.

algorithm; the configuration of operators in an FM system.

aliasing; the phenomenon of reflected frequencies around the half sampling rate of digital systems. (ref chap 2 - page 36)

amplitude; a physical measure of the energy of a simple or complex sound waveform. (ref chap 2 - page 29)

aperiodic; describes a waveform which is not repetitive and has no clear sense of pitch. (ref chap 2 - page 21)

bandwidth; the frequency space occupied by a spectrum. (ref chap 5 - page 91)

beating; acoustical effect of fluctuating loudness caused by the close proximity of two frequency components. (ref chap 5 - page 88)

Bessel, Bessel functions; *symbol j* : Graphs, or functions, which show the relationship between modulation index and the relative amplitude (yielded by a scaling factor) of frequency components in FM spectra (ref chap 4 - page 63; table of functions, appendix 4 page 179)

carrier; *symbol c*: the last operator (a digital sine wave generator) to be frequency modulated by other operators in any FM configuration. Single operators (in the context of FM synthesis, such as appear in algorithm 32 of the DX7) are also usually referred to as carriers. (ref chap 3 - page 46)

cascade; particular form of complex FM where three or more operators are acting one upon the other; sometimes called a stack. (ref chap 6 - page 127)

combination/combinatorial; describes side bands produced in complex FM as a result of the interaction of two or more modulators. (ref chap 6 - page 117)

cosine; the relationship between the adjacent and hypotenuse sides of a right-angled triangle; can be considered as a sine wave 90° out of phase. (ref chap 2 - page 22)

CX5; Yamaha music computer containing four-operator FM tone generation algorithm.

DAC; digital-to-analogue converter. (ref chap 2 - page 39)

decibel; a measure of loudness. (ref chap 2 - page 36)

denominator; the lower integer in a fraction. (ref chap 1 - page 16)

DX7; Yamaha "X" Series synthesizer with six-operator FM tone generation.

envelope; a time-based variable control which can be used to govern (in the case of the "X" synthesizers) the output of operators. (ref chap 5 - page 103)

FFT; Fast Fourier Transform, mathematical derivation of a spectrum from a complex waveform. (ref chap 1 - page 11)

FM, complex; a frequency modulation configuration consisting of more than carrier and/or more than one modulator. (ref chap 6 - page 113)

FM, simple; a frequency modulation configuration consisting of one carrier and one modulator. (ref chap 4 - page 55)

frequency; symbol Hertz, or Hz, or KHz (1000Hz) or c.p.s. (cycles per second): the number of times that a periodic wave repeats itself in one second. (ref chap 1 - page 10)

frequency deviation; *symbol* Δf : when considering frequency modulation as a form of vibrato, the amount by which the frequency travels above and below its non-modulated frequency, or centre frequency. (ref chap 3 - page 46)

function; the relationship between one variable and another. (ref cnap 1 - page 15)

function controls; refers to a level of "X" synthesizer programming which links physical controls, such as movement of the modulation wheel or foot pedal, to the output of an operator within the FM algorithm. (ref chap 1 - page 11)

fundamental; the first harmonic in a spectrum of which the remainder are integer multiples. (ref chap 1 - page 12)

graph; a visual expression of the relationship between two variables. (ref chap 1 - page 15)

harmonics; simple frequency components in a spectrum which are all integer multiples of the fundamental frequency (harmonic number one). Sometimes called overtones. (ref chap 1 - page 12)

Hertz; see frequency.

index, modulation index; *symbol* I : in an FM configuration, the amount by which a modulator acts on the operator(s) immediately below it. Relates to the output level of a modulator. (ref chap 4 - page 56)

inharmonicity; the effect of components within a spectrum that are not simple multiples of some given frequency.

log; a mathematical function often associated with the derivation of scales which approximate more closely certain perceptual processes than linear scales. (ref chap 2 - page 34, appendix 1 page 160)

modulator; *symbol* m : any operator in an FM configuration which has a frequency-modulating effect on subsequent operators. (ref chap 3 - page 49)

negative frequency; region of the frequency domain to the left of the origin. (ref chap 1 - page 12, chap 4 - page 61)

numerator; the upper integer in a fraction. (ref chap 1 - page 16)

operator; a digital sine wave generator. (ref chap 2 - page 38)

order; *symbol* k : refers to the side bands (pairs of frequency components) produced by FM synthesis. The carrier frequency alone is said to be of the zero order, the first sidebands of the first order and so on. (ref chap 4 - page 61)

origin; the point of a graph where the two axes cross. (ref chap 1 - page 15)

overtones; see harmonics.

partials; simple frequency components in a spectrum which may be, but are not necessarily, integer multiples of the fundamental. (ref chap 1 - page 11)

periodic; describes a waveform which is repetitive and has an unambiguous sense of pitch. (ref chap 2 - page 20)

phase; measured in degrees, refers to the starting point of a waveform. (ref chap 2 - page 28)

pitch; the perceptual equivalent of frequency. (ref chap 2 - page 29)

ratio, frequency ratio; (1) the number by which the keynote frequency is multiplied to give the frequency of an operator; (2) the ratio between the frequencies of operators in an FM configuration (*symbol c:m*). (ref chap 4 - page 59)

reflections; a term applied to frequency components which can be theoretically represented outside the positive frequency domain, but which are heard, 180° out of phase, within it. Also refers to the actual production, below the half sampling rate of digital systems, of frequencies which are theoretically above it. (ref chap 2 - page 38)

sampling rate; the number of times per second that digital systems produce a numeric value for a function. (ref chap 2 - page 36)

scaling; an "X" synthesizer programming parameter which allows operator output level or envelope rates to be linked to keynote number. (ref chap 5 - page 84)

scaling factor; see Bessel.

side bands; the frequency components which are produced by FM synthesis. (ref chap 4 - page 61)

sine; the relationship between the opposite and hypotenuse sides of a right-angled triangle; **sine wave, sinusoid;** a waveform calculated from a table of sine values which produces no harmonics, a "pure" waveform. (ref chap 2 - page 21)

sinusoid; see sine wave.

spectrum; a visual representation, in the frequency domain, of a particular timbre. (ref chap 1 - page 10)

symbols;
- "*J*" a particular Bessel function or curve
- "*I*" modulation index
- "*k*" order of side band
- "*c*" carrier frequency, or its ratio to keynote frequency
- "*m*" modulator frequency, or its ratio to keynote frequency
- "*n*" undefined final number in a series

time domain; usually a waveform representation of a timbre, rather than a spectral one (which is called **frequency domain**). (ref chap 2 - page 35)

trace; name given to a plot produced by a spectrum analyser. A 3D spectrum contains many traces. (ref chap 1 - page 13)

vibrato; a periodic fluctuation in pitch. (ref chap 3 - page 42)

waveform; a visual representation, in the time domain, of a particular timbre. (ref chap 1 - page 14)

APPENDIX 7

The Sampling Rate of the DX7

Given the parameters in "X"-ample 2.4 (page 37), we observe that around the note B3 (almost one octave above middle C) we can observe the beginnings of aliasing – a roughness in the sound due to the reflected harmonics being produced in the spectrum. (Remember that the half sampling rate need not be neatly placed on a harmonic frequency, and that therefore reflected components may be inharmonic.) The FM parameters of the sound were as follows ...

c:m = 1:5, Index = 12 (approx)

This means that, according to the rule for bandwidth, that there will be $I + 2$ sidebands, starting at the sixth harmonic, then the 11th then the 16th, and so on, as given by the expression $c + km$, which gives the positions of the upper sidebands.

We can assume that the roughness in the sound at note B3, which has a frequency of about 500 cycles per second, is due to reflections of the upper harmonics around the half sampling rate. At the frequency of note B3, the highest harmonic is $f\ (c + km) = 500\ (1 + 14 \times 5) = 35{,}500Hz$. If we estimated a half sampling rate at 25,000, this would lead to reflected components at 14,500Hz, very like to cause interference with the normal spectrum. This is in fact a reasonable estimate, as the actual half sampling rate of the DX7 is about 30,000Hz. The error is probably due to the fact that there are very low amplitude components produced beyond $I + 2$ which become more noticeable when they are reflected due to their inharmonicity, and also due to the interaction of very high frequency components. However, our calculation has enabled us to estimate that the sampling rate for the DX7 is *at least* 50,000Hz.

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